

Stitched (guest lecture, Toby Moskowitz; two phone recordings, the second in two parts, joined on their overlapping words with no hole; about five minutes of the opening are on neither file) · **Midterm 20 Oct:** unsettled — The review never placed the guest lecture: it was recalled three times as covered material, with nothing said about its place on the paper (Lecture 12 note §1.5). Its overlap with decks 04b and 05a is in through those blocks. Tony's working assumption of 14 Oct: examinable, weighted lower · **P9:** Passing — last confirming re-check 13 Oct. Edited after it — 16 Oct: its *Continues in* and *Forward* links re-pointed to the 13 and 15 Oct notes. Those edits were checked by the editing passes' own math, quotation and leak runs, not by a new audit. · **Adapted self-checks X36, X39, X44, X47 (the 6 Oct note's, repeated in this note's linked-problems block)** restate the claim of the item they adapt (16 Oct check). If you mean to sit the practice papers clean, sit them before reading these. Each idea is also in the course's own decks, workbook or recordings.

Financial Economics — Lecture 10: Guest Lecture (Toby Moskowitz) — From the CAPM to Anomalies, Betting Against Beta, Factor Investing, Sports Betting and Prediction Markets

RECORDED. This is the course's first guest lecture, given by Toby Moskowitz, named from the syllabus ("Guest Lectures: ... ii) Toby Moskowitz, Dean Takahashi '80 B.A., '83 M.P.P.M. Professor of Finance", *syl p. 6*), the course overview deck ("Guest Lectures 1. Toby Moskowitz (TBD)", *00 p. 12*) and Dávila's announcement on 1 October, "First we have Toby Moskowitz for the guest lecture" (10/1 [00:02]). The note is written from **two recordings of the meeting, the second saved in two parts, stitched on their overlapping words into one timeline; together they leave no hole.** Every time below is a time on that stitched timeline, written τ . To find a time in the original files, once: recording 2, part 1 is at the same time as τ (it covers τ 00:00–11:23); recording 1 is at τ minus 0:55.0 (it covers τ 00:55–71:52); recording 2, part 2 is at τ minus 12:23.8 (it covers τ 12:24–71:47). The text follows part 1 to  11:12, recording 1 from there to  68:07, and part 2 after that. **The recording starts mid-talk.** Its first decoded words, at  00:15, fall inside an aside the guest is already making; by one phone's own clock the first audio is at about 1:10 pm EDT, so **about five minutes after the scheduled 1:05 pm start are not on file:** the meeting's opening, any course business, the introduction of the guest and the opening of the talk. Nothing is rebuilt for them, because no material covers them (the Gap in §1). The talk runs to  59:42; a second voice then thanks the guest and opens the questions at  59:49, and the questions run to the talk's last words at  69:06. ECON 2251's instructor, Eduardo Dávila, was present — the guest addresses "Eduardo" at  40:39 — but **no words on file are the instructor's**, so nothing in this note is attributed to Dávila from this recording.

Thursday 8 Oct 2026 · 69:07 recorded (speech from τ 00:15) · guest lecture by Toby Moskowitz · ECON 2251 (course instructor Eduardo Dávila)

What this record is. The guest spoke from the guest's own slides, which are not in the course materials. So every chart, table or drawing the talk refers to — "this picture", "middle column", "I'll have to draw this" — is

known only from the words on the recording; each is described from them and marked *[transcript only]*, and a figure is redrawn only as a labelled schematic of what was said, never as the guest's slide. **No page of the course's own decks was shown on 8 October.** Where this note cites a course page it is *related, not shown*: the course's own written treatment of the same idea, which fixes the exact form of every formula the guest stated (the CAPM and the security market line, alpha, the three-factor model, the Sharpe ratio), with a link to the note of the meeting that taught it. The guest's own numbers — the betas 0.5 and 1.2, the Sharpe ratios 0.63, 0.4 and 1.5, the 22% loss on underdogs, the \$1 trillion of volume — have no written source; they enter as what was said, attributed to the guest. Where the two recordings render a number or a phrase differently, the note gives both renderings in a Warning and states neither as fact.

Sources used. Each is restated below where it is used; the codes are how the note cites them.

- The recording, cited by time on the stitched timeline:  10:25 . A time from another meeting carries its date, as in 10/1 [00:02].
- 04a p. N — the deck *CAPM* (34 pp.), taught 1 October (pp. 2–24) and, by inference, 6 October (pp. 25–33). Related, not shown: pp. 2, 4–9 (equilibrium, the market portfolio, the capital market line), 11–14 (the beta representation theorem, the security market line and its logic), 18–19 (the variance split), 21 (the security characteristic line), 26–29 (alpha, stock selection, hedge funds).
- 04b p. N — the deck *CAPM: testing and beyond* (21 pp.), taught by inference on 6 October. Related, not shown: pp. 4, 6–8 (the Fama–MacBeth two-step test and portfolio sorting), 10–12 (the beta-decile chart, "too flat", the size and value spreads), 16, 18–19 (the three-factor model, its test, momentum, "theoretical motivation absent").
- 03c p. N — the deck *Portfolio Selection with a Riskless Asset* (the export's copy; pp. 1–22 are identical to the build's): p. 4 (Bogle and Vanguard), p. 7 (the capital allocation line and the Sharpe ratio), p. 8 (the US market's Sharpe ratio 0.446), pp. 10–11 (loan-to-value, margin, leverage), p. 23 (the Madoff poster).
- 03d p. N — the deck *Portfolio Selection with Multiple Risky Assets*: p. 3 (the frontier), p. 4 (the tangency portfolio), pp. 8, 10–11 (equally weighted portfolios, independent and general), p. 14 (44% and 12%), pp. 15–16 (the estimation problem and the two ways forward).
- 03S p. N — the statistics review deck: p. 12 (the law of large numbers), p. 13 (tail risk, value at risk, the peso problem).
- 05a p. N — the deck *Market Efficiency* (28 pp.), **not yet taught** on 8 October; its pp. 1–22 are the Lecture 11 note (13 Oct, reconstructed)'s (the end evidenced by 15 October's opening, the start inferred), and pp. 23–27 the Lecture 12 note (15 Oct)'s. Related, not shown: pp. 3, 5–6 (the efficient market hypothesis, the random walk), 13–14 (post-earnings-announcement drift, overshooting), 15–18 (anomalies, data mining, the joint test, reactions to anomalies), 20–23, 25 (strong form, the Grossman–Stiglitz paradox, mutual fund costs, luck against skill).

- 01a p. 8 (basis points), 01b p. 11 (price impact), 01b p. 13 (short sales), 02b p. 12 (the holding-period return): earlier decks, recalled.
- 07b p. N (Arrow–Debreu securities, complete markets, risk-neutral probabilities) and 09a p. 7, 09d pp. 16–18 (moneyiness; the volatility smile): decks of **later blocks, not yet taught**, named as forward pointers only.
- handout 04 p. N — Dávila's CAPM handout, digested in the Lecture 9 note (6 Oct, reconstructed); named here for the theorem the guest restates.
- formula sheet p. N — the formula sheet the exam provides (posted; Dávila, 9/10 [66:16]). p. 1's *Midterm* list, items 6–11; p. 3's item 15 is on the *Final* list only.
- FF04 p. N — Fama and French (2004), "The Capital Asset Pricing Model: Theory and Evidence" (PDF page numbers), the optional reading digested in full in the Lecture 9 note; its formula pages come from a visual read. LT03 p. N (Lamont and Thaler 2003), Shiller03 p. N (Shiller 2003), WSJins (the WSJ insider-trading article): optional readings of the market-efficiency block, covered in full in the Lecture 11 note (13 Oct) (§6.3, §8, §9). Nobel13 p. N, Nobel13pop p. N (the 2013 Nobel scientific background and popular version): readings of a later block. Thorp06 p. 26 (Thorp on the Kelly criterion): an optional reading of the portfolio-choice block. Each is restated here only where the talk meets it, in a sentence or two.
- z04 p. 9, z06 p. 3 — optional decks (the APT; financial statements), named once each.
- syl p. N and 00 p. N — the syllabus and the course overview deck.
- Problems: no workbook item is owned by this meeting. The guest's five questions to the room, answered aloud, are its in-class exercises (four worked below; one is worked in the Lecture 9 note and linked), under the course's own basis for working practice problems: "There are no graded homeworks, but you are expected to work through the concept questions and the practice problems every week" (*syl p. 3*). No reading is assigned to the guest lecture (*syl p. 6; 00 p. 12*).

At a glance

Where this sits. The course has just built the CAPM — portfolio theory plus equilibrium, the tangency portfolio as the market, the security market line and beta (Lecture 8, 1 Oct) — and, by inference, has tested it and met the size and value anomalies and the three-factor model (Lecture 9, 6 Oct, reconstructed). The guest takes the same arc from a practitioner's side and runs it to its end. The CAPM's one usable gift is that an asset's expected return is pinned by its covariance with the tangency portfolio, which is estimable, where its historical average is not; the CAPM's answer for that portfolio, the market, is "great from a pedagogical standpoint" ⌚ 07:17 but empirically "doesn't explain enough" ⌚ 08:20. The early tests found the security market line too flat, and it is "even flatter today" ⌚ 10:36. Two responses follow. One repairs an assumption — costly borrowing makes the empirical line flatter — and turns the failure into a trade: buy low-beta stocks, short high-beta stocks, lever the long side to match the betas, *Betting Against Beta*. The other

collects every other gap between the model and the data — value, momentum, quality, carry, trend, accruals, sentiment — and combines them: thirteen factors with an average Sharpe ratio of about 0.63 give, equally weighted, about 1.5, because the Sharpe ratio of an average is not the average of the Sharpe ratios. Three schools explain the anomalies (risk, behaviour, data mining), and the way to tell them apart is to look where they cannot all apply: sports betting, where there is no macroeconomic risk and the outcome arrives tonight, and prediction markets, where a contract's price is a probability. There the same shapes reappear — the favourite–long-shot bias is the low-beta effect, the open-to-close reversal is over-reaction, the extreme-probability contracts are the option smile. The course's next lecture, 13 October, takes up the market-efficiency deck (Lecture 11 note, reconstructed) and puts the joint-hypothesis problem and data mining in the course's own words (§5.2); this talk had already used both.

After this lecture you must be able to:

- **State** what the CAPM gives an investor (expected return linear in covariance with the tangency portfolio), **explain** why a single stock's historical average is a poor estimate of its mean while a portfolio's is usable, and **say** why the market portfolio breaks the "chicken and egg" circle of estimating the tangency portfolio.
- **Describe** the early tests of the CAPM — portfolios sorted by beta, average excess return against beta times the market premium — and **explain** what it means for the fitted line to come out "too flat", with the course's own numbers.
- **Derive** how a wedge between the lending and borrowing rates flattens the empirical line, and **say** which investors are leverage-constrained and what that does to the betas they hold.
- **Construct** the Betting Against Beta position: given a long beta of 0.5 and a short beta of 1.2, **compute** the dollar ratio that makes the net beta zero, the borrowing it needs, and its leverage, margin and loan-to-value; and **explain** why zero beta is not zero risk and not an arbitrage.
- **Combine** Sharpe ratios: **show** why the Sharpe ratio of an equally weighted average of strategies exceeds their average Sharpe ratio, **compute** it for independent strategies ($\sqrt{N}$) and for correlated ones, and **say** what costs, leverage limits and overfitting do to a paper Sharpe ratio.
- **State** the three schools of thought on what drives anomalies, the joint-hypothesis problem, the data-mining arithmetic (100 strategies at the 5% level give five false positives), and the defence (out-of-sample tests).
- **Translate** the favourite–long-shot bias, the open-to-close test, the spread against the money line, and a prediction-market price into the course's language — beta, efficiency, over- and under-reaction, risk per dollar, and a state price that is not a physical probability.

Prerequisites. From the Lecture 8 note (1 Oct): the CAPM as an equilibrium model and its assumptions (§7–§8), the tangency portfolio as the market (§9), the capital market line with numbers (§10.2), the beta representation theorem and the security market line (§11.2–§11.3, §12.2), the variance split (§14), alpha as a regression intercept (§15.3), and the estimation problem (§4–§5.1). From the Lecture 9 note (6 Oct,

reconstructed): alpha and stock selection, the Fama–MacBeth test and the too-flat line, the size and value anomalies, and the three-factor model. From the Lecture 7 note (29 Sep): the tangency portfolio and two-fund separation (§6, §8), the many-asset frontier (§9.1, §9.4), diversification with independent and correlated returns (§11.2, §12), and the leverage cycle (§2). From the Lecture 6 note (24 Sep): the capital allocation line and the Sharpe ratio, and loan-to-value, margin and leverage. From the Lecture 5 note (22 Sep): the law of large numbers (§4.2) and value at risk (§4.3). From the Lecture 2 note (8 Sep): short sales (§10.2) and price impact (§9.3). From the Lecture 1 note (3 Sep): basis points (§5.3) and the course's plan for its guest lectures (§1). From statistics: the standard error of a sample mean, and the normal distribution's cumulative function, used in one illustration.

EXAM SIGNAL What the course's own sources say about this meeting and the exams

Nothing on file concerns the midterm of Tuesday 20 October, an exam, a quiz, the formula sheet or the schedule: all three recordings were searched, and no word on file is Dávila's; whatever was said in the five unrecorded minutes before the audio is unrecoverable (§1). What the course's own sources say stands. The syllabus: "All exams are based on material explicitly covered in class, handouts, and notebooks distributed via the course website." (*syl p. 5*). The course overview deck: "Exam questions can always be answered with the material covered in lecture" (*00 p. 9*), and readings are "Interesting and helpful but not required" (*00 p. 9*). The guest lecture is a class meeting of the course: the syllabus counts "25 lectures with attendance" (*syl p. 3*), and on the first day Dávila said of the two guest lectures, "The attendance counts for those guest lectures, so make sure you don't miss them" and that the guests would cover "some of the issues we will cover in a more conceptual way throughout the course" (9/3 [01:54]); on 1 October, announcing this one, "don't miss it" (10/1 [00:02]). Dávila twice pointed forward to this talk. On 24 September Dávila said, "I think when Toby comes in a couple weeks, he may talk about Sharpe ratios" (9/24 [65:47]), and on 29 September Dávila said, "Toby comes in at least, he may use those words, a tangency portfolio" (9/29 [27:04]); the guest did both (§§8.4 and 12, and §2, below). Where this note states a course formula the formula sheet the exam provides prints (that sheet: Dávila, 9/10 [66:16]), it says so in one clause: the portfolio mean and variance (*formula sheet p. 1, items 7–8*), the capital allocation line (*item 9*), the security market line with beta (*item 10*) and the security characteristic line (*item 11*) are all on the sheet's *Midterm* list. This note labels nothing as examinable or unexaminable.

Symbol table

Notation is normalized across the meeting. The guest spoke without writing a formula the recording preserves, so every symbol below is the course's where the course has one, and the note's own where it does not, marked so. The slides write expectation, variance and covariance in blackboard bold ($\mathbb{E}$, Var , Cov

); this note keeps $\mathbb{E}[\cdot]$ and writes $\text{Var}[\cdot]$ and $\text{Cov}[\cdot, \cdot]$, as the Lecture 5–9 notes do. The guest's letter T — "that object T"  07:37 — is the tangency portfolio, which the CAPM deck calls the efficient portfolio c (04a p. 11); the note writes T in the guest's sections and says so where c is meant. *Volatility* always means a standard deviation. The letters P_0, P_1 name a price at two dates in two places, the holding-period return (§11.4) and a bet's opening and closing prices (§17), and the table says which. V is the value bought in the leverage measures (the Lecture 6 note's and 03c p. 10's letter); the holding-period return of §11.4 writes $V(0)$ and $V(T)$, 02b p. 12's letters, for an investment's value at the start and end of the period, where T is a date and not the tangency portfolio. A few letters are local to one passage, and the table says where: a, b, c (§2.2), μ (§§8.4 and 12.3 against §17), s (§21.3).

Symbol	Meaning	Units	First used
R_i, R_p, R_M, R_f	the return on asset i ; on a portfolio; on the market portfolio; the risk-free return — 07b writes r^f in the later block named in §21 (07b p. 17)	per period, decimal	§2
$\mathbb{E}[\cdot], \text{Var}[\cdot], \text{Cov}[\cdot, \cdot]$	expectation, variance, covariance — slides write $\mathbb{E}, \text{Var}, \text{Cov}$	units of the argument; (per period) ²	§2
$\sigma_i, \sigma_p, \sigma_M, \sigma$	the volatility of asset i ; of a portfolio; of the market; of one strategy or stock where all are given the same volatility	per period, decimal	§2
w, w_i	a portfolio weight; with equal weights $w_i = 1/N$; $w > 1$ is leverage (03c p. 10)	fraction of wealth	§2
T, R_T	the tangency portfolio and its return — the guest's letter; 04a p. 11 writes the efficient portfolio c , 03d p. 4 "tangency portfolio"; in equilibrium T is the market	—; per period	§2
w_M^i, p_i, n_i	asset i 's weight in the market portfolio, $p_i n_i / \sum_j p_j n_j$; its share price; its shares outstanding (04a p. 6)	fraction; dollars; count	§2
$E, E_1, E_2; R_1, R_2; \sigma_1, \sigma_2$	an expected return as a coordinate, $\mathbb{E}[R_p]$, and the two assets' means, returns and volatilities, in the note's two-asset frontier (§2.2)	per period	§2
a, b, c (§2.2 only)	the coefficients of the portfolio variance written as a quadratic in its mean, $\text{Var}[R_p] = aE^2 + bE + c$ — letters local to §2.2's derivation: not the bet's b (§19), and not 04a p. 11's efficient portfolio c	pure number; per period; (per period) ²	§2
$\rho, \rho_{ij}, \bar{\rho}$	a correlation; the correlation of i and j ; an average pairwise correlation across strategies (§12.3)	pure number	§2
$\beta_i, \beta_{iT}, \beta_p$	asset i 's beta, $\text{Cov}[R_i, R_M] / \text{Var}[R_M]$ (04a p. 12; formula sheet p. 1 item 10); its beta with respect to T , $\text{Cov}[R_i, R_T] / \text{Var}[R_T]$ (04a p. 11's β_{ic}); a portfolio's beta, $\sum_i w_i \beta_i$	pure number	§3

Symbol	Meaning	Units	First used
α_i	asset i 's alpha, $\mathbb{E}[R_i] - R_f - \beta_i(\mathbb{E}[R_M] - R_f)$ (04a p. 27); the intercept of the security characteristic line (04a p. 21; formula sheet p. 1 item 11)	per period	§7
SR_i, SR_M, SR_N	the Sharpe ratio of i , $(\mathbb{E}[R_i] - R_f)/\sigma_i$ (03c p. 7); the market's (04a p. 9); the Sharpe ratio of an equally weighted portfolio of N strategies (§8.4, §12)	pure number	§8
μ, μ_i (§§8.4, 12.3)	the expected excess return of one strategy, all equal in §8.4; of strategy i in §12.3 — not §17's drift μ	per period	§8
σ_ε	the volatility of a stock's idiosyncratic (residual) return, in the variance split $\beta_i^2\sigma_M^2 + \sigma_\varepsilon^2$ (04a pp. 18–19)	per period	§8
$s_i, \bar{s}$	strategy i 's Sharpe ratio, and the average Sharpe ratio of N strategies, in the note's combination formula (§12.3)	pure number	§12
Y, SE	the number of years of data behind a historical average, and the standard error of that average, $\sigma/\sqrt{Y}$ (§3.2) — Y , not T , because T is the tangency portfolio throughout	years; per period	§3
N	a count: of assets in a portfolio, of strategies combined, of markets, of strategies tested	count	§3
$\bar{R}_i, \bar{R}_M, \bar{R}_f; \gamma_0, \gamma_1$	sample average returns; the intercept and slope of the second-step cross-sectional regression $\bar{R}_i - \bar{R}_f = \gamma_0 + \gamma_1\beta_i + \varepsilon_i$ (04b p. 7)	per period; per period, per unit of beta	§5
x, y (§5)	the axes of the early-tests picture as the guest described them: $x = \beta_i(\bar{R}_M - \bar{R}_f), y = \bar{R}_i - \bar{R}_f$	per period	§5
$R_{ZM}, \mathbb{E}(R_{ZM})$	the return on the zero-beta portfolio of Black's version of the CAPM, and its mean (FF04 p. 5)	per period	§6
δ	the borrowing wedge : the borrowing rate is $R_f + \delta$, $\delta > 0$ — the note's symbol for the guest's "wedge"; no value was given	per period	§6

Symbol	Meaning	Units	First used
$\beta_L, \beta_H; R_L, R_H; \alpha_L, \alpha_H$	the betas, returns and alphas of the low-beta portfolio bought and the high-beta portfolio shorted in Betting Against Beta (the worked problem's letters)	pure number; per period; per period	§8
x_L, Π, β_Π	the dollars of the low-beta portfolio bought per dollar shorted; the position's payoff per dollar shorted, in excess of what the investor's own \$1 would earn at R_f ; its beta (the worked problem's letters)	dollars; dollars; pure number	§8
B, V ; LTV, margin, leverage	the amount borrowed and the value bought; $LTV = B/V$, $\text{margin} = 1 - LTV$, $\text{leverage} = V/(V - B) = w$ (03c p. 10)	dollars; pure numbers	§8
$\Phi(\cdot)$	the standard normal cumulative distribution function, in the note's illustration of what a Sharpe ratio implies for losing years (§12.5)	pure number	§12
$R_{SMB}, R_{HML}; \beta_{iM}, \beta_{is}, \beta_{ih}$	the size and value factor returns and asset i 's loadings on the market, size and value factors (04b p. 16)	per period; pure numbers	§13
$R_{MOM}, R_{BAB}, R_{QMJ}$	the returns of a momentum, a low-risk (Betting Against Beta) and a quality factor — <i>the note's own labels for factors the guest named and the course gives no symbol</i> (04b p. 19 names momentum without one)	per period	§14
$R_B(t), \alpha_B; \beta_M, \beta_s, \beta_h, \beta_{mom}, \beta_{bab}, \beta_q; \varepsilon_B(t); R_M(t), R_f(t)$	Buffett's return in period t (public or private side) and its alpha; its loadings on the market, size, value, momentum, low-risk and quality factors; the residual; the market and risk-free returns in period t — §14's regression, in the note's labels	per period; pure numbers; per period	§14
$\varepsilon_i; e_i, \nu_i^2$	the residual of the second-step regression (§5.1, 04b p. 7); the estimation error in stock i 's historical average and its variance (problem B)	per period; (per period) ²	§5
$p_t, \mu, \varepsilon_{t+1}, \mathbb{E}_t[\cdot]$ (§17)	a price, its drift and its innovation in the random-walk model $p_{t+1} = \mu + p_t + \varepsilon_{t+1}$, and the expectation	dollars	§17

Symbol	Meaning	Units	First used
	given what is known at date t (05a p. 6) — this μ is a drift, not the mean excess return of §§8.4 and 12.3		
P_0, P_1	two prices: today's and next period's in the holding-period return (§11.4); a game's opening and closing betting prices (§17)	dollars; probability units	§11, §17
$D_1, \text{HPR}; V(0), V(T)$	next period's dividend and the holding-period return $1 + \text{HPR} = V(T)/V(0)$ (02b p. 12), with $V(0)$, $V(T)$ an investment's value at the start and end of the period (T a date here, not the tangency portfolio), in the note's reading of carry	dollars; per period	§11
$S_0, S_1; i_d, i_f$	the spot exchange rate today and next period (domestic currency per unit of foreign); the domestic and foreign interest rates, in the note's reading of the currency carry trade	domestic per foreign; per period	§11
p	the probability that the team bet on wins (the physical chance)	pure number	§17
b	a bet's net payout per dollar staked on a win: a \$1 stake returns $1 + b$	dollars per dollar	§19
ω	the outcome indicator of a bet or contract: 1 if it wins, 0 if not	pure number	§19
X	the net payoff of a \$1 stake, $(1 + b)\omega - 1$ (§19)	dollars	§19
$\text{SD}[\cdot]$	a standard deviation, the square root of the variance (§19.2)	units of the argument	§19
q	the price of a prediction-market contract that pays \$1 if its event happens (the note's letter; 07b writes q_k for a security's price)	dollars, between 0 and 1	§21
π^*, μ_s	a risk-neutral probability and a state price (07b pp. 6, 17; formula sheet p. 3 item 15), named as a forward pointer	pure number; dollars	§21
$q_k, d_k(s), \mu(s), \mathbb{E}^*[\cdot]$	07b's price of security k , its payoff in state s , the price today of a dollar in state s , and the expectation	dollars	§21

Symbol	Meaning	Units	First used
	under the risk-neutral probabilities, in $q_k = \sum_s \mu(s) d_k(s) = \mathbb{E}^*[d_k]/(1 + r^f)$ (07b pp. 13, 17), named as a forward pointer		
$s, S; q_s, \pi_s^*$ (\$21.3)	an outcome of an FOMC meeting and the number of outcomes; the price of the contract paying \$1 in outcome s , and its risk-neutral probability (the note's letters)	—; count; dollars; pure number	\$21
W_k, y_k, D, w_T^i	investor k 's wealth; the share of it put in T ; the total dollars all investors put in T ; T 's weight on asset i (problem A)	dollars; fraction; dollars; fraction	Worked problems A
I_k, K, Z	1 if strategy k looks significant and 0 if not; the number of strategies that do; a standard normal variable (problem D)	pure number; count; pure number	Worked problems D
bp	basis point, 1 bp = 0.01% (01a p. 8)	per period	\$21

Names. Names the recording garbled are written as the written sources spell them: Lintner, Fischer Black, Scholes, Eugene Fama, Jegadeesh and Titman, Bogle and Vanguard, Bernie Madoff, Fidelity. Names with no written source stand as heard: Andrea Frazzini and Lasse Pedersen, Mike Jensen, Warren Buffett, Berkshire Hathaway, Wells Fargo, Myron Scholes, David Booth, Dimensional Fund Advisors, Wayne Winston, Tice Jensen, Brian Kelly, Goldman Sachs, Citadel, Polymarket, Novig, ProfitX, the CFTC, the CME, the FOMC. AQR is the guest's firm, as said. The US prediction-market exchange the guest works with is never named by the recording clearly enough to write, so it is called the US exchange here. The authors of the quality paper, Buffett's insurer, the Fama student who founded AQR and the value-investing fund firm are left unnamed, because the recording does not make them out.

Picking up from the course's own material

The course's last meeting before this one was Tuesday 6 October, which by inference taught alpha, capital budgeting, the tests of the CAPM and the size and value anomalies (the Lecture 9 note, reconstructed). **This talk recaps nothing of it and never mentions it:** the guest names no meeting, and "the last couple weeks"

 03:09 names none either. What the talk assumes the class knows is the course's own material, restated where the talk uses it and linked here once:

- the mean–standard-deviation frontier of many risky assets, the tangent line from the risk-free rate, the tangency portfolio and the capital allocation line — the Lecture 7 note (29 Sep) and the Lecture 6 note (24 Sep); 03d pp. 3–5, 03c p. 7;
- equilibrium, the market portfolio and its value weights, the security market line and beta — the Lecture 8 note (1 Oct); 04a pp. 5–12;
- diversification: the variance of an equally weighted portfolio of independent returns falls as $1/N$, and with correlated returns it tends to the average covariance — the Lecture 7 note; 03d pp. 8–11;
- short selling, "sell something you don't own" — the Lecture 2 note (8 Sep); 01b p. 13;
- the capital allocation line's slope as the Sharpe ratio, and loan-to-value, margin and leverage — the Lecture 6 note; 03c pp. 7, 10–11.

The one place the guest asked whether the course had reached something — multifactor models, "I don't know if you've covered that yet"  36:29 — is evidence neither way about what 6 October taught; the note links the three-factor model to the Lecture 9 note as that note's inferred material.

1. The record opens mid-talk: what the talk is about

 00:15

This section states what the talk set out to do, as far as the recording preserves it, and marks what it does not preserve.

GAP Not recoverable — the first five minutes of the meeting, before τ 00:00

The earliest audio on file begins about five minutes after the scheduled 1:05 pm start: recording 1's phone stamped the end of its file at 14:21:52 EDT, its length places its start at about 13:10:55, and the measured 55-second lead of recording 2 puts τ 00:00 at about 13:10:00. Whether the meeting itself began at 1:05, and how accurate the phone's clock is, are not known. Recording 2's first 15.6 seconds carry no decoded words, and its first words, at  00:15, are an aside already under way — "Yes, tonight we're talking about baseball, tomorrow morning we'll be talking about momentum investing, but it's the same deal."  00:15 Recording 1's first words, at  00:55, are mid-sentence. **No introduction of the guest, no course business and no opening of the talk is on file, and nothing is rebuilt for the span:** no material covers it — the guest's slides are not in the course materials, no handout or reading belongs to the talk, and no other meeting's recording describes it. Nothing in this note treats  00:15 as the meeting's opening.

The framing. The first complete thought on file is the talk's plan. "Since you know the CAPM"  00:24, the guest starts where the field and the investment business started, "50-odd years ago"  00:24, and moves

from the CAPM to **anomalies**: patterns in returns "that seem to not be explained by the CAPM" ⌚ 00:34, which raise three questions — what they are, whether they are credible, and whether one can invest in them. The course's own definition is the same: anomalies "are defined relative to a benchmark model (e.g, CAPM)" (05a p. 15, not yet taught), and the course's reading on the subject, Lamont and Thaler's survey of anomalies (LT03), belongs to that later block. The guest's firm, AQR, "and not just AQR, but a bunch of other hedge funds", has built a thirty-year business running "trillions of dollars" ⌚ 00:49 on these anomalies [*the guest's figures*]. The talk's claim is that the same anomalies show up in two other markets: sports betting, treated briefly, and prediction markets, the guest's current research, "which I'm actually super into right now" ⌚ 00:49.

Prediction markets, in one paragraph of context. The guest is doing academic research on prediction markets and polled the room: not everyone may have heard of Gene Fama, but most had heard of the prediction-market platforms — the US exchange (unnamed here), Polymarket, Novig ⌚ 01:11. About twelve such exchanges exist and the number keeps growing. The School of Management is hosting an academic prediction-markets conference on 23 October, sponsored by AQR and the US exchange, whose head of research will attend; how to get an invitation is course logistics and is left out. At AQR the guest runs a group started in July, named on the recording as "Prediction Markets and Sports Networks" ⌚ 02:08, and "the hedge fund itself is trading in these markets and just starting to" ⌚ 02:14. The talk's purpose is to show "how some of the things that we've learned over the last several decades in financial markets actually apply to all these other markets" ⌚ 02:20.

Why it should carry over. Markets are driven by human behaviour and preferences — "risk, return preferences, psychological preferences sometimes" — and the same individuals take part in all of these markets, so the same things show up ⌚ 02:33. The course's own version of that thought is the behavioural reading of anomalies, "driven by investor psychology" (05a p. 18, item 6); the guest's talk is an argument for taking it seriously by looking where the alternative explanations cannot reach (§15). The guest's offer to teach a course on prediction markets next year, made in the same breath, is an announcement and not course content.

Self-check. The recording's first sentence is about baseball and momentum investing. Why does this note refuse to call it the lecture's opening?

Because the audio starts about five minutes after the scheduled start and its first words are inside an aside already under way; the meeting's opening, any course business and the introduction of the guest are not on file. A recording that starts late always opens mid-sentence, and treating its first beat as the opening would attribute to the meeting a beginning nobody recorded.

2. The CAPM recalled: the frontier, the tangency portfolio and equilibrium

🕒 03:09

This section restates, in the course's own forms, the picture the guest put up as known — "Okay, so you learned this, right, in the last couple weeks." 🕒 03:09 — and the one step the guest called economics rather than mathematics.

2.1 The frontier, the tangent line and the capital allocation line

The guest's figure, described from the words alone *[transcript only]*, has mean return on the vertical axis and the standard deviation of return on the horizontal axis; "I can square it, make it the variance on the x-axis if I want it to be a parabola" 🕒 03:18 . The curve is the set of efficient risk–return trade-offs: each point offers the highest expected return for its standard deviation, "or vice versa", the lowest standard deviation for its expected return. Every individual risky asset — "all individual stocks, bonds, commodities" 🕒 03:30 — lies inside the curve, and the curve is made of their optimal combinations. Adding a risk-free rate lets one "draw a nice little tangent line that identifies the one true optimal combination of all the risky assets called the tangency portfolio"; if everyone has the same inputs and computes the same moments of the return distribution, everyone wants to be on that **capital allocation line**. "Now that's just math, right? There's nothing, there's no economics in this. That is pure math." 🕒 03:54 .

The course's version is the same picture with its formulas (*related, not shown*). The many-asset frontier is 03d p. 3's figure — "Efficient Frontier" on the upper branch, "Inefficient Frontier" on the lower, the "Global Minimum Variance Portfolio" at the vertex, and a cluster of "Individual Assets" strictly inside the curve — taught on 29 September (§9.1 of that note). The tangency portfolio is the portfolio of risky assets with the highest Sharpe ratio (03d p. 4):

$$\max_{w_1, \dots, w_N} \frac{\mathbb{E}[R_p] - R_f}{\sigma_p} \quad \text{subject to} \quad w_1 + \dots + w_N = 1$$

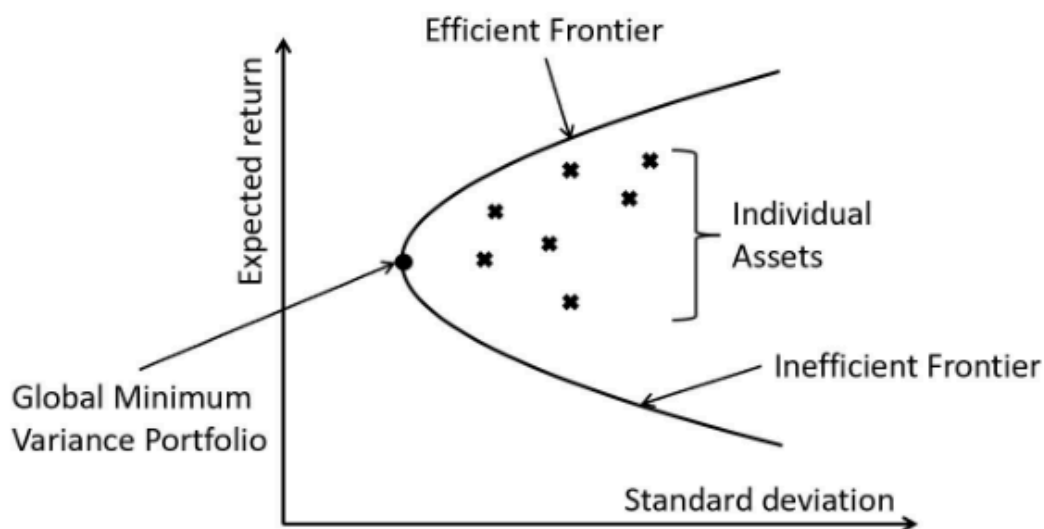
Why this is the right object: a line from R_f through any risky portfolio has slope equal to that portfolio's Sharpe ratio (03c p. 7), so the steepest such line, the one tangent to the frontier, passes through the portfolio with the highest Sharpe ratio. The line itself is the capital allocation line, which the exam's sheet prints as item 9 (formula sheet p. 1; 03c p. 7):

$$\mathbb{E}[R_p] = R_f + \frac{\mathbb{E}[R_T] - R_f}{\sigma_T} \sigma_p$$

Why this step follows: a portfolio with weight w in T and $1 - w$ in the risk-free asset has mean $R_f + w(\mathbb{E}[R_T] - R_f)$ and volatility $w\sigma_T$; eliminating w gives the line. "Assuming we all have the same inputs"

is the CAPM's sixth assumption, that "All investors perceive the same means, variances and covariances for returns" (04a p. 4), which the deck calls the "key assumption: homogeneous expectations" (04a p. 5); with it, every investor holds the same tangency portfolio in some mix with the risk-free asset (the Lecture 7 note's two-fund separation; 03d p. 5). The guest's "no economics in this" is the course's "CAPM = Portfolio Theory + Equilibrium" (04a p. 2): portfolio choice gives only the demand for securities, and the next step adds the supply.

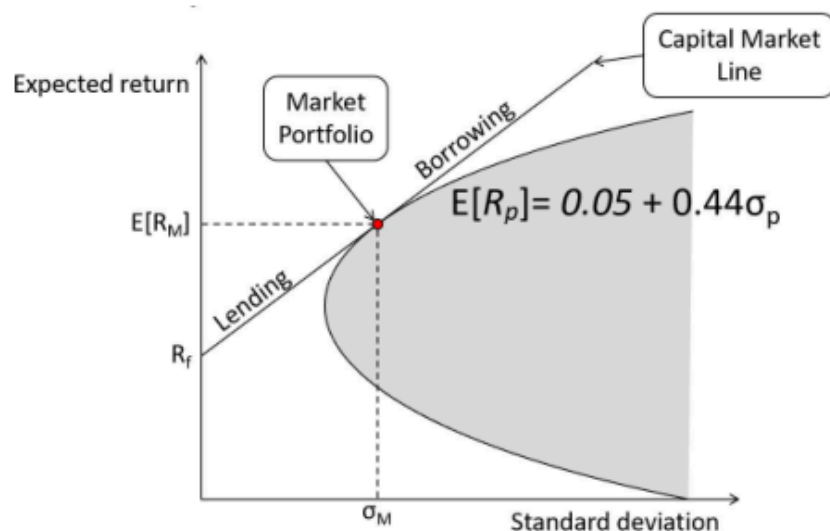
Investment Opportunity Set with Many Assets



3 / 20

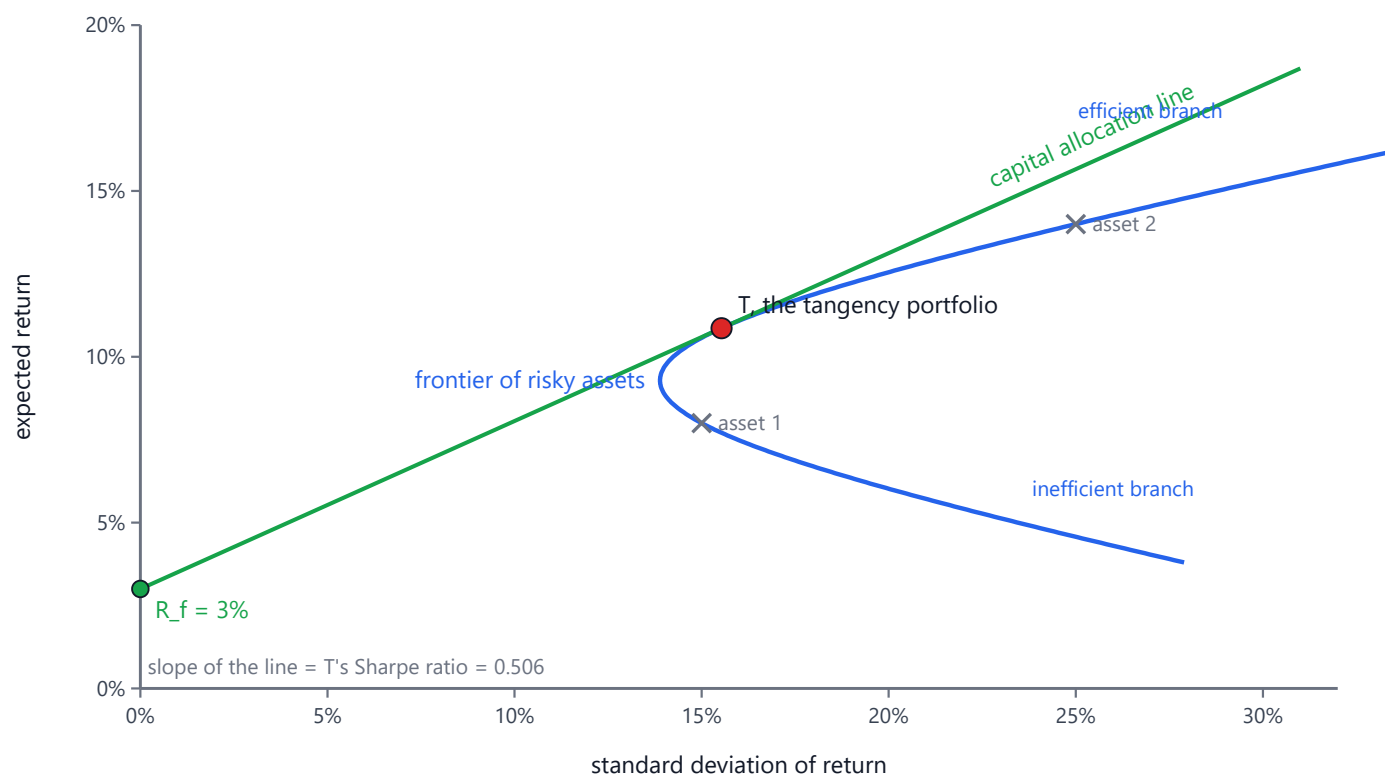
03d p. 3 — the course's own frontier with many assets: efficient and inefficient branches, the global minimum variance portfolio, individual assets inside the curve. Related, not shown on 8 October.

The $\mathbb{E}[R]$ -SD Frontier and the CML



- ▶ $\mathbb{E}[R_M]$, σ_M , R_f allow us to pin down the efficient frontier
 - ▶ No need to calculate the tangency: use the market portfolio!
 - ▶ Now we know that: tangency portfolio = market portfolio
- ▶ We have an estimate of the market price of risk
 - ▶ Ex: $\mathbb{E}[R_M] = 12\%$, $R_f = 5\%$, $\sigma_M = 16\%$
 - ▶ $SR_M = \frac{12\% - 5\%}{16\%} = 0.44$

The frontier and the capital allocation line (course shapes; illustrative inputs, not the guest's)



Redrawn: the shapes the guest described — the frontier in mean–standard-deviation space, the tangent from the risk-free rate, the tangency portfolio T and the capital allocation line; individual assets inside the curve. A schematic of the course's figures, not the guest's slide.

How to read this: the curved boundary is the best mean for each standard deviation that any mix of risky assets can reach; every single asset sits inside it. The straight line starts at the risk-free rate on the vertical axis and just touches the curve at T, the tangency portfolio; every point on that line is a mix of T and the risk-free asset, and no point above it is attainable. With a risk-free asset, every investor with the same inputs picks a point on this line, never on the curve itself. The redraw uses illustrative inputs for two risky assets (means 8% and 14%, volatilities 15% and 25%, correlation 0.2, risk-free rate 3%), chosen only to give the curve its shape; none is the guest's.

Interactive spec — The frontier of two risky assets and the capital allocation line from the risk-free rate

The sliders arrive with the site; the previewer shows the spec.

```
id: frontier-cal
title: The frontier of two risky assets and the capital allocation line from the risk-free rate
inputs:
  - {name: E1, label: "Expected return of asset 1", min: 0.00, max: 0.20, step: 0.005, default: 0.08, format: percent}
  - {name: E2, label: "Expected return of asset 2", min: 0.00, max: 0.25, step: 0.005, default: 0.14, format: percent}
  - {name: s1, label: "Volatility of asset 1", min: 0.02, max: 0.40, step: 0.01, default: 0.15, format: percent}
  - {name: s2, label: "Volatility of asset 2", min: 0.02, max: 0.50, step: 0.01, default: 0.25, format: percent}
  - {name: rho, label: "Correlation", min: -0.95, max: 0.95, step: 0.05, default: 0.2}
  - {name: Rf, label: "Risk-free rate", min: 0.00, max: 0.08, step: 0.005, default: 0.03, format: percent}
series:
  - {name: "frontier", x: "w*E1 + (1-w)*E2", x_values: "w from -0.5 to 1.5 step 0.02", y: "(w**2*s1**2 + (1-w)**2*s2**2 + 2*w*(1-w)*rho*s1*s2)**0.5", swap_axes: true}
  - {name: "capital allocation line", y: "Rf + ((wT*E1+(1-wT)*E2 - Rf)/((wT**2*s1**2 + (1-wT)**2*s2**2 + 2*wT*(1-wT)*rho*s1*s2)**0.5))*x", style: dashed, note: "wT = the weight on asset 1 that maximizes the Sharpe ratio, found by the renderer on the same grid"}
x_axis: {label: "standard deviation"}
y_axis: {label: "expected return"}
readout: ["the tangency portfolio's Sharpe ratio"]
```

2.2 Parabola or hyperbola: what the guest's "square it" means

The guest is right that the frontier is a parabola only when variance is on the horizontal axis; the course's own visual read of its frontier figure calls the mean–standard-deviation curve "parabola-shaped" (*03d p. 3, read*) as a description of its look, not a claim about its equation. The distinction, in the two-asset case the course's formulas cover (*formula sheet p. 1, items 7–8; 03a pp. 5–6*), is a two-line calculation.

DERIVATION Full derivation (not done in class) — variance is a quadratic in expected return, so the frontier is a parabola in (variance, mean) and a hyperbola in (standard deviation, mean)

Take two risky assets with means $E_1 \neq E_2$, volatilities σ_1, σ_2 and correlation ρ , and a portfolio with weight w on the first.

$$E = wE_1 + (1 - w)E_2 \Rightarrow w = \frac{E - E_2}{E_1 - E_2}$$

Why this step follows: the portfolio mean is linear in w (formula sheet p. 1, item 7), so it can be inverted for w whenever the two means differ.

$$\text{Var}[R_p] = w^2\sigma_1^2 + (1 - w)^2\sigma_2^2 + 2w(1 - w)\rho\sigma_1\sigma_2$$

Why this step follows: the two-asset variance formula (formula sheet p. 1, item 8).

$$\text{Var}[R_p] = aE^2 + bE + c, \quad a = \frac{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}{(E_1 - E_2)^2} \geq 0$$

Why this step follows: substitute $w = (E - E_2)/(E_1 - E_2)$ into the variance; every term is at most quadratic in w , hence in E , and the coefficient of E^2 collects the three w^2 terms. (The constants b and c are the remaining coefficients; their exact forms are not needed for the shape.) The numerator of a is $\text{Var}[R_1 - R_2] \geq 0$, so the parabola opens toward larger variance.

So in the (variance, mean) plane the frontier is the parabola $\text{Var} = aE^2 + bE + c$, which is what the guest meant by squaring the axis. In the (standard deviation, mean) plane it is $\sigma = \sqrt{aE^2 + bE + c}$, that is $\sigma^2 - aE^2 - bE = c$: a conic with a σ^2 term and an E^2 term of opposite signs, which is a **hyperbola**, opening to the right, with the global minimum-variance portfolio at its vertex. With many assets the same two facts hold (the mean is linear in the weights and the variance quadratic), so the many-asset frontier has the same two shapes. **EXPLANATION**

WARNING Said in class vs written — 03:09 and 04:22

The guest called the curve "a parabola" twice: once correctly conditioned on squaring the axis, and once, "If you look at the mathematics of a parabola, you get these equations done" ⌚ 04:22, as the name for the frontier's equations. The course's figures (03d p. 3; 04a p. 9) plot mean against **standard deviation**, where the curve is a hyperbola, and the derivation above is why. The note never calls the mean–standard-deviation curve a parabola. Nothing in the economics turns on the name.

2.3 Equilibrium: the tangency portfolio must be the market

The frontier's equations themselves were passed over as known — "you get these equations done" 04:22 ; the course's are the tangency condition of handout 03 §7, worked in the Lecture 7 note. The economics enters with one question: "The economics is, what is the tangency portfolio in equilibrium? Economics is supply equals demand." 04:27 . If everyone holds the same risky portfolio, in different combinations with the risk-free asset, what must that portfolio be? The room answered, and the guest repeated it: "Thank you. The market portfolio." — "the market portfolio of everything weighted in proportion to how we value the economy" 04:50 .

The course's statement is the deck's: "In equilibrium supply = demand", so "the tangency portfolio must be the portfolio of all existing risky assets, the "market portfolio"" — supply being the shares outstanding, demand the tangency portfolio (04a p. 5). Its weights are market-capitalization weights (04a p. 6):

$$w_M^i = \frac{p_i n_i}{\sum_j p_j n_j} = \frac{\text{market capitalization of asset } i}{\text{total market capitalization}}$$

The deck's example — two assets with prices 2 and 4 and 25 and 50 shares, capitalizations 50 and 200 of a total 250, weights 20% and 80% — and the mechanism that enforces the equality (if demand for an asset exceeds its supply, its price is bid up until the ratio of desired weights equals the ratio of capitalizations, 04a p. 7) were taught on 1 October (§9 of that note). The guest's question is the first in-class exercise of this meeting, worked under *Worked problems* as a self-check that derives the aggregation in full: everyone's risky dollars are the same portfolio, so adding them up asset by asset and equating to what exists forces that portfolio's weights to be value weights.

Self-check. The guest said the tangency portfolio is "pure math" and the market portfolio is "economics". What is the one assumption that turns the first into the second?

Homogeneous expectations plus market clearing. If every investor computes the same moments, every investor's risky holding is the same tangency portfolio T (two-fund separation). Supply must equal demand asset by asset, and since all risky demand is a multiple of T 's weights, T 's weights must match the shares that exist, valued at market prices: T is the market portfolio with weights $w_M^i = p_i n_i / \sum_j p_j n_j$ (04a pp. 5–6).

Self-check. Why does every investor end up on the capital allocation line rather than on the frontier?

Because any point on the frontier other than T is dominated: the line from R_f through T is the steepest line through any risky portfolio, so for every standard deviation it offers a higher expected return than the frontier does (03d p. 4; 03c p. 7). An investor who wants more or less risk than T moves along the line by lending or borrowing at R_f , not along the curve.

3. What the CAPM gives an investor: covariance beats averages, and the chicken-and-egg problem

⌚ 05:00

This section establishes why the equilibrium result matters to someone who has to invest money: it replaces an unestimable quantity, a stock's expected return, by an estimable one, its covariance with the tangency portfolio — and why that still leaves a circle that only economics can break.

3.1 Expected return is linear in covariance with the tangency portfolio

"Suppose I'm investing in AQR," the guest says: run the mathematics, and "if I just do the tangency portfolio, there's actually a linear combination between the expected return on every asset in the economy and its covariance with the tangency portfolio" ⌚ 05:00. The course proved this on 1 October as the **beta representation theorem** (§11.2 of that note; handout 04 §1): for an efficient portfolio c — the deck's letter for the guest's T — and any risky asset i ,

$$\mathbb{E}[R_i] - R_f = \beta_{iT} (\mathbb{E}[R_T] - R_f), \quad \beta_{iT} = \frac{\text{Cov}[R_i, R_T]}{\text{Var}[R_T]}$$

(04a p. 11, with c for T). It follows from the fact that, at an efficient portfolio, the ratio of the marginal gain in mean to the marginal gain in variance from adding a little of any asset — $(\mathbb{E}[R_i] - R_f) / 2 \text{Cov}[R_i, R_T]$ — "must be the same across all securities" (04a p. 11), the equal-ratio condition handout 04 pp. 1–2 proves by local deviations. The guest's "linear combination" between expected return and covariance is the same equation read with the covariance as the variable **EXPLANATION**:

$$\mathbb{E}[R_i] - R_f = \underbrace{\frac{\mathbb{E}[R_T] - R_f}{\text{Var}[R_T]}}_{\text{one number for the whole market}} \text{Cov}[R_i, R_T]$$

Why this step follows: substitute the definition of β_{iT} and move the constant $\text{Var}[R_T]$ into the slope. The slope is the same for every asset — the price of a unit of covariance — so expected excess return is a straight line in covariance with T through the origin. That is "day one" investing in the guest's phrase: "identify the tangency portfolio, then look at the covariances with it, and I'm off and running. I can figure out what the expected return is of any stock or bond in the economy" (06:08). The course lists exactly this as the CAPM's second application: find an investment's beta, compute its expected return (04a p. 26).

3.2 Why the historical average of a single stock is "absolute garbage"

Why not just estimate each stock's expected return from its past? Because "if you take the historical average return of a single stock, it is absolute garbage" (05:19): "signal-to-noise ratios in financial markets are tiny" (05:28), so the average of any single series is "mostly just an average of noise" (05:34). Second moments are different — "the covariance of those returns historically with something is a more stable instrument and a more precise one" (05:41) — so covariance with the tangency portfolio "is a far better predictor of your future expected returns than the historical past. And that seems weird, but that's just true" (05:52), because the model is "filtering out all that noise" (06:02). The course says the first half in its own words: "Estimates of means are imprecise over short periods", "Garbage in—garbage out" (03d p. 15), which is the first of the three practical problems with mean-variance optimization taught on 1 October (§4 of that note).

How imprecise, with the course's own numbers (*explanation; the guest gave none*). If a stock's annual returns are independent draws with volatility σ , the standard error of its average over Y years is $\sigma/\sqrt{Y}$. The course's typical single stock has a volatility of 44% and a 500-stock portfolio 12% (03d p. 14); its beta regressions use "a relatively short time period, e.g., 5 years/60 months" (04b p. 6). Over five years,

$$\text{SE}_{\text{one stock}} = \frac{44\%}{\sqrt{5}} = 19.68\%, \quad \text{SE}_{500 \text{ stocks}} = \frac{12\%}{\sqrt{5}} = 5.37\%$$

Why this step follows: $\sqrt{5} = 2.2361$; divide each volatility by it. A single stock's five-year average has a standard error of about twenty percentage points — three to five times the whole market risk premium, which the course puts at "4% to 6% in practice" (04a p. 14). The average tells you almost nothing about the mean. The second self-check under *Worked problems* develops this: estimation errors that are uncorrelated across stocks average away in a portfolio exactly as idiosyncratic risk does (03d p. 8's algebra applied to the errors), which is the guest's point at (09:28) (§5.2).

GAP Not recoverable — why second moments are more stable and more precise

The guest asserted it ("that's just true") and gave no reason; no course page states one. The note's own account, labelled as such (**EXPLANATION**): a mean is a *level*, and the precision of a level estimated from returns improves only with the calendar length of the sample — five years of daily data pin the average no better than five years of annual data. A covariance or a variance is a *second moment*, and its precision improves with the *number of observations*, so sampling the same five years daily instead of annually multiplies the information. That is why betas can be estimated from five years of data (04b p. 6) while means cannot (03d p. 15). Whether this is the reason the guest had in mind is not on the recording.

3.3 The chicken-and-egg problem, and how economics breaks the circle

"The problem is we have a chicken and egg issue, which is how do you calculate the tangency portfolio?"

⌚ 06:24 . To compute T one needs every mean, variance and covariance; once one has T , one can back out the means from the covariances. "So which moves first?" ⌚ 06:36 The course counted the inputs on 03d p. 15: with N assets, unrestricted mean-variance analysis needs

$$2N + \frac{N(N-1)}{2} \text{ estimates: } N = 2 \Rightarrow 5, \quad N = 500 \Rightarrow 1,000 + 124,750 = 125,750$$

Why this step follows: N means, N variances, and one correlation for each of the $N(N-1)/2$ pairs; for $N = 500$, $500 \times 499/2 = 124,750$ pairs. The course's two ways forward are a statistical one, factor models, and an economic one, the CAPM (03d p. 16; the Lecture 8 note's §5.1); the guest's answer is the economic one. "If I know what the tangency portfolio has to be under some model, in this case a very simple supply and demand framework, I know the market has to be the tangent, now if I have a proxy for the market, I can back out all these things that are really difficult to estimate" ⌚ 06:43 . The deck says the same: "No need to calculate the tangency: use the market portfolio!" (04a p. 9); "Use a proxy for the market, e.g., a value weighted stock index" (04b p. 6); an investor "can free-ride on other investors and use the market portfolio as the optimal mutual fund of risky assets" (04a p. 8).

What AQR does, in the guest's description. Both AQR and academic finance try to "figure out what that tangency portfolio should be without having to calculate everything, using some economics. A mixture of modeling and a mixture of data" ⌚ 07:03 . The CAPM gives one answer; "Turns out it's not a very good answer, a very helpful answer. It's great from a pedagogical standpoint to understand how markets work" ⌚ 07:17 , but "a lot of things are much better than the market portfolio at backing out returns, variances, and co-variances" ⌚ 07:29 , and each of them is "trying to get a better proxy for that object T" ⌚ 07:37 . How: complicated models, "eight-figure data budget and a bunch of PhDs and computer scientists looking for information like crazy to help identify that thing" ⌚ 07:46 [the guest's description of the firm; no written

source]. The course's verdict on the CAPM is the same in substance — "CAPM is rejected in the data", a "Qualitative success" and a "Quantitative failure" (04b p. 11) — and its sixth assumption is introduced with the line "All assumptions are inaccurate/wrong, but useful" (04a p. 4), which the guest will echo at 11:28 (§6).

Self-check. A colleague proposes estimating the tangency portfolio of 500 stocks directly from ten years of monthly returns (*the ten years illustrative, the note's own*). Give two reasons, in the course's words, why the output would be unusable, and say what the CAPM offers instead.

First, the inputs: 125,750 parameters (03d p. 15), of which the 500 means are "imprecise over short periods" — a 44%-volatility stock's ten-year average has a standard error of $44\%/\sqrt{10} \approx 13.9\%$, larger than any plausible premium; "Garbage in–garbage out". Second, the circle: the tangency portfolio needs the means, and the means are what one hoped to read off the tangency portfolio. The CAPM supplies the tangency portfolio from economics instead of statistics — in equilibrium it is the market — so a value-weighted index can proxy for it, and each stock's expected return follows from its covariance with the index, which is estimable from a few years of data.

4. The CAPM equation and its empirical record

08:02

This section fixes the equation the guest skipped past on a slide and records the verdict the guest gave on it.

"Okay, so that's that linear equation I talked about, so we'll skip over this, right?" 08:02 [transcript only: the slide is not in the build]. In words, the expected return on any stock in excess of the risk-free rate is "just its beta times the tangency portfolio in excess of the risk-free rate, but if the tangency is the market, you have the Sharpe, Lintner, and CAPM" 08:07. The written form is §3.1's theorem with the market in the place of T , the **security market line**, which the exam's sheet prints as item 10 (formula sheet p. 1; 04a p. 12):

$$\mathbb{E}[R_i] = R_f + \beta_i (\mathbb{E}[R_M] - R_f), \quad \beta_i = \frac{\text{Cov}[R_i, R_M]}{\text{Var}[R_M]}$$

Why this follows from §3.1: in equilibrium the tangency portfolio is the market (§2.3), so β_{iT} becomes β_i and $\mathbb{E}[R_T]$ becomes $\mathbb{E}[R_M]$. The reading's version is the same line, "(Sharpe-Lintner CAPM) $E(R_i) = R_f + [E(R_M) - R_f]\beta_{iM}$ " (FF04 p. 6). On the credit: the course names Sharpe, "and also by Lintner, Mossin and Treynor" (04a p. 2), and Fama and French refer "to the Sharpe-Lintner-Black model as the CAPM" (FF04 p. 2); the guest's Sharpe, Lintner is the usual short form, and Black's version is §6's subject.

The record. "empirically, the record is that that equation just doesn't explain enough" ⌚ 08:20. The guest's division of the failure is between and within asset classes [*the guest's reading; no course page says this*]: the CAPM "does a great job of telling you why stocks and bonds have different long-run average returns" ⌚ 08:25, "pretty good at explaining differences across asset classes" ⌚ 08:30, but "not very good at explaining differences within an asset class, like why small stocks have better returns than large stocks, or value stocks do better than growth stocks" ⌚ 08:33. The course's own numbers for the within-class failure are the spreads of 04b p. 12, taught by inference on 6 October (\$6.1 of that note): over July 1926 – October 2010 the tenth book-to-market decile (value) averaged 16.41% a year against 10.16% for the first (growth), a **value spread of 6.25% per year**, and the smallest size decile 17.45% against 10.35% for the largest, a **size spread of 7.10% per year** (04b p. 12). The deck's conclusion matches the guest's: "CAPM is rejected in the data. The estimated SML is too flat (although its slope is positive)" — a qualitative success, "high-beta stocks earn a higher return to compensate for their risk", and a quantitative failure, "the additional return for beta is not as high as the model predicts" (04b p. 11). *Too flat* is the next section.

5. The early tests: the SML comes out too flat

⌚ 08:43

This section reconstructs the early-tests picture the guest showed, fixes its axes in the course's own test, explains why the test is run on portfolios, and records the finding that the line is too flat and "even flatter today".

5.1 The picture: Black, Jensen and Scholes, 1972

"here's a picture of what the early tests of the CAPM looked like" ⌚ 08:43 [*transcript only*], built from the data of Black, Jensen and Scholes (1972), whom the course's reading lists among the early tests (FF04 pp. 8–9). The guest's aside: Black and Scholes are known for the option-pricing model (the options block, 09d, later in this course), but "In 1972, the options markets didn't exist yet, they were running regressions of the CAPM equation" ⌚ 08:58, and "Mike Jensen is pretty famous, too, by the way" ⌚ 09:07 — famous, the aside went on, though without a Nobel Prize. The picture's axes, in the guest's words: "On the x-axis, you have beta times the average market return" ⌚ 09:12, and "On the y-axis, you have the actual average excess return on these portfolios, averaged over time" ⌚ 09:20.

The course's form of the test is the second step of the Fama–MacBeth procedure (04b p. 7; the Lecture 9 note's §5.5): regress average excess returns on betas,

$$\bar{R}_i - \bar{R}_f = \gamma_0 + \gamma_1 \beta_i + \varepsilon_i, \quad \text{the CAPM predicts } \gamma_0 \rightarrow 0, \gamma_1 \rightarrow \bar{R}_M - \bar{R}_f$$

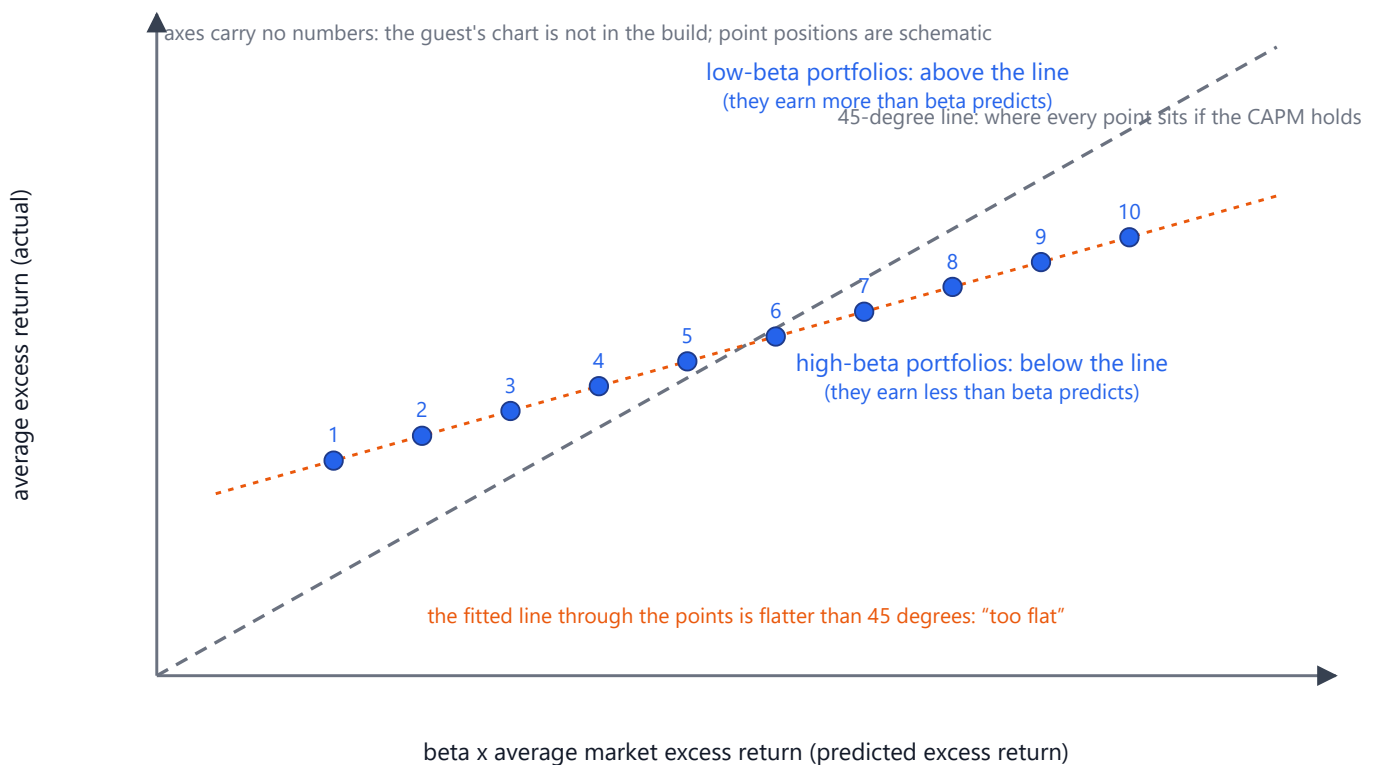
With the horizontal axis scaled as the guest describes, the CAPM's prediction is a line of slope one

EXPLANATION:

$$x \equiv \beta_i (\bar{R}_M - \bar{R}_f), \quad y \equiv \bar{R}_i - \bar{R}_f \Rightarrow \text{under the CAPM, } y = x$$

Why this step follows: with $\gamma_0 = 0$ and $\gamma_1 = \bar{R}_M - \bar{R}_f$ the fitted value of y is exactly $\beta_i(\bar{R}_M - \bar{R}_f)$, which is x ; so every portfolio should sit on the 45-degree line through the origin. "Everything should follow along the 45-degree line here, once I take out the risk-free rate" 🕒 10:25 is this statement. The guest's "beta times the average market return" 🕒 09:12 is read as beta times the average market excess return, since only then does subtracting $\bar{R}_f$ from the vertical axis give the 45-degree line.

The early tests as described: points flatter than the 45-degree line (schematic, no data)



Redrawn: the early-tests picture as the guest described it — on the horizontal axis beta times the average market excess return, on the vertical axis the portfolio's actual average excess return; the 45-degree line is where every point would sit if the CAPM held. The points are schematic: they show a flatter line, not any data. Reconstructed from the description; the guest's chart is not in the build.

How to read this: a point on the dashed 45-degree line is a portfolio whose average excess return equals what its beta says it should earn. Points above the line at the left and below it at the right trace a flatter line than 45 degrees: low-beta portfolios beat their CAPM prediction, high-beta ones fell short of it. The drawing has no numbers because the guest's chart carries none the recording preserves.

5.2 Why the test is run on portfolios: estimation error diversifies too

The guest paused on the choice of portfolios rather than stocks: "averaging an individual stock return is not a very good measure of its mean. For a portfolio, it's much better. Why is that?" ⌚ 09:28 . The room's answer is not decoded; the guest's completion of it is: "not just risk, but also estimation error, right?" — "it's also about estimation error, which also diversifies away, right? It should be uncorrelated to each other" ⌚ 09:50 . In theory one thinks in terms of risk, "because that's the way we write it down in terms of theory, but in reality, in the world we live in, where I'm trying to estimate this in AQR" ⌚ 09:50 , estimation error matters as well. The course says it of betas on the portfolio-sorting page: one forms portfolios "because betas are estimated more precisely for portfolios. This make sense if measurement noise is uncorrelated" (04b p. 8, as printed); the deciles are the ten percent of stocks with the lowest first-step betas up to the ten percent with the highest, "portfolio sorting by beta" (04b p. 8; the Lecture 9 note's §5.6). The arithmetic behind "diversifies away" is the law of large numbers — sample averages of independent random variables converge to the mean, so that "Randomness disappears" by averaging (03S p. 12) — applied to the errors rather than the returns; it is the second worked self-check under *Worked problems*, with §3.2's numbers.

5.3 "Too flat", and "even flatter today"

"despite my best efforts to come up with relatively stable portfolios with not a lot of estimation error, the capital asset pricing model does not match the data. This security market line is too flat, right?" ⌚ 10:12 . The original paper's data ran from the 1950s to 1972; the data go back to 1926; the chart the guest had ended in 2010, and the guest extended it to 2026 for the guest's own class — "It's super easy with Claude" — with the result that "it's even flatter today than it ever has been" ⌚ 10:36 [transcript only; the 2026 update is the guest's].

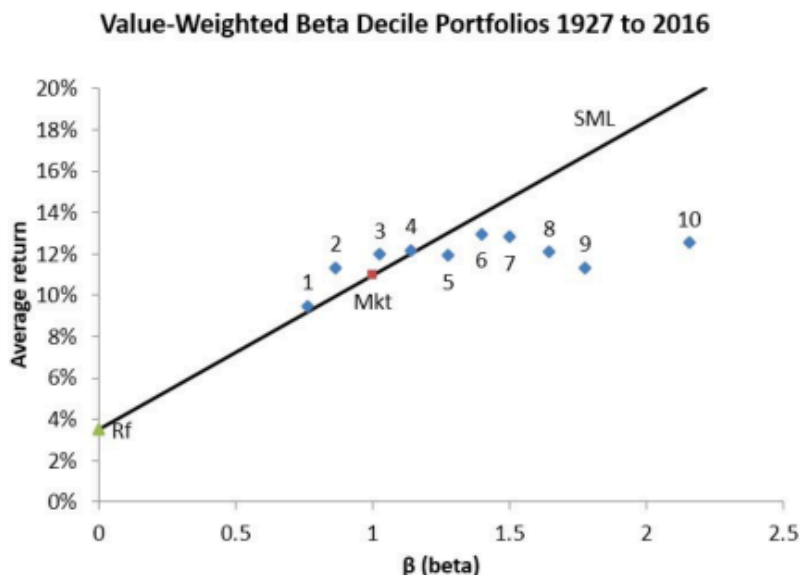
GAP Not recoverable — the guest's early-tests chart and its 2026 update

The chart's points, its sample and the size of its flattening are not on the recording. **The course's own chart of the same kind is 04b p. 10**, value-weighted beta-decile portfolios 1927–2016, which this note embeds below with its own numbers; those numbers belong to the course's chart and are never transferred to the guest's, which covers other years.

The course's chart and its reading (*related, not shown; the Lecture 9 note's §5.8*): betas rise from about 0.76 (decile 1) to about 2.16 (decile 10), but average returns run only from about 9.5% to 12–13%, far flatter than the drawn SML; "Lowest-beta portfolio: predicted return is 9.2% while actual return is 9.5% (alpha is +0.3%)" and "Highest-beta portfolio; predicted return is 19.5% while actual return is 12.5% (alpha is –7%)" (04b p. 11). The reading reports the same shape for 1928–2003: "a positive relation between beta and average return, but it is too "flat."" (FF04 p. 9); the predicted return on the lowest-beta portfolio is 8.3% a year against an actual 11.1%, and on the highest-beta portfolio 16.8% against 13.7% (FF04 p. 10). And the guest's

"even flatter today" is the reading's "the relation between average return and beta for common stocks is even flatter after the sample periods used in the early empirical work on the CAPM" (FF04 p. 13).

Second-step Regression (sorting by beta)



- ▶ What should be true about these points if the CAPM is true?
- ▶ What should be the intercept? And the slope?
- ▶ Does the CAPM work?
- ▶ Where are the alphas?
- ▶ Which stocks do you want to buy? And to sell?
- ▶ Can you find an arbitrage opportunity?

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04b p. 10 — the course's own second-step chart: value-weighted beta-decile portfolios, 1927–2016, with the SML through the T-bill rate and the market. The course's chart, not the guest's; its numbers are the course's.

How to read this: each diamond is a decile of stocks sorted by beta, plotted at its average beta and its average return; the line is what the CAPM predicts. The low-beta deciles sit above the line and the high-beta deciles well below it, so the fitted line through the diamonds is flatter than the SML. This is the course's version of the picture the guest described, over the course's sample.

5.4 What the profession did next

"what the academic profession has really spent time doing is figuring out what explains the gaps between these things" ⌚ 10:57 — "What other theories, models, and data can tell us why these things don't line up" ⌚ 11:05 ; "Covariance at the market isn't explaining nearly enough" ⌚ 11:12 , because the differences in

average returns are big. The rest of the talk is a tour of the answers; the course's own tour is 04b pp. 12–19 and FF04 pp. 14–17, both the Lecture 9 note's.

Self-check. In the early-tests picture, where would a portfolio with a positive alpha sit, and what does "too flat" say about the alphas of low- and high-beta portfolios?

Above the 45-degree line: its average excess return exceeds $\beta(\bar{R}_M - \bar{R}_f)$, and the vertical distance to the line is its alpha (04a p. 27's definition with sample averages). A line flatter than 45 degrees means low-beta portfolios sit above it (positive alphas) and high-beta portfolios below it (negative alphas) — the course's +0.3% and –7% for deciles 1 and 10 (04b p. 11), and, in substance, the reading's finding that the regression intercepts are positive for low-beta portfolios and negative for high-beta ones (FF04 pp. 9–10).

Self-check. Why does sorting stocks into ten beta portfolios make the test more reliable, and what assumption does that rest on?

Each stock's beta and average return are measured with error. If those errors are uncorrelated across stocks, averaging fifty or more stocks in a decile shrinks the error's variance by the number of stocks (the law of large numbers, 03S p. 12), while the decile's beta keeps the spread the sort created. The assumption is the uncorrelated measurement noise 04b p. 8 names; a common error, such as a shared market shock in a short sample, does not average away.

6. Where the model is wrong: costly borrowing and the borrowing wedge

🕒 11:28

This section rebuilds the drawing the guest made, states the assumption it relaxes, and derives the implication the guest called testable: a wedge between the lending and borrowing rates flattens the empirical line.

6.1 "All models are wrong, but where is the model wrong?"

The first way to explain the gaps is to ask which assumption fails: "obviously the model's wrong, all models are wrong, but where is the model wrong?" 🕒 11:28 — the course's own framing of its assumptions, "All assumptions are inaccurate/wrong, but useful" (04a p. 4). Perhaps the CAPM with the market portfolio "is the right model, but some of the assumptions that go into it are incorrect. So one assumption is that you

can borrow and lend at the risk-free rate all you want. And this is what Fisher Black came up with after writing that 1972 paper" ⌚ 11:40 — Fischer Black, as the reading spells it. The reading says the same of the assumption: "Unrestricted risk-free borrowing and lending is an unrealistic assumption" (FF04 p. 6), and Fischer Black's 1972 paper develops a version of the CAPM without risk-free borrowing or lending (FF04 p. 6). In the course's own picture, 04a p. 9's capital market line has a "Lending" segment from R_f to the market and a "Borrowing" segment beyond it, both at the one rate R_f .

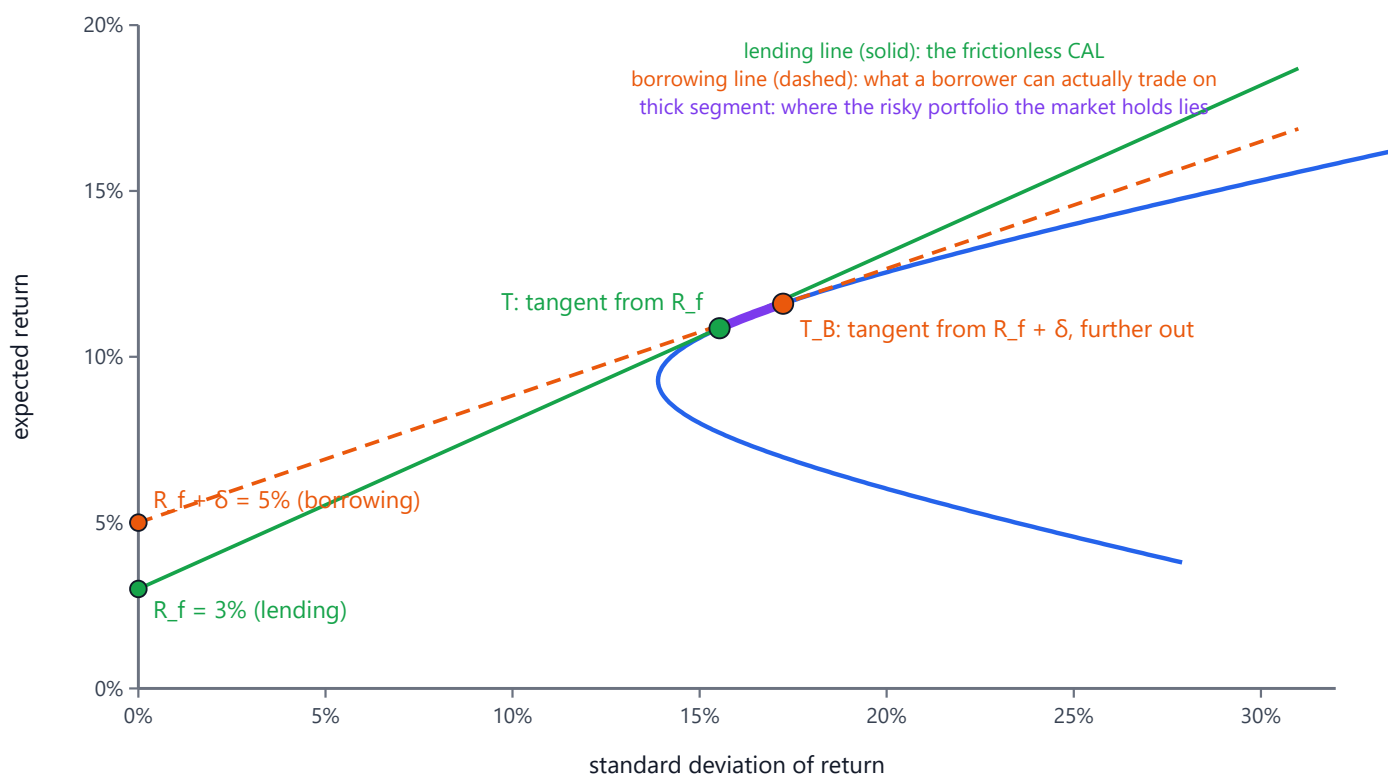
6.2 The drawing: a wedge lifts the borrower's line and moves its tangency out

The guest drew it by hand ("I'll have to draw this, just with the arrow") ⌚ 12:10 [transcript only]. The risk-free rate sits on the vertical axis; from it a tangent line reaches the frontier at the tangency point. "Now imagine that borrowing costs aren't free, right? There's like a wedge. It jumps up to here." The line one would love to draw is the lower one, "but you can't actually trade there"; one trades on a line that starts at the higher rate and meets the frontier "right there, right, at a further point" ⌚ 12:35. Leverage is not free: one posts collateral and is charged interest; and "there's lots of investors out there like pension funds or the Yale endowment that can't borrow, at least not, they can't leverage their portfolio. The university can borrow, but the endowment really can't." ⌚ 12:55. With a constraint or a cost on leverage, the line is "going to intersect somewhere on the efficient frontier where the market portfolio, what people can actually hold, is actually different than the tangency portfolio" ⌚ 13:14. The course's optional reading on the Kelly criterion draws the same line between lending and borrowing rates: a large, well-connected investor might borrow at a little above the Treasury-bill rate and other investors only at higher rates, and once the borrowing rate is set above the lending rate, the leverage the reading finds best for long-run growth in its S&P 500 example falls sharply (Thorp06 p. 26).

GAP Reconstructed from the lecture audio — 12:10

The drawing is not on any slide in the build and no photograph exists. The figure below is rebuilt from the description: a frontier, a lending line from R_f tangent at one point, and a borrowing line from $R_f + \delta$ tangent further out. The size of the wedge was not stated; the drawing uses an illustrative δ and says so.

The borrowing wedge, reconstructed from the description (illustrative delta = 2%)



Redrawn: the guest's borrowing-wedge drawing — the lending line from R_f touches the frontier at T; the borrowing line from the higher rate $R_f + \delta$ touches it further out, at T_B. The portfolio investors as a whole can hold lies on the frontier between the two tangency points. Reconstructed from the description; illustrative values, none the guest's.

How to read this: the lower line is the capital allocation line of the frictionless model, which an investor can only trade along by lending (to the left of T) or borrowing at R_f (to the right). If borrowing costs $R_f + \delta$ instead, the upper line is the best a borrower can do, and it touches the frontier at a riskier portfolio T_B. Nobody trades on the lower line's borrowing segment, so the risky portfolio the market as a whole holds is not T: it lies on the frontier between T and T_B, and its expected return is reached without the leverage the frictionless model assumes.

Interactive spec — Lending at R_f , borrowing at $R_f + \delta$: where the two lines touch the frontier

The sliders arrive with the site; the previewer shows the spec.

```
id: borrowing-wedge
title: Lending at  $R_f$ , borrowing at  $R_f + \delta$ : where the two lines touch the frontier
inputs:
  - {name: delta, label: "Borrowing wedge  $\delta$  (no value was given in the talk; 0 is the frictionless CAPM)", min: 0.00, max: 0.06, step: 0.005, default: 0.00, format: percent}
  - {name: Rf, label: "Risk-free (lending) rate", min: 0.00, max: 0.08, step: 0.005, default: 0.03, format: percent}
series:
  - {name: "frontier (illustrative two-asset frontier, as in frontier-cal)", x: "w*0.08 + (1-w)*0.14", x_values: "w from -0.5 to 1.5 step 0.02", y: "(w**2*0.15**2 + (1-w)**2*0.25**2 + 2*w*(1-w)*0.2*0.15*0.25)**0.5", swap_axes: true}
  - {name: "lending line from  $R_f$ , tangent at T", y: " $R_f + SR_T * x$ ", note: "SR_T = the highest Sharpe ratio on the frontier measured from  $R_f$ "}
  - {name: "borrowing line from  $R_f + \delta$ , tangent at T_B", y: " $R_f + \delta + SR_B * x$ ", style: dashed, note: "SR_B = the highest Sharpe ratio on the frontier measured from  $R_f + \delta$ ; T_B lies further out than T whenever  $\delta > 0$ "}
x_axis: {label: "standard deviation"}
y_axis: {label: "expected return"}
readout: ["the gap between T_B and T along the frontier as delta grows"]
```

6.3 The testable implication: a flatter line

"this is something that's testable, right?" The test: "the standard CAPM says you get this line that goes through the risk-free rate, but if there's a wedge between what the theory says you can borrow at and something that's a bit higher, you will, in fact, get a flatter line. That's kind of cool, right?"  13:26. The course's reading prints the version of the model that delivers this, Black's zero-beta CAPM, which replaces the risk-free rate by the expected return on a portfolio uncorrelated with the market (*FF04* p. 5):

$$\mathbb{E}(R_i) = \mathbb{E}(R_{ZM}) + [\mathbb{E}(R_M) - \mathbb{E}(R_{ZM})] \beta_{iM}$$

and says of it that the Black version "can accommodate a flatter tradeoff of average return for market beta" (*FF04* p. 20). The guest's wedge is a concrete reading of that intercept.

DERIVATION Full derivation (not done in class) — a borrowing wedge δ raises the pricing line's intercept by δ and lowers its slope by δ ; both lines pass through the market point

Write the zero-beta portfolio's expected return as the risk-free rate plus a wedge, $\mathbb{E}(R_{ZM}) = R_f + \delta$ with $\delta > 0$. This is the note's own reading of the guest's picture: when borrowing costs $R_f + \delta$ and some investors cannot borrow at all, the return that substitutes for the riskless rate in the pricing relation sits above R_f (explanation; the guest did not write this step).

$$\mathbb{E}(R_i) = R_f + \delta + [\mathbb{E}(R_M) - R_f - \delta] \beta_{iM}$$

Why this step follows: substitute $\mathbb{E}(R_{ZM}) = R_f + \delta$ into Black's equation (FF04 p. 5).

$$\mathbb{E}(R_i) = \underbrace{(R_f + \delta)}_{\text{intercept, up by } \delta} + \underbrace{[\mathbb{E}(R_M) - R_f - \delta]}_{\text{slope, down by } \delta} \beta_{iM}$$

Why this step follows: regroup the constant and the coefficient of β_{iM} . Against the Sharpe–Lintner line $R_f + (\mathbb{E}(R_M) - R_f) \beta_{iM}$ (FF04 p. 6; 04a p. 12), the intercept is higher by δ and the slope lower by δ : the line is flatter.

$$\beta_{iM} = 1 : R_f + \delta + (\mathbb{E}(R_M) - R_f) - \delta = \mathbb{E}(R_M)$$

Why this step follows: set $\beta_{iM} = 1$ in the line above; the δ terms cancel. So the two lines cross at the market's own point $(1, \mathbb{E}(R_M))$, as they must, since the market has beta one under either model (04a p. 13). Below $\beta = 1$ the flatter line lies above the CAPM's — low-beta assets beat their CAPM prediction — and above $\beta = 1$ it lies below. That is the shape of §5's picture.

Numerical check (explanation; the two rates are the course's, the wedge is illustrative (the note's own), none is the guest's): with 04a p. 9's $R_f = 5\%$ and $\mathbb{E}(R_M) = 12\%$, a wedge of $\delta = 2\%$ turns the line $5\% + 7\% \beta$ into $7\% + 5\% \beta$. At $\beta = 0.5$ the CAPM predicts 8.5% and the wedge line 9.5%; at $\beta = 1.5$, 15.5% against 14.5%; at $\beta = 1$ both give 12%.

6.4 Bonds too

"So that should not only be true in stocks, but it should be true in bonds as well. And you can see that, this is actually for bond portfolios, you can see the same thing happens in bonds as well"  13:51 [transcript only].

GAP Not recoverable — the bond-portfolio chart

The chart's content is not on the recording beyond the claim that bond portfolios show the same flattening. No course source covers it.

Self-check. Why does a borrowing cost make the empirical line flatter rather than simply shifting it up?

Because the market point is pinned: the market has beta one and earns $\mathbb{E}(R_M)$ whatever the borrowing rate. A higher effective riskless return ($R_f + \delta$) raises the intercept, and a line that starts higher but still passes through the same market point must have a smaller slope — by exactly δ , as the derivation shows. A pure upward shift would require the market itself to earn more.

7. The prescription: buy the cheap, short the dear

🕒 14:05

This section records the guest's turn from testing to trading, in the course's own definitions of alpha and short sales, and flags one phrase, "net risk neutral", that means something narrower than it sounds.

Theories prescribe. "the nice thing about finance is they give you a prescription for what to do if the model is false" 🕒 14:05 . Back to the picture: if you believe the CAPM, everything should fall on the 45-degree line, and it doesn't; so, as someone working at AQR, "Which of these assets look overpriced and underpriced to you? Which ones look like they're offering better returns than they should?" 🕒 14:27 . The room's reply is not decoded; the guest gives the answer: "Yeah, you want to buy. Be confident." 🕒 14:51 . Stocks or portfolios offering more return than their risk warrants "look like a good deal to me. I'll buy those all day long" 🕒 14:56 ; and at the same time, those offering "much lower returns than they should, those prices are too high. I want to short those guys as well" 🕒 15:03 . "everyone knows what short-selling is? You sell something you don't own?" 🕒 15:24 — the course's definition word for word, "Short sale: sell something you don't own", with its mechanics, borrowing the security and later replacing it, and its practical requirements, that the proceeds stay with the broker and the seller posts margin (01b p. 13; the Lecture 2 note's §10.2).

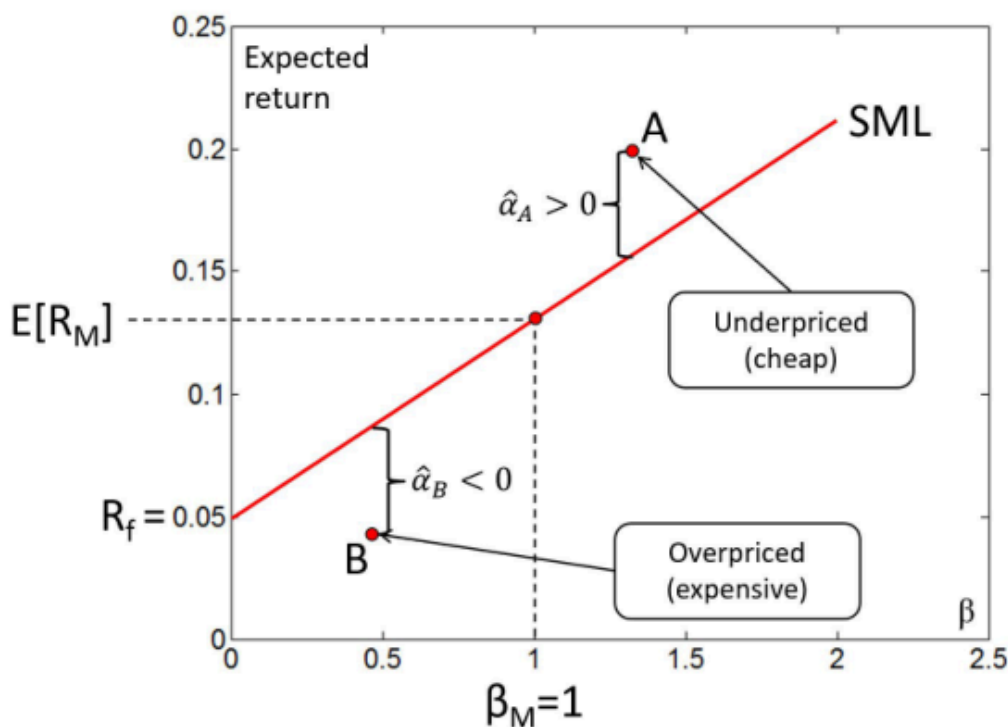
The course's name for "better returns than they should" is alpha (04a p. 27; the Lecture 9 note's §2.2):

$$\alpha_i = \mathbb{E}[R_i] - R_f - \beta_i [\mathbb{E}[R_M] - R_f]$$

An asset above the SML has $\alpha > 0$ and is "Underpriced (cheap)"; one below it has $\alpha < 0$ and is "Overpriced (expensive)" (04a p. 28); "Fund managers try to buy positive-alpha stocks and (short) sell negative-alpha stocks; the CAPM predicts that all alphas are zero" (04a p. 27). The logic connecting price and expected return is the SML's: a high price today means a low expected return, so offering too low a return and being priced too high are one statement (04a p. 14). The guest's question is the course's own, printed under its decile chart — "Which stocks do you want to buy? And to sell?" (04b p. 10) — and is worked in the Lecture 9 note as that page's exercise, with the answer buy the low-beta portfolios and short the high-beta ones; the link block under *Worked problems* restates it.

Stock Selection

- Does a positive or negative alpha imply that there is an arbitrage?
 - An $\alpha \neq 0$ represents an abnormal return



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04a p. 28 — the course's own picture of alpha: A above the SML, "Underpriced (cheap)", and B below it, "Overpriced (expensive)". Related, not shown on 8 October.

How to read this: the line is the SML; the vertical distance from a dot to the line is its alpha. A earns more than its beta warrants and is a buy; B earns less and is a sell or a short. The guest's prescription is the two dots at once: long A, short B.

The combination. "If I do that in the right combination, I make money, and I can actually balance the risks so that I'm net risk neutral. That's cool, right? That sounds like a hedge fund strategy, right? And it is."

🕒 15:35 . The course said the same of its stock-selection page: "I just described how hedge funds operate" — "produce a model of returns; try to beat the model" (04a p. 29; the Lecture 9 note's §3).

WARNING Said in class vs written — 15:35: "net risk neutral" means zero net beta, not no risk, and not an arbitrage

"net risk neutral" 🕒 15:35 is the guest's shorthand for a position whose **market exposure nets to zero**: the long and short legs are sized so that their betas cancel (§8.1 makes the sizing exact). It does not mean the position is riskless — each leg keeps its own idiosyncratic shocks, and the alphas it collects are averages that can disappoint — and it is not the course's "risk-neutral probabilities" of a later block (07b p. 17), which are a pricing device, not a trading position. Nor is it an arbitrage. The course asks on the same pages, "Does a positive or negative alpha imply that there is an arbitrage?" (04a p. 28; 04b p. 4), and the Lecture 9 note answers it (§2.4) with the workbook's own question on exactly this pair of trades: a long-positive-alpha, short-negative-alpha position can lose money, so it is a bet with a favourable expected payoff, not a sure profit.

Self-check. The guest's two trades are to buy what offers more than its risk warrants and to short what offers less. Rewrite both in terms of alpha and in terms of price.

Buy $\alpha_i > 0$: the asset's expected return exceeds $R_f + \beta_i(\mathbb{E}[R_M] - R_f)$, so its price is too low for its risk (cheap). Short $\alpha_i < 0$: the expected return falls short of the SML, so the price is too high (expensive). Under the CAPM every alpha is zero and neither trade exists (04a pp. 27–28).

8. Betting Against Beta

🕒 15:48

This section builds the one strategy the talk works out in numbers — long low-beta stocks, short high-beta stocks, levered until the betas match — fixes its arithmetic in the course's formulas, and records the evidence the guest described and the diversification argument that follows from running it in many markets.

8.1 The paper and the strategy

The paper is *Betting Against Beta*, by two AQR colleagues, one former: Andrea Frazzini, "a graduate of this institution in the PhD program at the Econ Department", and Lasse Pedersen 🕒 15:48 [the paper is not in

the course materials]. The strategy is the prescription of \$7 applied to the too-flat line of \$5: "if I buy the low beta stocks and short the high beta stocks" (15:48) and adjust the two sides so that each carries the same risk, "I can make what looks like really good money on a zero investment strategy" (16:18). Low-beta stocks offer more return than their beta warrants and high-beta stocks less — the recording garbles the sentence, and the course's own numbers for it are the +0.3% and −7% alphas of the lowest and highest beta deciles on its 1927–2016 chart (04b pp. 10–11; §5.3). Balancing the risks is the problem, because "The low-beta stocks, on average, are lower risk than the high-beta stocks" (16:42); and then the line that sums up the evidence: "So beta, by the way, predicts risk really well. It just doesn't predict returns all that well." (16:47). The course's variance split gives a reason for the first half (*explanation: the guest gave none*) — a stock's total variance is $\beta_i^2 \sigma_M^2 + \sigma_\varepsilon^2$, so beta measures its systematic risk, "the right measure of risk within a portfolio (not volatility!)" (04a pp. 18–19; the Lecture 8 note's §14) — and the too-flat line is why the second half holds.

8.2 Making the two legs even: the leverage match

"So, how do I make those risks the same? What's the key? It's the same thing that might be causing this problem in the first place." (16:49) — that is, the leverage of \$6. In numbers: a long portfolio whose average beta is about 0.5 and a short portfolio whose average beta is about 1.2 (the guest's betas, the same in both recordings). "How do I make those even?" (17:15) A student: "You buy more of the lower beta stocks relative to the company you're shorting." (17:19). The guest: "That's right. I buy more of the low-beta stocks in exact proportion to match that 1.2 beta, so I'm evenly matched. And how am I going to do that if I don't have a lot of money? I'm going to borrow it and use leverage. That is a classic hedge fund strategy." (17:24).

The arithmetic is this meeting's one numerical in-class exercise, worked in full under *Worked problems* (the question at 16:49). Its result: a position's beta is the dollar-weighted sum of its legs' betas, with the short leg negative — $\beta_\Pi = x_L \beta_L - \beta_H$, which follows from beta's definition and the covariance rule the exam's sheet prints (*formula sheet p. 1, items 6 and 10*) and is the reading's statement that a portfolio's beta is the weighted average of its assets' betas (FF04 p. 8, footnote 4; printed with a typo the Lecture 9 note rules on). Setting it to zero with the guest's betas gives

$$x_L \times 0.5 - 1.2 = 0 \Rightarrow x_L = \frac{1.2}{0.5} = 2.4$$

buy \$2.40 of the low-beta portfolio for every \$1 of the high-beta portfolio shorted, paid for with **\$1 of the investor's own money and \$1.40 borrowed** — the short sale's \$1 pays for none of it, because "Short sale proceeds must remain with the broker" and the short seller must also post collateral (01b p. 13) — so the long leg is a levered purchase with loan-to-value $1.4/2.4 = 58.33\%$, margin 41.67% and leverage 2.4 (03c p. 10's definitions). The same proportion appears if each leg is scaled to a beta of one — $1/0.5 = 2.0$ long against $1/1.2 = 0.8333$ short, a ratio of 2.4 — which is how the guest described the paper's test. With

equal dollars on each side instead, the position would have beta $0.5 - 1.2 = -0.7$, a net short on the market, which is the risk the matching removes. Once matched, the expected payoff is $2.4\alpha_L - \alpha_H$: the alphas, and nothing from the market. Zero beta is not zero risk and not an arbitrage (§7's Warning); the worked problem says so in its last step.

WARNING Said in class vs written — 16:18: "a zero investment strategy"

The guest's words, as said: the trade makes "what looks like really good money on a zero investment strategy" ⌚ 16:18. The course's account of short sales in practice differs: the proceeds stay with the broker and the short seller posts collateral (01b p. 13), so the short leg cannot pay for the long leg and the position ties up the investor's own money — in this note's arithmetic the long leg's \$1 down payment, plus collateral the page does not size. The note does the arithmetic on the course's terms and keeps the guest's phrase as the guest's.

8.3 The paper's test: alpha by beta decile

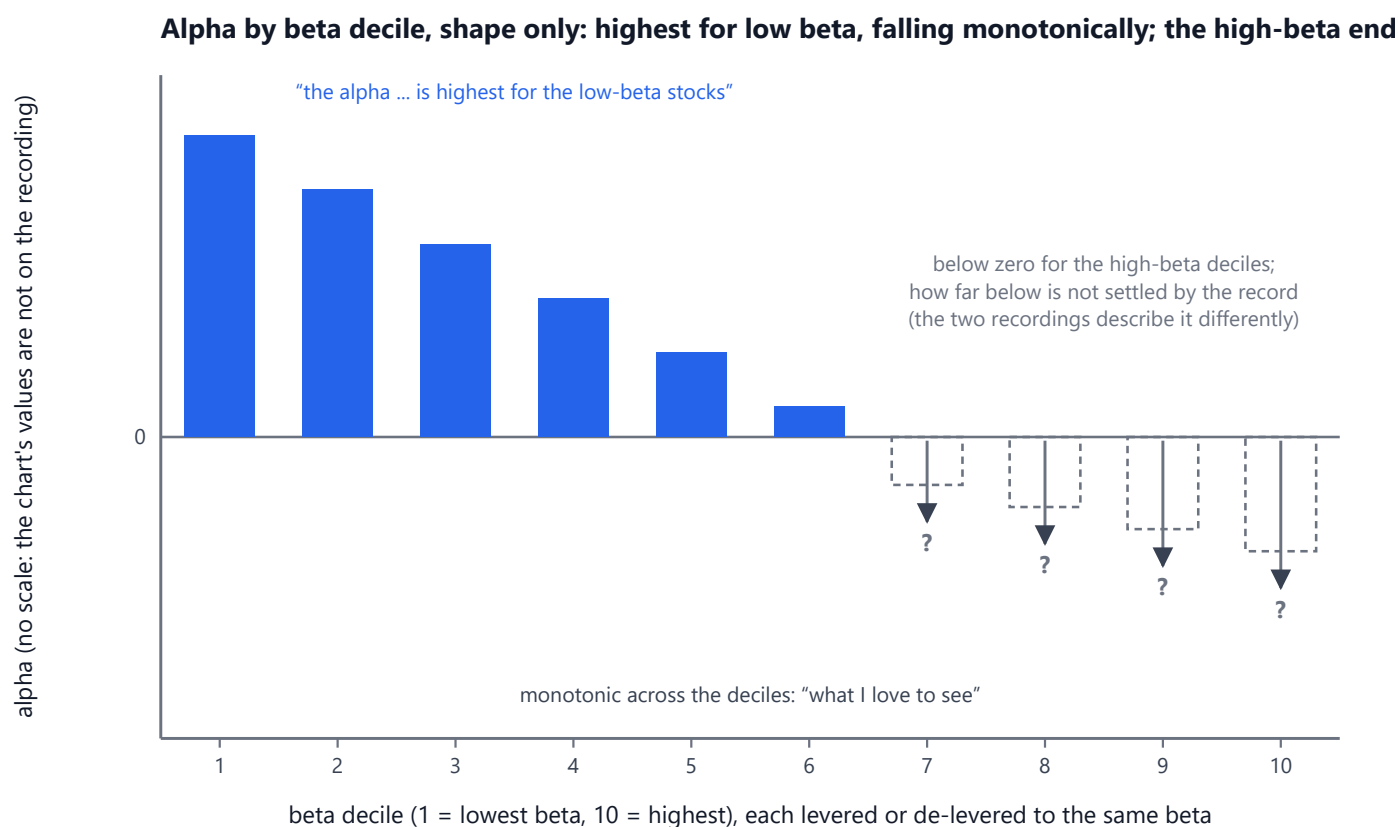
"So, that betting against beta strategy is what these guys tested in their paper" ⌚ 17:41 [transcript only]. Sort US stocks into deciles from lowest to highest beta — the course's own "portfolio sorting by beta" (04b p. 8) — then look at their risk-adjusted returns with the beta netted out, "so I neutralize it by either leveraging or de-levering, so I get the equal dollars here" ⌚ 18:01. The finding, as the guest described the chart: the alpha, "the excess return, is highest for the low-beta stocks", and lower for the high-beta stocks — how much lower is where the two recordings differ (the Warning below). What the guest stressed is the pattern across the deciles: "this is what I love to see, monotonicity. That tells me I didn't probably get lucky. There's something going on, right? You don't see something systematic like that." ⌚ 18:07. And the method it illustrates: "This is what a quant manager does. Look for systematic patterns based on an economic reason, in my opinion, and then exploit that." ⌚ 18:26.

WARNING Said in class vs written — 18:07: the size of the high-beta deciles' alpha

The two recordings render the clause after "is highest for the low-beta stocks" differently: one hears "maybe negative for the high-beta stocks", the other "big and negative". The chart is not in the build, so the record does not settle which was said, and the note states neither. What both agree on: alpha is highest for the low-beta deciles, falls monotonically across the deciles, and is lower at the high-beta end. The redraw below shows that shape and deliberately gives the high-beta end no size. **The course's chart (04b p. 10) is a different chart** — different years, raw returns rather than beta-neutral ones — and its -7% for decile 10 is never transferred to the guest's.

GAP Not recoverable — the Betting Against Beta decile chart

The chart's values, sample and the construction of the beta-neutral deciles beyond the words above are not on the recording and the paper is not in the course materials.



Redrawn: alpha by beta decile as the guest described the chart — highest for the lowest-beta decile, falling monotonically across the deciles. Shape only: the vertical axis has no scale, and the high-beta end's level is left open because the two recordings describe it differently. Reconstructed from the description; not the guest's chart.

How to read this: each bar is one decile of US stocks sorted by beta, after each decile has been levered or de-levered to the same beta; its height is the alpha, the return left after the market's contribution is removed. The bars step down from left to right without a reversal — the "monotonicity" the guest prized — and cross zero somewhere in the middle. Where the right-hand bars end is not drawn, because the recording does not say.

8.4 Fifteen years, forty markets, and diversification across strategies

AQR has run the strategy "for at least 15 years, I think about 15": buy the low-beta deciles, short the high-beta ones, "and it's actually been one of our most profitable" trading strategies — "Super simple, right? Yeah, there's bells and whistles to it, but it's the basic idea" 🕒 18:35 [the guest's account of the firm]. Then the step further: it should work in global stock markets too, and "This is across about 40 different global

markets, same pattern" ⌚ 18:54 *[transcript only]*; also in Treasury securities, credit indices and credit default swaps, corporate debt, equity index futures across the world, commodities, "and even country bonds" ⌚ 19:04 . The paper also tries currencies, "which doesn't make any sense to me" — "I don't understand what a beta of a currency is" — and "they don't get much there" ⌚ 19:17 .

"and you guys learned about diversification, right?" ⌚ 19:27 . The strategy exploits "some mispricing or some distortion in markets. We believe that CAPM is kind of right, but there's these distortions" ⌚ 19:33 , and running it in 40 countries, in fixed income, credit, equity futures and commodities "works in that way as well" ⌚ 19:58 — as diversification. The guest's version of the course's result: a stock and a bond diversify each other, "but if I buy a market-neutral hedge fund strategy designed for stocks and do another similar one in bonds, they also diversify each other" ⌚ 20:01 . In numbers, the Sharpe ratio of the strategy in any one of these markets is "maybe about 0.4, 0.5", and run in "roughly eight different markets, all of these are about the same, and roughly independent, you easily more than double that Sharpe ratio" ⌚ 20:01 . "That's kind of the math that works for you. That's what a quant does." ⌚ 20:38 — someone "trading tens of thousands of securities with small edges in each of them, then when you add them up, they add up to a lot. Very different than making a bet on a single stock and getting it right." ⌚ 20:41 .

WARNING Said in class vs written — 20:01: whether the 0.4–0.5 is net of costs

Before "maybe about 0.4, 0.5" one recording hears "that would cost us" and the other recording renders the words as net of cost is. The figure 0.4–0.5 is agreed; whether the guest said it is net of trading costs is not settled by the record, and the note does not say either. The doubling arithmetic below does not depend on it.

The course's formula for "the math that works for you" is the equally weighted portfolio of independent returns, $\sigma_p^2 = \frac{1}{N} \times (\text{average variance})$ (03d p. 8; the Lecture 7 note's §11.2), applied to strategies rather than stocks.

DERIVATION Full derivation (not done in class) — **N equal, independent strategies: the combined Sharpe ratio is $\sqrt{N}$ times one strategy's; eight markets at 0.4–0.5 give 1.13–1.41, "more than double"**

Take N strategies, each with the same expected excess return μ and the same volatility σ , so each has Sharpe ratio $SR = \mu/\sigma$ (03c p. 7), and independent of one another, as the guest assumed ("roughly independent"). Hold them in equal weights $1/N$ (explanation: the guest's "all of these are about the same" is the equal-weights case).

$$\mathbb{E}[R_p] - R_f = \sum_{i=1}^N \frac{1}{N} \mu = \mu$$

Why this step follows: the portfolio's mean is the weighted sum of the means (formula sheet p. 1, item 7), and N equal weights $1/N$ on the same μ sum to μ .

$$\sigma_p^2 = \frac{1}{N^2} \sum_{i=1}^N \sigma^2 = \frac{\sigma^2}{N}$$

Why this step follows: with independent returns every covariance is zero, so the variance formula (formula sheet p. 1, item 8) keeps only the $w_i^2 \sigma_i^2$ terms, N of them at $(1/N)^2 \sigma^2$; this is 03d p. 8's line with every variance equal.

$$SR_N = \frac{\mu}{\sigma/\sqrt{N}} = \sqrt{N} SR$$

Why this step follows: divide the mean excess return by the volatility $\sigma/\sqrt{N}$; the numerator did not change and the denominator fell by $\sqrt{N}$ — the guest's "the denominator gets smaller and smaller and the numerator is linear" of §12.

Numerical check, the guest's inputs: $\sqrt{8} = 2.828$; at 0.4 per market, $SR_8 = 0.4 \times 2.828 = 1.131$; at 0.5, 1.414. "Easily more than double" holds with room to spare; the excess over doubling is what eight independent markets give beyond four. The assumption that carries the result is independence — §12.3 shows what an average correlation does to it.

Self-check. Why must the long leg of Betting Against Beta be bigger than the short leg, and what pays for the difference?

Because beta is a per-dollar exposure and the long leg's is smaller: to cancel \$1 of a 1.2-beta short, one needs $1.2/0.5 = 2.4$ dollars of the 0.5-beta long. The investor's own \$1 pays for part of it and the remaining \$1.40 is borrowed — the short sale's \$1 stays with the broker (01b p. 13) — which is why the guest calls it a leverage trade and why investors who cannot borrow cannot run it (§9).

Self-check. Four independent markets each run the strategy at a Sharpe ratio of 0.5 (*four markets illustrative, the note's own; 0.5 is the guest's upper figure*). What is the combined Sharpe ratio, and what would it be if the four were perfectly correlated?

Independent: $\sqrt{4} \times 0.5 = 1.0$. Perfectly correlated: the combined volatility is the same as each one's, so the Sharpe ratio stays 0.5 — diversification across strategies works only because they are not the same bet.

9. Who is leverage-constrained, and who is not

🕒 20:57

This section states the guest's account of which investors can and cannot lever, and derives the two facts the account rests on: that the average beta held must be one, and that an investor who cannot borrow and wants more return must hold a beta above one.

"are we the first ones to figure this out? No" 🕒 20:57 : the pattern follows from who is constrained by leverage [*the guest's account; the institutional facts have no course source*]. Individual investors "can't borrow cheaply, right? We can't even buy on margin of Fidelity if we have an account. It's expensive" 🕒 21:10 — buying stock on margin is the course's leverage of 03c pp. 10–11 — and "Many mutual funds are simply not allowed to use leverage" 🕒 21:21 . "Because they can't use leverage, in order to get their risk-return" profile "that they want, they go further out on the efficient frontier, they take on more risk because they can't use leverage. So, on average, mutual funds and individual investors hold stocks that have a beta greater than one. The average beta should be one." 🕒 21:28 . On the other side are "the firms that actually have access to cheap leverage" 🕒 21:46 : leveraged-buyout firms, which take public firms private with a great deal of leverage, "the kinds of stocks that they buy or apply leverage to, they have way lower betas" 🕒 21:53 ; and Berkshire Hathaway, whose owner the room was asked to name — "Warren Buffett buys stocks that have betas that are like 0.6, 0.7, way lower than one, and then leverages them up" 🕒 22:19 , using "one of the cheapest sources of leverage in the world" 🕒 22:19 , Buffett's insurance company, which the recording does

not name clearly. The aside: Buffett "is out there on TV saying leverage is an evil thing" while being "the biggest leverage user in the world, as an individual"  22:37 ; the guest doubts Buffett believes it, "another discussion for another day"  22:46 . The course's decks name Buffett once, as the example attached to the value spread (*04b p. 12*); the guest's later verdict on Buffett's record is \$14.

DERIVATION Full derivation (not done in class) — the market-weighted average beta is exactly one, and an investor who cannot borrow and wants a mean above the market's must hold a beta above one

The average beta is one. The market portfolio's return is $R_M = \sum_i w_M^i R_i$ (04a p. 6).

$$\text{Cov}[R_M, R_M] = \text{Cov} \left[\sum_i w_M^i R_i, R_M \right] = \sum_i w_M^i \text{Cov}[R_i, R_M]$$

Why this step follows: covariance is linear in its first argument (formula sheet p. 1, item 6); this is the reading's footnote 3 line by line (FF04 p. 6).

$$1 = \sum_i w_M^i \frac{\text{Cov}[R_i, R_M]}{\text{Var}[R_M]} = \sum_i w_M^i \beta_i$$

Why this step follows: divide both sides by $\text{Var}[R_M] = \text{Cov}[R_M, R_M]$ and use beta's definition (04a p. 12). So the value-weighted average beta of all stocks is one, the market's own beta (04a p. 13). If some investors hold betas above one on average, others must hold betas below one: the two groups the guest names.

Why the constrained go "further out". An investor who can borrow at R_f reaches any expected return along the capital market line by leveraging the market, $\mathbb{E}[R_p] = R_f + w(\mathbb{E}[R_M] - R_f)$ with $w > 1$ (04a p. 9's "Borrowing" segment; 03c p. 11). One who cannot borrow is confined to $w \leq 1$ and the frontier of risky assets. To reach a mean above $\mathbb{E}[R_M]$ without borrowing, the investor must hold a portfolio of risky assets with $\mathbb{E}[R_p] > \mathbb{E}[R_M]$; by the security market line (04a p. 12) such a portfolio has

$$\beta_p = \frac{\mathbb{E}[R_p] - R_f}{\mathbb{E}[R_M] - R_f} > 1$$

Why this step follows: solve the SML for the beta of a portfolio with mean $\mathbb{E}[R_p]$; a mean above the market's gives a beta above one. This is the course's logic read backwards: high-beta stocks are "bad at diversifying risk" and so carry a high expected return (04a p. 14), which is exactly what an investor barred from leverage must buy to raise expected return. (explanation)

Numerical check (the guest's betas): an investor with cheap leverage who buys stocks of beta 0.6–0.7 and wants the market's exposure levers them by $1/0.7 = 1.429$ to $1/0.6 = 1.667$ — holding \$1.43 to \$1.67 of stock per dollar of own money.

Self-check. If mutual funds and individuals hold betas above one on average, who holds the low-beta stocks, and what does Betting Against Beta assume about them?

Someone must, since the value-weighted average beta is one. The guest's answer is investors with cheap leverage — leveraged-buyout firms, Buffett — who buy low-beta stocks and lever them. The strategy assumes that the constrained investors' demand for high-beta stocks pushes their prices up (low alpha) and leaves low-beta stocks cheap (high alpha), and that a levered investor can collect the difference.

10. The history of the low-beta finding

🕒 22:53

This section records the guest's story of where the finding came from and why it lay unused, as told; none of it has a course source beyond the names.

The low-beta effect "was one of the first sort of anomalies" 🕒 22:53 . Fischer Black found it in 1972 in the paper with Jensen and Scholes, while working for Wells Fargo, which, in the guest's telling, "were the first index manager, well before the S&P 500, well before Vanguard exists" 🕒 23:08 , offering one of the first stock-market index products in the late 1960s and early 1970s 🕒 23:15 [*the guest's account; the dating of the S&P 500 is as said*]. What Wells Fargo offered was a simple market-cap-weighted index, the CAPM's prescription — hold the market, with value weights (04a pp. 6, 8). Black "was adamant that what they should do is tilt away from the high-beta stocks and towards low-beta stocks, because that's what the data is telling them" 🕒 23:30 , was outvoted, and left the meeting angrily 🕒 23:39 . The guest's source is someone who was in the room, heard as Wayne Winston (the firm named is not decoded). Also present: Myron Scholes; David Booth, who founded Dimensional Fund Advisors, "currently a trillion-dollar money manager" 🕒 23:51 ; and Jack Bogle, who founded Vanguard 🕒 24:02 (the recording garbles both names; the written sources spell them) — the course's own deck quotes "John Bogle (founder of Vanguard)" on asset allocation (03c p. 4), and 05a p. 23 names the Vanguard S&P 500 Index Fund as its example of a passive fund. "A bunch of these guys went off to run and start index funds" 🕒 24:04 , and the result "lay dormant for a long time until" Frazzini and Pedersen "looked at it in 2010 or so, and then hedge funds started using it" 🕒 24:11 ; it became one of AQR's biggest sources of profit. "Such a simple strategy." 🕒 24:21 .

Self-check. Black wanted the index tilted toward low-beta stocks in the early 1970s. Why is that the same trade as Betting Against Beta, forty years later?

A tilt away from high-beta toward low-beta stocks within a market-cap index is a long position in low-beta stocks and a short position (relative to the index) in high-beta ones. Betting Against Beta takes the two legs explicitly, matches their betas with leverage, and runs the trade market-neutral; the economic bet — the SML is too flat — is the one Black read off the 1972 data.

11. Other anomalies: value, momentum, quality, carry, trend, accruals, sentiment

🕒 24:22

This section lists the other gaps between the CAPM and the data that the guest named, each with the course's own written version where one exists, and the guest's claim about them that the course's deck contradicts.

"What else fills the gap between what the CAPM says and what the data says?" 🕒 24:26 . The first answer was §56–9's: betas mispriced because of leverage, which the guest promised to come back to. The rest follow.

11.1 Value

Value investing, "another thing that Warren Buffett made famous" 🕒 24:39 : "Buying something that looks cheap based on some ratio of fundamental value to market value" 🕒 24:43 — book-to-market or earnings-to-price, "those types of things" 🕒 24:49 . "Fama-French made this famous" 🕒 24:52 , and value strategies became the main strategy of a fund-management firm the recording does not name clearly; David Booth "was a student of Eugene Fama", and the firm moved from California to Austin, Texas, "because of taxes" 🕒 25:01 . The course's version is 04b p. 12 — "Fama & French (1992): Firms' returns depend on their size (market cap) and book-to-market (B/M) ratio"; high book-to-market stocks are value stocks and low ones growth stocks, with the 6.25%-a-year value spread of \$4 — and the value factor R_{HML} , long high book-to-market and short low, of 04b p. 16 (*the Lecture 9 note's §6.1 and §7.1*). The course's return-predictability detour names the same kind of ratios, price–earnings and dividend–price (05a p. 7, not yet taught).

11.2 Momentum, and "defensive"

"Momentum investing is the idea that recent winners tend to beat recent losers." 🕒 25:10 . In the same breath the guest gave Betting Against Beta its house name: AQR calls it a "defensive strategy" 🕒 25:18 —

"we're talking about buying low-risk assets and shorting high-risk assets" 🕒 25:26, "It's low-risk in that sense." 🕒 25:31.

WARNING Said in class vs written — 25:26: a clause the two recordings render differently

Between "shorting high-risk assets" 🕒 25:26 and "It's low-risk in that sense" 🕒 25:31 one recording hears "within 11-year line, right?" and the other "but then leveraging them up." 🕒 25:26. The first has no support anywhere and is not stated; the second repeats a point the guest made in clear words at 🕒 17:24, that the long leg is levered, which this note takes from there. Neither rendering is treated as what was said here.

"just like defensive investing, both value and momentum don't just work for stocks, in which they were originally studied, but bonds, currencies, commodities. You actually see it in private equity as well. It's one of the key indicators of private equity, value investing." 🕒 25:33. The famous academic study of momentum is Jegadeesh and Titman (1993) — the reading names "the momentum effect of Jegadeesh and Titman (1993)" (*FF04 p. 17*) — and the guest's origin story for AQR: a PhD student of Fama's (unnamed; the recording garbles the name) was studying momentum combined with value in 1994, skipped the academic job market for Goldman Sachs "and start trading this stuff", made money, "decided that you didn't want to make all the Goldman guys rich, you want to make yourself rich" 🕒 26:02, and founded AQR 🕒 26:16 [*the guest's account*].

Horizons, by source. The guest gave none. The course's deck defines Carhart's momentum factor on the past twelve months — "sort stocks by their return over the previous 12 months", go long the winners with the highest past-twelve-month return and short the losers with the lowest — and adds a short-term reversal factor, stocks with high returns over the past month tending to underperform (*04b p. 19*; *the Lecture 9 note's §7.4*). The market-efficiency deck lists "Momentum (last 3 to 12 months)" and "Long-term reversal (3 to 5 years)" among its anomalies (*05a p. 15*). The Fama–French reading measures the winners over the same span: stocks that beat the market over the last three to twelve months tend to keep doing well for the next few months, and the losers to keep doing poorly (*FF04 p. 17*). The Nobel background describes Jegadeesh and Titman's strategy as buying stocks that have done well and selling those that have done poorly over the past three to twelve months (*Nobel13 p. 43*), and Shiller's survey reports their finding that stocks with exceptionally high six-month returns beat those with exceptionally low six-month returns "by 12 percent over the following year", with the momentum reversing over longer periods (*Shiller03 p. 14*). These are different cuts of one effect, not a conflict.

11.3 Quality

Quality investing: "Profitable, stable, well-run firms earn higher risk-adjusted returns than junk firms." 🕒 26:24. It sounds obvious, and the guest's correction is the talk's most quotable line: "Don't confuse a

good company with a good investment, okay? If it's a great, well-run company, but it's overpriced, still a terrible investment, right?" (26:35). A second sentence follows, on a company that is still a great investment if bought at the right price; the word describing that company is one the two recordings hear differently, so whether the sentence is the maxim's mirror image is not settled (the Warning below). "That's been Warren Buffett's speech, for instance." (26:49) The anomaly is that the market does not price quality fully: profitable, stable, well-run firms "are still underpriced. The market doesn't fully appreciate them" (26:51), and junk firms are "still overpriced. So you can actually make money from that." (27:00). There is a paper on it — the recording garbles the authors, so they are not named — and a professor at Rochester who consults with a fund firm (heard as "Dimension" (27:08)); "a lot of this work ends up finding its way into markets" (27:18). The course has no page on quality. Its version of the maxim is the logic of the security market line, that a high price means a low expected return (04a p. 14), and the holding-period return's dependence on the price paid (02b p. 12): the return on a company is a return on its price, however good the company.

WARNING Said in class vs written — 26:44: the second half of the maxim

"And if it's a" — one recording hears "cracking", the other recording renders the word as crappy — "company, it's still a great investment if you can get it at the right price." The first word would make both halves praise the company; the second would make the sentence the mirror image of the one before it. The record does not settle the word, and the note quotes neither and adopts neither reading.

11.4 Carry

"Defensive, I already talked about. And then I've got something outside of stocks, which is carry. This is high-yielding assets outperform low-yielding assets." (27:22). The classic application is currencies; it also shows up in bonds, commodities and even equities, where it resembles value: "High dividend yield, paying stocks, that's a high carry." The guest's definition: "I think of carry as what's the return you get if prices don't change" (27:34) — popular in private equity "because prices never change, they've never been traded" (27:34), so cash flow or interest is computed from carry. The currency version, in the second recording's words: "If you buy one currency and short the other, you're earning the interest rate of the currency. That's the carry, as if you're putting the money in one market by borrowing it from the other." (28:05). The puzzle: "you don't just earn the interest rate differential, it actually predicts the future price change, and that's the part that we're trying to understand as a field" (28:16). The Nobel background states the theory carry violates: under the expectations hypothesis "the so-called carry trade, which involves borrowing in a low-interest currency and investing in a high-interest currency, should not yield positive excess returns, as the higher interest rate should be offset by currency depreciation" — and the hypothesis is rejected in the data (Nobel13 p. 22); the same reading lists carry, with momentum, among the strategies with very high Sharpe ratios (Nobel13 p. 30).

DERIVATION Full derivation (not done in class) — the return if prices don't change: for a stock it is the dividend yield; for a currency pair it is the interest differential, and it is an excess return only if the exchange rate fails to move as the expectations hypothesis says

A stock. The course's holding-period return is $1 + \text{HPR} = V(T)/V(0)$ (02b p. 12). For a share bought at P_0 that pays a dividend D_1 and sells for P_1 ,

$$\text{HPR} = \frac{P_1 + D_1}{P_0} - 1 \xrightarrow{P_1=P_0} \frac{P_0 + D_1}{P_0} - 1 = \frac{D_1}{P_0}$$

Why this step follows: set the end price equal to the start price, as the guest's definition says, and the fraction collapses to the dividend yield. So "high dividend yield ... that's a high carry" is exact (explanation).

A currency pair. Let S_0 be today's spot rate (domestic currency per unit of foreign), i_d and i_f the two one-period interest rates with $i_d > i_f$. Borrow one unit of the low-rate foreign currency, convert it at S_0 , invest at i_d ; at the end, convert back at S_1 to repay $1 + i_f$ units of foreign currency.

$$\text{profit (domestic)} = S_0(1 + i_d) - S_1(1 + i_f)$$

Why this step follows: the domestic deposit grows to $S_0(1 + i_d)$; the foreign debt of $1 + i_f$ costs $S_1(1 + i_f)$ in domestic currency at the end-of-period rate.

$$S_1 = S_0 : \text{profit} = S_0(i_d - i_f)$$

Why this step follows: substitute $S_1 = S_0$ and cancel the $S_0 \times 1$ terms. If the exchange rate does not move, the trade earns the interest differential on the amount converted — the guest's "carry".

$$\mathbb{E}[S_1] = S_0 \frac{1 + i_d}{1 + i_f} \Rightarrow \mathbb{E}[\text{profit}] = S_0(1 + i_d) - S_0 \frac{1 + i_d}{1 + i_f}(1 + i_f) = 0$$

Why this step follows: this is the expectations hypothesis of Nobel13 p. 22 — the high-rate currency is expected to depreciate by exactly the differential — substituted into the profit; the two terms cancel. The anomaly is that in the data the expected profit is not zero, and the guest's "it actually predicts the future price change" is the statement that the differential forecasts S_1 in a direction the hypothesis rules out. (explanation)

11.5 Trend, accruals, sentiment

Three more, briefly, as the guest gave them  28:27 . **Trend following** is "a cousin of momentum"  28:29 : instead of going long winners and short losers across assets, one follows the trend in each asset on its own;

it "works phenomenally well among futures contracts" (28:36), and is "a whole business called managed futures investing" (28:39). **Accounting anomalies** such as accruals: "firms that are fudging with their accruals and changing their income statement, that's usually a bad signal" (28:46) — the accrual method itself is on the course's optional financial-statements deck (z06 p. 3). **Sentiment measures**, "the latest thing" (28:54): "patterns tied to investor over and under reaction, and different preferences that you can try to measure" (29:02), and measurement keeps improving as data grow — text, "film and images, as well as sound bites" (29:12), not just the words of earnings calls "but the tone of voice and all that stuff" (29:20). The course's own exhibits of over- and under-reaction are its event studies: the continued drift after an earnings surprise, read as under-reaction, and the overshoot and fall-back after a CNBC mention (05a pp. 13–14, *not yet taught*); the Nobel background links under-reaction to momentum and over-reaction to reversals (Nobel13 p. 43).

11.6 The same economics, new measurement

These factors "help explain the gaps between our starting model of the CAPM and what we see in the data" (29:23) — the course's own gap-filling is 04b pp. 12–19 and FF04 pp. 14–17, the Lecture 9 note's. The ideas "have been around for decades, but the way we apply them today is using current technology" (29:35): AQR's measures of value go beyond book-to-price — still used — to adjustments for intangibles and the digital economy and "clever text measures" (29:52) of value against price. "So the concept's the same, but the way you measure it, it evolves through time, right?" (29:56) Value "has been around for more than a century, and it was really valuable when the railroads were being introduced" (30:11); how value is measured today — "stocks like Nvidia versus Ford Motor Company" — "is using different measures, but the concept is the same" (30:25). The job is not only to invent new factors; often it is "to apply the same economics to different ways of measuring" and implementing them, "and it works pretty well" (30:45).

WARNING Said in class vs written — 30:04: "good economic theories behind each of these" against the course's "theoretical motivation absent"

The guest: "this is kind of where the economics come in. There's good economic theories behind each of these, and then how you implement them obviously changes over time" (30:02). The course's deck, closing its own tour of the factor models: **"Big problem: theoretical motivation absent"** — "What are the underlying sources of risk you expose yourself to with such strategies?" — unclear, the deck says, for the three- or four-factor model, but clear in the context of the CAPM (04b p. 19). The two are a disagreement of view between the guest and the course's deck, not a conflict of fact; the note records both and adjudicates neither. §13 is the guest's own account of what the theories might be.

WARNING Said in class vs written — 30:19: a sentence only one recording carries

After "really valuable when the railroads were being introduced", one recording hears "In the last 10 years, value investing worked really well, but how do we calculate" before "value was dependent on that economy at the time"; the other hears "So, the whole idea of the". Only one file renders a claim about value's last ten years, so the note makes no such claim. The agreed point stands: value's measurement depended on the economy of its era.

Self-check. Which of the guest's factors has a symbol in the course's own notation, and which do not?

Size (R_{SMB}) and value (R_{HML}) are the two new factors of 04b p. 16, and momentum is named on 04b p. 19 as Carhart's fourth factor without a symbol. Quality, carry, trend, accruals and sentiment appear on no course page; "defensive" is the guest's name for Betting Against Beta. Where this note writes a symbol for momentum, low-risk or quality, it is the note's own label.

12. Combining the factors: Sharpe ratios

⌚ 30:53

This section is the talk's central piece of arithmetic: why a set of factors with an average Sharpe ratio of about 0.63 gives, combined, a Sharpe ratio of about 1.5, what that rests on, and what halves it in practice — with the course's formulas supplying every step the guest called "simple intuition".

12.1 The factor zoo and thirteen themes

The academic literature holds "like three or four hundred different papers that talk about new signals"

⌚ 30:53

that predict markets, which "you can kind of classify or break those down into about a dozen"

⌚ 31:08

themes — the course's own remark is the deck's "seek and thou shall find" (05a p. 16). The slide listed eight [transcript only]: the many flavours of value, quality and momentum are related and collapse together. "What's neat is each one of these produces a pretty good Sharpe ratio, better than what the CAPM says you should be able to get." ⌚ 31:24 . *Why that is a strong claim (explanation):* in the CAPM the highest Sharpe ratio any portfolio can reach is the market's, because the capital market line through the market is the steepest attainable line (04a p. 8); a strategy with a higher Sharpe ratio than the market's is, by itself, evidence against the model.

The chart the guest showed *[transcript only]* plotted "what the returns look like for each one of these", "going back to 1970 to today", each factor rescaled "to the same volatility, so we can compare apples to apples", and their combination 🕒 31:42. Rescaling to a common volatility is leverage, and leverage leaves a Sharpe ratio unchanged: with weight w in a strategy, $\mathbb{E}[R_p] - R_f = w(\mathbb{E}[R_i] - R_f)$ and $\sigma_p = w\sigma_i$ (03c p. 11), so their ratio is SR_i for every w EXPLANATION. The data come from a public database built by colleagues at the School of Management, named as Tice Jensen and Brian Kelly, with Lasse Pedersen, published in the Journal of Finance "a couple years ago" 🕒 32:26, hosted on GitHub where one can vary the parameters and test the portfolios, and open to crowdsourced additions 🕒 32:44. They replicated "something like 157 factors" 🕒 32:47 and "boiled them down to about 13 different themes" 🕒 32:54, and the guest plotted each theme's returns 🕒 31:42.

GAP Not recoverable — the factor chart and the factor database

The chart of the thirteen themes' cumulative returns is not on the recording beyond its description, and the 157-factor database is not in the course materials. The only factor definitions the course prints are size, value (04b p. 16) and momentum (04b p. 19); every other factor name in this note is the guest's.

12.2 The numbers: 0.63, 0.4, and 1.5

"the average Sharpe ratio of those 13 factors is about 0.63" 🕒 33:04 — "That's pretty good. The Sharpe ratio on the market is about 0.4. So this is like 50%, more than 50% larger than what you get from just the market portfolio, market risk." 🕒 33:10. And then: "But an equal-weighted portfolio of all 13 gives you a Sharpe ratio of 1.5. So that is two and a half times higher at the same volatility. That's what we try to do as a company." 🕒 33:22 *[the guest's figures; no course source]*.

Checked EXPLANATION: $0.63/0.4 = 1.575$, so "more than 50% larger" is exact. The course's own examples of the market's Sharpe ratio are of the same size as the guest's "about 0.4": $(12\% - 5\%)/16\% = 0.4375$, printed 0.44 (04a p. 9), and the US market's 0.446 (03c p. 8); an estimate against two worked examples, not a conflict.

WARNING Said in class vs written — 33:22: "two and a half times higher"

Against the thirteen factors' average, $1.5/0.63 = 2.38$; against the market's 0.4, $1.5/0.4 = 3.75$. The guest did not say which comparison was meant. That 2.38 is the nearer to "two and a half" is the note's own arithmetic EXPLANATION, not a statement about what the guest meant; the note gives both.

12.3 Why the combination wins: the Sharpe ratio of the average is not the average of the Sharpe ratios

"It's non-linear. Sharpe ratios is mean divided by standard deviation. The average of that is different than the Sharpe ratio of the average. The Sharpe ratio of the average is a much more powerful thing because of diversification. The denominator gets smaller and smaller and the numerator is linear."  33:37 . "That's all quant investing is. I haven't done anything fancy."  33:58 . The course's formula for the denominator is the general equally weighted portfolio (03d pp. 10–11; the Lecture 7 note's §12):

$$\sigma_p^2 = \frac{1}{N} \overline{\text{Var}} + \left(1 - \frac{1}{N}\right) \overline{\text{Cov}}$$

DERIVATION Full derivation (not done in class) — N strategies with the same volatility, average Sharpe ratio $\bar{s}$ and average pairwise correlation $\bar{\rho}$: the equal-weighted Sharpe ratio is $\bar{s}\sqrt{N} / \sqrt{1 + (N-1)\bar{\rho}}$; for $0.63 \rightarrow 1.5$ at $N = 13$ the implied $\bar{\rho}$ is 0.108

Give every strategy the same volatility σ (the guest rescaled them so), strategy i an expected excess return $\mu_i = s_i\sigma$ with s_i its Sharpe ratio, and the pairs an average correlation $\bar{\rho}$. Hold equal weights $1/N$ (explanation: the assumptions the guest did not state, made explicit).

$$\mathbb{E}[R_p] - R_f = \frac{1}{N} \sum_{i=1}^N \mu_i = \sigma \bar{s}, \quad \bar{s} \equiv \frac{1}{N} \sum_i s_i$$

Why this step follows: the mean is linear in the weights (formula sheet p. 1, item 7); factor out the common σ — this is the "numerator is linear".

$$\sigma_p^2 = \frac{1}{N} \sigma^2 + \left(1 - \frac{1}{N}\right) \bar{\rho} \sigma^2 = \sigma^2 \frac{1 + (N-1)\bar{\rho}}{N}$$

Why this step follows: 03d p. 10's formula with every variance σ^2 and every covariance $\rho_{ij}\sigma^2$, whose average is $\bar{\rho}\sigma^2$; then put the two terms over the common denominator N : $\frac{1}{N} + \frac{N-1}{N} \bar{\rho} = \frac{1+(N-1)\bar{\rho}}{N}$. This is the "denominator gets smaller".

$$\text{SR}_N = \frac{\sigma \bar{s}}{\sigma \sqrt{(1 + (N-1)\bar{\rho})/N}} = \frac{\bar{s} \sqrt{N}}{\sqrt{1 + (N-1)\bar{\rho}}}$$

Why this step follows: divide the mean by the square root of the variance; σ cancels; invert the fraction under the root. With $\bar{\rho} = 0$ this is $\bar{s}\sqrt{N}$ SR; with $\bar{\rho} = 1$ it is $\bar{s}$, no gain at all; in between, the gain is capped at $\bar{s}/\sqrt{\bar{\rho}}$ as N grows — the course's result that the variance converges to the average covariance (03d p. 11), in Sharpe-ratio form.

The guest's numbers. Independent factors would give $0.63 \times \sqrt{13} = 0.63 \times 3.6056 = 2.2715$, above the 1.5 the guest reported, so the thirteen themes are not independent. The correlation that makes the formula give exactly 1.5:

$$1 + 12\bar{\rho} = \left(\frac{0.63\sqrt{13}}{1.5}\right)^2 = \frac{0.3969 \times 13}{2.25} = 2.293 \Rightarrow \bar{\rho} = \frac{1.293}{12} = 0.108$$

Why this step follows: square both sides of $\text{SR}_N = \bar{s}\sqrt{N} / \sqrt{1 + (N-1)\bar{\rho}}$ and solve for $\bar{\rho}$ with $N = 13$, $\bar{s} = 0.63$, $\text{SR}_N = 1.5$. An average pairwise correlation of about 0.11 across the themes is what the guest's two figures imply together — positive, as the course says the limiting average covariance must be (03d p. 10; the Lecture 7 note records that the strict claim holds only in the limit).

This is the note's inference from the guest's numbers under its stated assumptions, not a figure the guest gave.

12.4 On paper, and in practice: "cut that in half"

"Now, that's on paper, right? That's just academic." Trading it meets "transactions costs, there's leverage constraints, there's also the fact that I'm looking historically and there's possibly I'm overfitting the data, data mining" ⌚ 34:13. The guest's rule of thumb: "in my experience, you could cut that in half, and that's probably about what your clients will experience. So 1.5 Sharpe ratio ended up being about 0.75 or 0.8 Sharpe ratio, neutral to the market, that's really valuable." ⌚ 34:31. $1.5/2 = 0.75$, as said **EXPLANATION**. The course's figures for the first of the three costs are the active-fund numbers of 05a p. 23 — expense ratios "on the order of 1% per year" and trading costs "of about 0.5 to 1% per year" on top — and its treatment of the third, over-fitting and the in-sample/out-of-sample distinction, is 05a pp. 16–17 and §13.3 below; leverage constraints are §§6 and 9.

12.5 Buffett's 0.6, Madoff's 3, and what a Sharpe ratio implies

"So the richest investor of all time is Warren Buffett. His net worth is, what's that, \$100 billion?" ⌚ 34:46 The room was asked to guess Buffett's Sharpe ratio: "His Sharpe ratio historically is about 0.6. That's it." ⌚ 35:04 [the guest's figures]. Sustaining 0.6 for a long time, "that's how you become \$100 billion" ⌚ 35:08 — how long is a number the recordings disagree on (the Warning below) — "And that's easier said than done. You have to stick with it, right? There are years where you can lose a lot of money." ⌚ 35:15. People claim Sharpe ratios of two or three: "The only three Sharpe ratio I've ever seen in practice is Bernie Madoff's, which is making it up, right? It doesn't exist" ⌚ 35:30 — a Sharpe ratio of three means "they should not lose money in 100 years" ⌚ 35:38, and no manager has done that; "But a 0.6 Sharpe ratio for 50 years is massive wealth, okay? 0.8, even better." ⌚ 35:45. The course's own Madoff reference is the film poster on 03c p. 23, shown on 29 September. A statistical reason to doubt such a record is 03S p. 13's (explanation: the note's link; the guest gave none beyond the fabrication): a record with no losses can hide tail risk, the "peso problem", which is why variance and the Sharpe ratio do not summarize such a distribution (the Lecture 5 note's §4.3).

WARNING Said in class vs written — 35:08: how many years of a 0.6 Sharpe ratio

"If you do a 0.6 Sharpe ratio for, in his case," — one recording hears "eight years", the other "80 years" — "that's how you become \$100 billion". The record does not settle it and the note states neither.

GAP Not recoverable — wealth from a Sharpe ratio

No arithmetic connects a Sharpe ratio to a net worth: that needs the return, the volatility, the leverage and the starting capital, none of which the guest stated. The note attempts none under either reading of the years.

DERIVATION Full derivation (not done in class) — under independent, normal annual returns the chance of a losing year is $\Phi(-SR)$: about one year in 741 at $SR = 3$, one in 44 at $SR = 2$, and one in four at $SR = 0.6$

Assumption, which the guest did not state: a strategy's annual excess return is normal with mean μ and volatility σ , the same every year and independent across years, so $SR = \mu/\sigma$ (explanation; the normal model is the course's in 03S p. 13's value-at-risk remark).

$$\Pr(\text{excess return} < 0) = \Pr\left(\frac{R - \mu}{\sigma} < -\frac{\mu}{\sigma}\right) = \Phi(-SR)$$

Why this step follows: standardize the return; the standardized variable is standard normal, and the threshold $-\mu/\sigma$ is minus the Sharpe ratio.

$$SR = 3 : \Phi(-3) = 0.00135, \quad SR = 2 : \Phi(-2) = 0.0228, \quad SR = 0.6 : \Phi(-0.6) = 0.274$$

Why this step follows: the standard normal table. So a true Sharpe ratio of three would lose money in about 0.135 of a year per century — one year in $1/0.00135 \approx 741$ — and the chance of a century with no losing year at all is $(1 - 0.00135)^{100} = 0.874$; that is the arithmetic behind "should not lose money in 100 years". A Sharpe ratio of two loses about one year in 44. And a Sharpe ratio of 0.6, Buffett's by the guest's figure, loses money in about 27% of years, roughly one in four: "There are years where you can lose a lot of money." The normal assumption understates the chance of large losses for strategies with tail risk (03S p. 13), so these are illustrations of the guest's remark, not a correction of it.

Self-check. Thirteen strategies each have a Sharpe ratio of 0.63. Why does their equal-weighted combination not have a Sharpe ratio of 0.63, and what two numbers bracket what it can have?

Because the combination's mean is the average of the means but its volatility is less than the common volatility unless the strategies are perfectly correlated. The bracket is 0.63 (perfect correlation, $\bar{\rho} = 1$) to $0.63\sqrt{13} = 2.27$ (independence, $\bar{\rho} = 0$); the guest's 1.5 sits inside it, at an average correlation of about 0.11.

Self-check. A manager reports a Sharpe ratio of 3 over twenty years with no losing year (*the twenty years illustrative, the note's own*). What two reasons does the lecture give for doubt?

Under normal returns a true Sharpe ratio of 3 makes a losing year a once-in-seven-centuries event, so twenty clean years are not surprising if the ratio is genuine — but no manager has ever sustained such a ratio, and the one reported case the guest has seen, Madoff's, was fabricated. A further, statistical reason, which the note links from the course **EXPLANATION**, is the peso problem (03S p. 13): a history with no losses can hide a rare large loss that the variance, and so the Sharpe ratio, does not capture.

13. What drives the anomalies: three schools of thought

🕒 35:52

This section sets out the three explanations the guest tabulated — risk, behaviour, data mining — each beside the course's own written treatment, and records the one the guest called the live debate.

"We do care what's driving these anomalies, what gets you that Sharpe ratio, because what we want to know is, is it real? Or, where are the bodies buried, right? What's the risk?" 🕒 35:52 . The slide was a table of "three schools of thought here on what drives deviations from the CAPM" 🕒 36:11 [transcript only]. The course's counterpart is 05a p. 18's list of six "Reactions to Anomalies" and the reading's section on explanations, irrational pricing or risk (FF04 pp. 14–17; the Lecture 9 note's §9.5).

13.1 Risk: the CAPM is the wrong model, and there are several factors

"One is classic risk, which is, hey, maybe the CAPM is the wrong model, and there's multiple sources of risk, and what you're picking up is new risk premium that markets are compensating for. It works the same as the CAPM, except instead of a single factor, you now have multiple factors. I don't know if you've covered that yet, but..." 🕒 36:16 . The course's version is the Fama–French three-factor model, taught by inference on 6 October (§7.1 of that note): two new sources of systematic risk, the size factor R_{SMB} (long small stocks, short big) and the value factor R_{HML} (long high book-to-market, short low), and the augmented security market line (04b p. 16)

$$\mathbb{E}[R_i] - R_f = \beta_{iM}(\mathbb{E}[R_M] - R_f) + \beta_{is} \mathbb{E}[R_{SMB}] + \beta_{ih} \mathbb{E}[R_{HML}]$$

"there is something inherently, systematically risky about holding a small or a value stock" (04b p. 16). Its test is the CAPM's with three right-hand-side variables, and the deck reports a "Superior fit: R^2 goes from 25% to 75%" and "Alphas no longer different from zero (success!)" (04b p. 18; the Lecture 9 note's §7.3); the reading prints the same model and its time-series form (FF04 pp. 15–16), and the optional APT deck is the general case, a model that "generalizes to multiple factors" (z04 p. 9). The guest's "new risk premium" is the

course's "compensated for systematic covariation of your stock with small stocks and value stocks" (04b p. 16).

The joint hypothesis problem. "and that's the joint hypothesis problem, right?" (36:37). To know whether markets are efficient — "meaning they reflect full information, and there's not really a lot of, you know, profit opportunities or mispricing" — "I have to know, do I have the right model, right? Am I comparing it to the right thing?" (36:40). So apparent mispricing relative to the CAPM means one of two things: "Either markets are totally inefficient, and the CAPM is the right model, or the CAPM is the wrong model, and markets might be much closer to efficient" (37:01). The course states it under the same name on a page not yet taught: "The 'joint-test problem'" — "is the market inefficient or did you adjust for risk incorrectly?"; "We faced this problem when testing whether the CAPM holds": is the market inefficient because the CAPM tests failed, or efficient because the true model is the Fama–French three-factor model? (05a p. 16). Efficiency itself is Fama's 1970 definition, prices that fully reflect all available information (05a p. 3).

13.2 Behaviour: mistakes, lottery preferences, and limits to arbitrage

"The other middle column here says why markets might be efficient, so we could be missing some risk factors, we could also just be not giving investors too much credit and giving the forces of arbitrage too much credit" (37:10) [transcript only: the column]: "people make mistakes, they're dumb in some instances, they overreact to certain information, they underreact to others, they bet on emotions, they're not just thinking about risk and return, they think of preferences" (37:10). The example is Betting Against Beta: described as a leverage story, "and that's probably partly true. But it's also consistent with lottery preferences. People like risky assets because they like a small probability of a big payoff." A small sure payoff is not exciting: "So people overpay for lotteries. We know this in economics, right?" (37:37). The investor willing to take the other side sells the tickets: "I very rarely have to pay up, and I make my steady premium. It's boring, but it makes a lot of money." (38:12). But behavioural stories "really also rely on limits to arbitrage" (38:17): if everyone were willing to sell lottery tickets and no one to buy them, "it doesn't work. You need that equilibrium of who's on the other side of the trade." (38:24).

The course's written versions, each a sentence (*related, not shown*): the sixth reaction to anomalies, that they "are the result of market mispricing (market inefficiency), driven by investor psychology" (05a p. 18); Shiller's observation that "People have always been very attracted to lottery tickets" (Shiller03 p. 19); and the limits-to-arbitrage question Lamont and Thaler ask of their own examples, "What prevents arbitrageurs from enforcing the Law?", answered with short-sale constraints and the "noise trader risk" that a mispricing "could get worse before it gets better" (LT03 p. 10). The Nobel popular summary makes the same point about credit constraints that stop rational investors from betting against less rational ones (Nobel13pop p. 4). The statistical face of selling lottery tickets is the course's own caution: a strategy that delivers small steady gains and occasional large losses is exactly the kind whose variance misstates its risk (03S p. 13; *explanation*).

13.3 Data mining, and out-of-sample testing

"And then the last thing you do have to worry about is data mining. We're looking historically" (38:33) to find patterns, and "Those patterns may not actually be real, right? If I look at 100 different stock strategies and I have a 95% significance level that I'm testing, just by random chance, five are gonna look good, right? That doesn't mean anything, it's just random" (38:40). The course prints the same count: in a large population of unskilled managers, "5 out of 100 will perform well enough to be significant at the 5% level" (05a p. 25); the guest's "a 95% significance level" (38:45) and the course's "significant at the 5% level" are one test, a 5% chance of a false positive, described by its confidence or by its size.

$$\text{expected false positives} = 100 \times 0.05 = 5; \quad \Pr(\text{at least one}) = 1 - 0.95^{100} = 0.9941$$

*Why this step follows: each of 100 null strategies passes with probability 0.05, so the expected count is the sum, 5, whether or not the tests are independent; the probability that none passes is $0.95^{100} = 0.0059$ if they are independent — an assumption the slide does not state — so almost surely at least one apparent discovery is noise (explanation). The arithmetic, the spread of the count, and what a second independent sample does to the false positives (it cuts their expected number to $100 \times 0.05^2 = 0.25$) are the fourth worked problem under *Worked problems*, the guest's question at 38:33.*

"So I have to, what's my best weapon against data mining? How do I know I've found something real?" (38:57) The room answered; the guest repeated: "Out-of-sample testing, right? Because if it's just noise, it won't repeat." (39:11) And out-of-sample "can take many forms. Different markets, different asset classes, different time periods" (39:17). The course: "in-sample predictability need not imply out-of-sample predictability" (05a p. 16), illustrated by its over-fitting figure, a fifth-order polynomial that tracks fifty noisy points of a sine wave and then shoots off once the estimation window ends (05a p. 17); and the reading's standard response to data dredging, to look for the same findings in other samples (FF04 p. 14). "The stuff that I put up on the board has passed all of these tests" (39:27) [transcript only].

13.4 The live debate: risk or behaviour

"But we still have a very active and healthy debate, both in academia and practice, between the first two columns. Is it risk, or is it behavioral? And the world isn't so bright-lined as one or the other, it's some combination of the two. And we're trying to sort out what those are." (39:30). The reading calls the same standoff a "timeworn impasse" (FF04 p. 17), and the course's own conclusion on its efficiency deck is that the efficient market hypothesis "seems to hold partially" (05a p. 22).

Self-check. Low-beta stocks beat their CAPM prediction. Give the risk-school and the behaviour-school explanation of the same fact, and the one observation the guest said would help tell them apart.

Risk school: the CAPM is the wrong model; a low-beta stock's premium compensates for exposure to some other priced factor, so the excess return is a risk premium (04b p. 16's logic). Behaviour school: investors overpay for high-beta, lottery-like stocks and limits to arbitrage stop the mispricing from being traded away, so the excess return is a mispricing. The observation: a market with no macroeconomic risk, such as sports betting, where a risk premium cannot be the answer (§15).

Self-check. A researcher reports a strategy with a statistically significant backtest. What question does the guest ask first, and why does the course's "5 out of 100" settle nothing by itself?

"How do I know I've found something real?" — has it worked out of sample, in other markets, asset classes or periods? The count of 5 in 100 only says that significance at the 5% level is expected for a few strategies among many tested; it cannot say which five. Only a sample the strategy was not fitted to can.

14. Other markets, and Buffett's alpha

🕒 39:44

This section records why the guest turned to sports and prediction markets, and the factor decomposition of Buffett's record that preceded them, with its two charts flagged as unrecoverable and its two disputed phrases carried as such.

Why other markets. "so that brings me to other assets, other geographies, new markets. That's why I started looking at sports markets and prediction markets." 🕒 39:44 If the anomalies are "really about human behavior or risk, maybe they should show up in these other markets. And some of these we can rule out, right? Risk probably isn't the primary driver of sports markets. There could be preferences and beliefs."

🕒 39:56 Prediction markets sit "somewhere in between" 🕒 40:14 . The reading's prescription of testing in other samples (FF04 p. 14) is the method; §15 gives the reasoning.

Buffett's alpha. First, "a really cool paper that my colleagues looked at at one point" 🕒 40:17 : thinking of the world in terms of the factors — "value, momentum, quality, defensive, carry, all these different things" — one "can take the world's most famous investors and decompose their returns into those groups" 🕒 40:22 . The paper, *Buffett's Alpha*, is not in the course materials, and the guest skipped part of it ("I'm going to skip over this, but you can make these slides available if you want, Eduardo") 🕒 40:39 [transcript only]. The

construction: Berkshire Hathaway holds publicly traded stocks, which it must report to the SEC, and private businesses whose returns are not observed; "but we know that his total firm is traded on the New York Stock Exchange, so we can back out, implicitly, what his private businesses must be worth" ⌚ 40:50 . The identity behind that **EXPLANATION**: the market value of Berkshire equals the value of its public holdings plus the value of its private businesses, so the second is the first less the observable part. Then "you just run a regression of Warren Buffett's returns", for the public holdings and for the private remainder, "on the various factors academics have used, size, value, momentum, low risk," a factor the recording does not make out, "and quality" ⌚ 41:12 . The course's form of such a regression is its three-factor test, "Like the SCL but with three right hand side variables" (04b p. 18); with the guest's list it reads, in the note's own labels for the factors the course has no symbol for **EXPLANATION**,

$$R_B(t) - R_f(t) = \alpha_B + \beta_M(R_M(t) - R_f(t)) + \beta_s R_{SMB}(t) + \beta_h R_{HML}(t) + \beta_{mom} R_{MOM}(t) + \beta_{bab} R_{BAB}(t) + \beta_q R_{QMJ}(t) + \varepsilon_B(t)$$

with R_B Buffett's return (public or private) and α_B what the factors leave unexplained — the course's alpha as the measure of selection skill (04a p. 27). The factor the recording does not make out has no term, so the equation has six factors, the market's among them, where the guest's list had one more. The result, in the second recording's words: "lo and behold, we explain most of the performance of Warren Buffett's portfolio, at least on the public side." ⌚ 41:37 . "We don't explain a lot of it on the private side" ⌚ 41:46 — and the guest's verdict on that is a phrase the recordings reverse (the Warning below).

WARNING Said in class vs written — 41:46 and 42:33: two phrases the recordings render with opposite meanings

After "We don't explain a lot of it on the private side", one recording hears "which doesn't make any sense", and the other recording renders the clause with the opposite sense, that it kind of makes sense. And after "He's great at thinking about the factors that he wants in his stocks", one hears "And I don't think he would agree with that", while the other recording renders it as the guest thinking Buffett would agree. The record settles neither, so the note says nothing about whether the private side's poor fit made sense to the guest, and nothing about whether Buffett would agree. What is agreed stands: the factors explain most of the public portfolio and little of the private one, and the guest's reading is that Buffett's holdings load on book-to-price, quality and low beta.

WARNING Said in class vs written — 41:50: the chart's start year

"This is what the overall stock market did" — one recording hears "from 1983 to", the other "from 1980 through" — "the end of this analysis, I think it was around 2015." The note states no start year.

GAP Not recoverable — the Buffett charts

The chart of the market against Buffett's public stock, which did "way better than the market" ⌚ 42:01, and the plotted quantitative strategy "that used the same factors as Buffett used, but in proportion to what Buffett and his portfolio had done" ⌚ 42:08 — the sentence breaks off — are known only from these words.

The verdict. "I give Warren Buffett a lot of credit for being one of the earliest value and quality investors who uses cheap leverage, but to call him a good stock picker is a misnomer. Not a good stock picker. He's great at thinking about the factors that he wants in his stocks." ⌚ 42:17. Buffett professes to love the companies held; in the guest's reading, the love is for their book-to-price ratios, their quality metrics and their low beta — "And that's a great strategy, no question about it. It just doesn't do it in a quantitative way." ⌚ 42:43 The quantitative model "does a little bit better, only because it's better diversified, but it's the same thing. So there's no mystery." ⌚ 42:48. The decks' one line on Buffett, "(Ex: Buffett)" beside the value spread (04b p. 12), is the same attribution.

Self-check. In the factor regression of Buffett's public returns, what would a large positive alpha have meant, and what does "we explain most of the performance" say about it?

A large alpha would be return the six factors cannot account for — stock-picking skill in the course's sense (04a p. 27). "Explain most of the performance" means the factor loadings absorb most of the average return, leaving a small alpha: the record is largely early, levered exposure to value, quality and low beta, which is the guest's "not a good stock picker".

15. Why study sports and prediction markets

⌚ 42:56

This section states the guest's case that sports betting is a laboratory the joint-hypothesis problem cannot spoil, and derives the one line of the course's model that makes the case precise.

"Why study other markets? Well, I would argue that financial markets are really hard to test because of what I talked about earlier, the joint hypothesis problem. I need to know what the right model is. Is it the CAPM? Is it the Fama-French model? Is it something else" ⌚ 43:00 — the course's own two-horned question (05a p. 16). Sports betting markets are "a pretty clean laboratory in one sense. They're idiosyncratic bets, so I don't really have to worry about the risk model. I mean, you know, last night there were four baseball games. None of those are tied to the macro economy" ⌚ 43:17 — who won "has nothing to do with the macro economy" ⌚ 43:35, and has no covariance with the things the economy cares about ⌚ 43:41.

DERIVATION Full derivation (not done in class) — a bet with zero covariance with the market has beta zero, so every asset-pricing model of the CAPM family prices it at the risk-free rate: any systematic average gain or loss is mispricing (or fees), never a risk premium

$$\text{Cov}[R_{\text{bet}}, R_M] = 0 \Rightarrow \beta_{\text{bet}} = \frac{\text{Cov}[R_{\text{bet}}, R_M]}{\text{Var}[R_M]} = 0$$

Why this step follows: the definition of beta (04a p. 12) with a zero numerator — the guest's point that the bets have no covariance with what the economy cares about.

$$\mathbb{E}[R_{\text{bet}}] = R_f + 0 \times (\mathbb{E}[R_M] - R_f) = R_f$$

Why this step follows: the security market line (04a p. 12) at beta zero. The course's reason is that only systematic risk is compensated — a bet's variance is entirely idiosyncratic, and idiosyncratic risk earns nothing in a diversified portfolio (04a pp. 18–19; 03d pp. 12–13). The same holds with several factors: a bet whose outcome is unrelated to every macro factor has every loading zero, so a multifactor model also prices it at R_f . Hence in a betting market the risk school of §13.1 has nothing to say: if underdogs lose 22% on average (§19), no risk premium can be the reason. What is left is behaviour, beliefs, and the bookmaker's fee. **EXPLANATION**

The second virtue: "they have a short terminal date" ⌚ 43:46 ; the words that follow, about the outcomes, are garbled in one recording, and the other renders them as the outcomes being known. "I don't know what the true value of Microsoft is until Microsoft decides to sell everything off one day and disappear, then I get a terminal value. Every sporting contest, there's an outcome. Who won, who lost, contracts pay off, uncertainties resolve. Beautiful for testing, right? It's a really nice laboratory." ⌚ 43:53 . Prediction markets are "really similar, and actually more liquid, and there's a lot more things to trade, not just sports": real-world events, elections, "all kinds of stuff" ⌚ 44:11 .

Self-check. Why does a predictable loss on underdog bets refute the risk explanation outright, when a predictable premium on small stocks does not?

Small stocks might covary with some priced source of risk, so their premium can be a reward for bearing it — the joint-hypothesis problem leaves the question open. A bet on tonight's game has no covariance with anything the economy cares about: its beta on every factor is zero, so any model in the CAPM family prices it at the risk-free rate. A systematic loss on such bets cannot be compensation for risk; it has to be a mispricing, a belief error, or the fee.

16. The favourite–long-shot bias

⌚ 44:23

This section records the first pattern the guest looked for in sports betting and the three explanations offered for it, and says which of them the setting rules out.

"And my question here from an academic perspective is, do we see some of the same patterns?" ⌚ 44:23 .

The most famous bias in sports is the **favourite–long-shot bias**, which "came from horse racing, so betting on the favorite is profitable because it's underpriced, betting on the long-shot, which everyone wants to do, is a bad strategy" — the second recording adds that it is a lottery ticket — "and it's overpriced" ⌚ 44:29 .

"Well, that economic parallel is very similar to this low-risk effect we started with, where the CAPM first failed, back in 1972." ⌚ 44:48 — the low-beta deciles above the line and the high-beta deciles below it (04b pp. 10–11; FF04 pp. 8–9), with the long shot in the place of the high-beta stock.

The leading explanations, in the guest's order ⌚ 44:23 :

1. **A preference for lotteries.** "Some people say it is actually a preference for lotteries." People know they overpay for them "but they just have those preferences. That's fine." — Shiller's "People have always been very attracted to lottery tickets" (*Shiller03 p. 19*), and §13.2's behaviour column.
2. **Beliefs.** "Others say it's about beliefs. They just believe that the underdog has a much bigger chance than they actually do." — a belief error rather than a taste.
3. **Leverage constraints.** "Leverage constraints is what we talk about in financial markets. I'm not sure that matters so much for sports betting markets." — §§6 and 9's story, which the setting weakens.

"This is kind of a nice way to differentiate some of these theories. But risk, in a macroeconomic sense, is not what's driving this." ⌚ 45:21 — §15's derivation is the reason: a bet has beta zero, so no risk premium can explain a systematic loss on long shots.

Self-check. Map the favourite–long-shot bias onto the beta-decile chart: which bet plays the role of the high-beta decile, and what sign does its "alpha" have?

The long shot. It is the high-risk, high-payoff-if-it-wins bet that bettors demand too much of, so its price is too high and its average return is negative — the high-beta decile below the SML. The favourite is the low-beta decile above the line: underpriced, with a positive average return before fees.

17. The open-to-close test

⌚ 45:29

This section states the efficiency test the guest ran on betting prices, fixes the efficiency benchmark in the course's random-walk model, and derives what reversal and continuation of a price move say about over- and under-reaction.

The test. From "One of the first papers I looked at in this area" ⌚ 45:36: for a baseball game — "There's White Sox and Guardians" ⌚ 45:46 playing tonight — take "the opening price" ⌚ 45:36 and "the closing price right before betting activity ends, which is right when the game starts" ⌚ 45:51, for "just a bet on who wins" ⌚ 45:57, and see whether the move from open to close predicts anything ⌚ 46:00. "If markets are perfectly efficient, there should be no predictability. The closing price should be the price that is your best forecast of the outcome." ⌚ 46:04. The course's version is the random walk of its efficiency deck (05a p. 6, *not yet taught*): $p_{t+1} = \mu + p_t + \varepsilon_{t+1}$ with $\mathbb{E}_t[\varepsilon_{t+1}] = 0$ and, over short horizons, $\mu \approx 0$, so "Current price is best predictor of future price $\mathbb{E}_t[p_{t+1}] = p_t$ " and returns are unpredictable; weak-form efficiency says the history of past prices, "Trends in prices" included, cannot be traded on (05a p. 5).

The example. "So if the closing odds say that the White Sox are 58% likely to win, and if that game were played 1,000 times, it would win" ⌚ 46:14 — a count the two recordings render differently ⌚ 46:19. "That's the idea, right?" ⌚ 46:19

WARNING Said in class vs written — 46:19: the count of wins in 1,000 plays

One recording hears "500 major games", the other "580 euros". Neither is stated as the guest's figure. The arithmetic the sentence sets up is $0.58 \times 1,000 = 580$ (*explanation; the course's expected value and law of large numbers, 03S pp. 4, 12*): if the closing price is the true probability, 58 wins in 100 plays on average, 580 in 1,000. One recording renders a count of 500, which the note records without adopting.

What would break it. Prices can move "for non-information reasons, like people just like one team more than the other, they're not based off of anything fundamental, or they're chasing momentum irrationally, or they have a preference for the underdog, right? Irrationally, so they're overpricing them." ⌚ 46:26 Then "the price that moves from P0 to P1 would either reverse, if it's an overreaction, or continue, if it's an underreaction, or would be predictable, if prices are wrong, okay?" ⌚ 46:41. And: "you can move for complete non-informational reasons. I call that overreaction, okay?" ⌚ 46:53.

DERIVATION Full derivation (not done in class) — with a closing price P_1 on a contract that pays 1 if the team wins, efficiency means the move $P_1 - P_0$ carries no information about the outcome beyond P_1 ; over-reaction makes the outcome lean against the move, under-reaction with it

Let $\omega = 1$ if the team wins and 0 otherwise, P_0 the opening price and P_1 the closing price of a contract paying ω (a bet's price expressed as a probability; §21 makes the price–probability reading precise). (explanation: the note's formalization of the guest's words)

$$\text{efficient close: } \mathbb{E}[\omega \mid P_0, P_1] = P_1 \Rightarrow \mathbb{E}[\omega - P_1 \mid P_1 - P_0] = 0$$

Why this step follows: "the closing price should be ... your best forecast of the outcome" is the statement that the close already contains everything the open and the move contain (05a p. 6's $\mathbb{E}_t[p_{t+1}] = p_t$ applied to the terminal payoff); then the forecast error $\omega - P_1$ cannot be predicted by the move.

$$\text{over-reaction: } \text{sign } \mathbb{E}[\omega - P_1 \mid P_1 - P_0] = -\text{sign}(P_1 - P_0)$$

Why this step follows: if the move overshoots, a price that rose from open to close now sits above the true probability, so the average outcome falls short of it — the move reverses, in the guest's word, and betting against the move is profitable.

$$\text{under-reaction: } \text{sign } \mathbb{E}[\omega - P_1 \mid P_1 - P_0] = +\text{sign}(P_1 - P_0)$$

Why this step follows: if the move is incomplete, a price that rose still sits below the true probability, so outcomes exceed it on average — the move continues, and betting with it is profitable. Either way the forecast error is predictable from the move, which is "if prices are wrong". The course's two exhibits of the same two shapes are the drift after an earnings surprise (under-reaction) and the overshoot after a CNBC mention (over-reaction) (05a pp. 13–14), and the Nobel background's pairing of under-reaction with momentum and over-reaction with reversals (Nobel13 p. 43).

Self-check. A team's price rises from 50 to 60 cents between open and close, and over many such games teams in that situation win 55% of the time (*illustrative numbers, the note's own, not the guest's*). Over-reaction or under-reaction, and what is the profitable side?

Over-reaction: the close of 60 overstates a 55% chance, so the outcome leans against the move. Betting against the move — on the other team at 40 cents, which wins 45% of the time — has a positive expected payoff before fees, while betting with the move loses.

18. Momentum in betting markets

🕒 47:02

This section records the finding the guest reported from the open-to-close test, and flags its chart as unrecoverable.

The guest measured "one thing I could measure very well in sports markets, which is momentum, people chasing past returns and extrapolating that too far": bettors see a small sample and think "this team's on a hot streak, I think they're going to continue to win" 🕒 47:02. "The reality is there is really not much momentum in sports, but betting markets act as if there is" 🕒 47:20. The chart *[transcript only]* covered three contracts on the same games — "who wins and by how much, called the spread", "just who wins, called the money line", and "the over-under, which is the totals, that's just how many total points are scored" 🕒 47:30 — "Very different aspects of the game, all see the same pattern. People are chasing recent performance, pushing the price up too far", from open to close, "and then it reverses" 🕒 47:44 — "because it's an overreaction. It's not predictive of anything." 🕒 47:57. The same holds across sports ("NHL, baseball, MLB, NFL, and NBA, in college sports you see the same" 🕒 47:56) and the lower panel showed the NBA alone 🕒 47:56. In §17's terms the sign of the forecast error is opposite to the move: over-reaction, and so a reversal. The course's own overshoot exhibit is the CNBC chart (05a p. 14); Shiller describes the same mechanism in stocks, investors who extrapolate from a short run of past prices (*Shiller03 pp. 11–12*).

GAP Not recoverable — the sports-momentum chart

The chart's magnitudes, its sample and the lower NBA panel are known only from the description above.

19. Spread against money line: risk and return by decile

🕒 48:18

This section rebuilds the two betting contracts the guest compared, derives why one of them has flat risk across games and the other rising risk, and records the returns by decile with the bookmaker's fee netted out.

19.1 Two contracts on the same outcome

Betting contracts are derivatives: "I think of them as derivative contracts, much like prediction markets are, this is a bet on the same outcome, who wins the game" 🕒 48:22 — the course's security that pays in one state is the Arrow–Debreu security of a later block (07b p. 6). The example, with the baseball team swapped

for football: "let's say the Giants play, let's say they're three and a half point favorites of the Cardinals" ⌚ 48:50. **The spread** "adjusts the points so that it's roughly a 50-50 bet" ⌚ 48:59: to win a bet on the favourite, "I have to win by at least four points" ⌚ 49:05, and the underdog's backer wins if the underdog wins or loses "by less than four points" ⌚ 49:12. *The four (explanation)*: winning margins are whole numbers, and the smallest whole number above 3.5 is 4; the half point in the spread is what makes a tie on the bet impossible. **The money line** keeps the bet as "who wins" ⌚ 48:22 and adjusts the payoffs instead: "rather than adjust the point spread and make it a 50-50 bet so we each make the same amount of money, we can just adjust the payoffs" ⌚ 49:21 — a bet on the favourite pays if it wins by any margin, "but I make less than the dollar I put in" ⌚ 49:33; a bet on the underdog pays more: "I put in a dollar, I make more than that, \$1.25" ⌚ 49:40. "So I can either adjust the spread so that it's 50-50 bet and same dollar payoff, or I can adjust the payoffs. Because one is more likely to happen than the other." ⌚ 49:48.

WARNING Said in class vs written — 49:40: the favourite's payoff per dollar

"So if I put in a dollar, I make" — one recording hears "\$0.08", the other "\$0.80". The record does not settle it; the note states only what is agreed, that a dollar on the favourite returns less than a dollar of profit and a dollar on the underdog returns \$1.25. No inference from the two quoted payoffs to one reading is drawn.

19.2 Risk by decile: flat for the spread, rising for the money line

The chart *[transcript only]* uses "the exact same games. Same contest. Everything is controlled for." ⌚ 49:59, with games sorted "from the biggest underdogs, a decile one, to the biggest favorites, decile 10" ⌚ 50:06 — the course's portfolio sorting (04b p. 8) with games for stocks. In orange, the spread contract: "the standard deviation is flat because they are forced to have the same risk because I've adjusted the point spread so it's the same risk" ⌚ 50:15. In blue, the money line: "if these guys win, I make a lot more money. These guys, they make a lot less, but this is the standard deviation of payoffs, right? The risk is monotonic" ⌚ 50:25 — "This is a highly implied levered bet, and this is a low levered bet. So I'm going to test this betting against, you know, risk kind of idea, this defensive strategy." ⌚ 50:40.

GAP Not recoverable — the spread-against-money-line chart

The guest's chart of the two contracts' standard deviations and returns by decile is not in the course materials; its values, its sample of games and its sports are known only from the words above and in §19.3. The figure below is the note's own illustration from a formula, not the guest's chart.

DERIVATION Full derivation (not done in class) — a \$1 stake at fair odds has variance $(1 - p)/p$: the money line's risk per dollar falls as the favourite gets stronger, while the spread fixes p at about $1/2$ and so fixes the risk

Stake \$1 on an outcome with win probability p that pays a net b on a win and loses the stake otherwise: the net payoff is $X = (1 + b)\omega - 1$ with ω a Bernoulli indicator (explanation; 03S p. 5's variance of a random variable).

$$\text{Var}[X] = (1 + b)^2 \text{Var}[\omega] = (1 + b)^2 p(1 - p)$$

Why this step follows: X is a constant times ω plus a constant, and the variance of a Bernoulli variable with probability p is $p(1 - p)$.

$$\text{fair odds (zero expected profit): } pb - (1 - p) = 0 \Rightarrow b = \frac{1 - p}{p}$$

Why this step follows: the expected net payoff $pb + (1 - p)(-1)$ set to zero and solved for b . A bookmaker's fee makes b smaller than this; the fair case shows the shape.

$$\text{Var}[X] = \left(1 + \frac{1 - p}{p}\right)^2 p(1 - p) = \frac{1}{p^2} p(1 - p) = \frac{1 - p}{p}, \quad \text{SD}[X] = \sqrt{\frac{1 - p}{p}}$$

Why this step follows: $1 + (1 - p)/p = 1/p$; square it and cancel one p . The standard deviation falls as p rises: a long shot at $p = 0.2$ has $\text{SD} = \sqrt{4} = 2$ per dollar, an even bet at $p = 0.5$ has $\text{SD} = 1$, a strong favourite at $p = 0.8$ has $\text{SD} = 0.5$.

The two contracts. The spread moves the line until $p \approx \frac{1}{2}$ for every game, so its risk per dollar is about 1 for every decile: the flat orange line. The money line leaves p where the matchup puts it, so risk per dollar rises from the strongest favourites to the biggest underdogs: the rising blue line. The implied leverage the guest spoke of is the course's $\sigma_p = w\sigma_i$ (03c p. 11): a long-shot bet packs several even-bets' worth of volatility into one dollar, as a levered position does. **EXPLANATION**

Risk per dollar at fair odds: the money line falls with p, the spread is flat (note's illustration)



Redrawn: the standard deviation of a 1 stake at fair odds against the win probability p — the money line's risk per dollar, falling from long shots to favourites — against the spread's flat risk at $p = 1/2$. The note's own illustration from the formula above, with p on the horizontal axis; it is not the guest's decile chart.

How to read this: move left along the horizontal axis toward small win probabilities, the long shots, and the money-line curve climbs steeply — a dollar on a 20% shot carries twice the risk of a dollar on an even bet. The spread's horizontal line is the even bet for every game. The guest's deciles would sit along the curve from its right end (the biggest favourites, decile 10) to its left (the biggest underdogs, decile 1).

Interactive spec — Risk per dollar staked at fair odds, as a function of the win probability

The sliders arrive with the site; the previewer shows the spec.

```
id: moneyline-sd
title: Risk per dollar staked at fair odds, as a function of the win probability
inputs:
  - {name: p, label: "Win probability of the team bet on", min: 0.05, max: 0.95, step: 0.01, default: 0.5}
series:
  - {name: "money line: SD of a $1 stake", x: p, x_values: "p from 0.05 to 0.95 step 0.01", y: "((1-x)/x)**0.5"}
  - {name: "spread: the even bet", y: "1", style: dashed}
x_axis: {label: "win probability p"}
y_axis: {label: "standard deviation of the net payoff per dollar"}
readout: ["((1-p)/p)**0.5", "(1-p)/p"]
```

19.3 Returns by decile: underdogs lose 22%

"What are the actual returns of the markets? It's pretty cool. These guys are way overpriced. You will lose 22% of your money on average when you bet on underdogs, right?" ⌚ 50:53 . On favourites the bettor loses about 1–2% net, by the guest's figures below; the guest's phrase for it is one the recordings render differently (the Warning below) — "which sounds like, well, you know, no one's winning money on sportsbooks. That's correct." ⌚ 51:06 The sportsbook charges about 10%, "It will wipe you out. But if you bet on the underdogs, you'll lose 10% plus another 12%, brutal. On the favorites, you're losing 10%, but you're actually making about 8 or 9%, right?" ⌚ 51:14 *[the guest's figures]*.

The arithmetic **EXPLANATION**: underdogs, $-10\% - 12\% = -22\%$, the headline loss; favourites, $-10\% + 8\% = -2\%$ to $-10\% + 9\% = -1\%$, a small loss. The pattern is the favourite–long-shot bias of \$16 with the fee netted out: favourites are the better bet by 20 points, and before the fee they would earn 8–9% on average while underdogs lost 12%.

WARNING Said in class vs written — 51:06: how favourites fare

"And on favorites, you'll" — one recording hears "not break even", the other renders it as about break even — "which sounds like, well, you know, no one's winning money on sportsbooks." The figures that follow fit both: a net loss of one or two percent is both "not" and "about" breaking even. The note states the arithmetic and neither phrase as the guest's.

Self-check. Why can the guest compare risk across deciles with the spread but not with the money line, and why is that comparison the test of betting against risk?

The spread forces every game to an even bet, so risk per dollar is the same in every decile and any return difference is not a risk difference. The money line's risk per dollar rises toward the underdogs, as $\sqrt{(1-p)/p}$ does, so the underdog deciles are the implicitly levered, high-risk bets. If high-risk bets earn less (−22%) than low-risk ones (about −1% to −2%), that is the low-beta effect in a market with no macro risk: the "defensive" pattern.

20. The same pattern in options

🕒 51:27

This section records the guest's one financial-market parallel to the long-shot bias, as a pointer to a later block.

"by the way, this is actually very cool too, you can do the same thing with stock options in financial markets, and you see the same thing" 🕒 51:27 [transcript only]: in index options on the S&P 500 and in single-stock options such as Apple's, "the deep out of the money ones way overpriced" 🕒 51:27 ; what the guest said of the ones near the money, "where the strike price and the underlying are the same" 🕒 51:57 , is garbled on the recording, so the note does not say how those are priced. The guest called the pattern a smile — the two recordings render the word before it differently — and went on: "This is what it looks like in sports markets. That is the extreme probabilities at much different prices than the ones in the middle. Even though it's the same underlying asset, the same thing is true for the options markets. I don't think that's an accident. It's just the way humans are." 🕒 52:07 . The course's options block, not yet taught, defines moneyness (09a p. 7) and states the fact the guest describes as the **volatility smile**: implied volatility is higher for in-the-money and out-of-the-money options than for at-the-money options (09d p. 16). The guest's chart of it is not in the build, and no figure is drawn here; the shape belongs to that block's note.

Self-check. What plays the role of the "long shot" in an options market, and what is the common thread the guest draws?

The deep out-of-the-money option: a small-probability, large-payoff claim that is "way overpriced", as the long shot is. The thread is that the extreme probabilities trade at prices out of line with the middle ones in both markets, with the same underlying — in the guest's view the same human preference for lottery-like payoffs, not an accident of either market.

21. Prediction markets

🕒 52:36

This section records the talk's last and newest subject: what prediction markets are, why their prices read as probabilities and what that reading leaves out, what the contracts on the Federal Reserve add, and the one finding the guest reported from trading inside a game.

21.1 Brand new, and already large

"So, prediction markets, brand new market. Many of you probably trade on these things." 🕒 52:36 . The guest's own household no longer may, under AQR's compliance rules, since the firm now trades them — "But I like looking at them, and we're trading them." 🕒 52:51 Prediction markets "have been around really since 2021. They've took off like crazy in 2025." 🕒 52:54 The US exchange the guest works with "were doing about \$60 billion in trading volume last year, 2025. This year, the year isn't even over, and they're very close right now to breaking \$1 trillion in trading volume. That is massive." 🕒 53:01 *[the guest's figures]*. Much of it is sports, but other contracts are gaining ground: weather contracts, elections — "The elections will be big this November" 🕒 53:27 — and macro contracts; some of the papers at the School of Management's conference in two weeks look at Fed funds futures and the exchange's contract on the FOMC 🕒 53:35 . The exchanges: the US exchange, "the most famous in the U.S." 🕒 53:47 ; "Polymarket, the most famous internationally" 🕒 53:49 ; Novig, ProfitX and others. The idea is old: "prediction markets have been around since 1988. There's the Iowa Futures Exchange, which never took off." 🕒 53:56 What changed is technology — tokenization, "just today's generation with everything on their phone" — "right place, right time" 🕒 54:05 ; "this is an idea that's at least 30 years old" 🕒 54:16 .

21.2 Prices as probabilities, and what that reading leaves out

The contracts: the FOMC contract ("The Fed cuts rates in June. That's a macro contract" 🕒 54:21), politics ("The Democrats win the House in 2026. You can bet on that" 🕒 54:28) 🕒 54:19 *[transcript only: the slide of contracts]*. "And there are implied probabilities, because the contracts are, they range from zero to one, and

you get a dollar if you're right." ⌚ 54:28 A contract on the Democrats winning the House "might be selling right now for, you know, 56 cents in the dollar, which implies a 56% chance that happens" ⌚ 54:40 . "so it tells you what the market thinks. So this is really useful for thinking about probabilities." ⌚ 55:09 . The guest's favourite, as an aside: a contract on whether Jesus returns by 2027, which "actually gives you a better return than the risk-free rate" ⌚ 54:59 and is "four times higher than the Jets winning the Super Bowl" ⌚ 55:03 — no price is on the recording, so nothing is computed from it.

WARNING Said in class vs written — 54:51: the guest's own view of the 56-cent price

After "56 cents in the dollar, which implies a 56% chance that happens", one recording hears "I wouldn't do that", and the other recording renders the words as the guest thinking the chance is actually higher than that; both continue "but something like that". The record does not settle it, and the note says nothing about the guest's own estimate.

WARNING Said in class vs written — 54:28: an implied probability is a price, and the course's later block will call it a risk-neutral probability

A contract paying \$1 in one state and nothing otherwise is the course's **Arrow–Debreu security**, and its price is a **state price** μ_s (07b p. 6, a later block). Scaled so that they add to one, state prices become **risk-neutral probabilities** π^* , which the course's deck says in so many words "are not physical probabilities" (07b p. 17); the exam's sheet prints the pricing identity $q_k = \sum_s \mu(s) d_k(s)$ on its *Final* list only (formula sheet p. 3, item 15). For a single "yes" contract with price q **EXPLANATION**:

$$q = \frac{\pi_{\text{yes}}^*}{1 + R_f} \Rightarrow \pi_{\text{yes}}^* = q(1 + R_f) \approx q \quad \text{over a short horizon}$$

Why this step follows: a sure dollar is worth $1/(1 + R_f)$ today, and the contract pays one dollar in the "yes" state only; with weeks to settlement R_f is small, so the price and the risk-neutral probability nearly coincide (07b p. 17's $q_k = \mathbb{E}^*[d_k]/(1 + r^f)$, with its r^f the note's R_f , in the one-state case). So 56 cents implying 56% is a statement about the market's price, not a measurement of the chance p . The gap between the two is exactly the favourite–long-shot bias of §§16 and 21.4: for long shots $q > p$, and the contract is "overpriced". The course's own version is that prices are the best forecast available (05a p. 6), which is true of q only if the market has no bias.

21.3 A distribution of outcomes: the FOMC contracts

"There's a really nice paper that's gonna be at our conference by a couple of people at the Board of Governors, Federal Reserve" ⌚ 55:15 , using the exchange's contracts "to help you understand what the

FOMC, what the Central Bank or the Federal Reserve is going to do in terms of rate-cutting" ⌚ 55:24 . What the contracts add over interest-rate futures: the exchange "will tell you, will the Fed cut by zero, 25 bips, 50 bips, or raise it by 25 or 50, so you actually get the distribution of potential outcomes, too, which is really nice, right? That's kind of what option markets can do, and now we have a market for these things in lots of different ways." ⌚ 55:36 . A basis point is one hundredth of one percent, $1 \text{ bp} = 0.01\%$, so 25 bp is a quarter point and 50 bp a half (01a p. 8; the Lecture 1 note's §5.3). Politics ranges "from actual elections to what Trump is going to say at whatever speech he's giving out" ⌚ 55:59 , and trade policy; "But you also have sports as well." ⌚ 56:15 .

DERIVATION Full derivation (not done in class) — the prices of an exhaustive set of mutually exclusive \$1 contracts add up to the price of a sure dollar, about 1; so a set of FOMC contracts is a probability distribution over the meeting's outcomes

Suppose the possible outcomes of a meeting are $s = 1, \dots, S$ (cut 50, cut 25, hold, raise 25, raise 50), one and only one of which will happen, and a contract $k = s$ pays \$1 in outcome s **EXPLANATION**.

$$\sum_{s=1}^S q_s = \sum_{s=1}^S \frac{\pi_s^*}{1 + R_f} = \frac{1}{1 + R_f} \sum_{s=1}^S \pi_s^* = \frac{1}{1 + R_f}$$

Why this step follows: each price is its risk-neutral probability discounted (the one-state pricing of §21.2); the risk-neutral probabilities of an exhaustive, mutually exclusive set add to one, which is the course's "They add up to one" (07b p. 17). Holding every contract pays \$1 whatever happens, and a sure dollar costs $1/(1 + R_f)$ — the course's point that any asset is priced from the Arrow–Debreu securities (07b p. 6), with the sure dollar the simplest such asset.

With a short horizon the sum is about 1, so the five prices read directly as a distribution over the five outcomes — "the distribution of potential outcomes" — which a single interest-rate futures price, an average over the outcomes, cannot give. The course's test of this, "check this", is left for that block.

21.4 The long-shot bias inside the game

"we took a look at the sports contracts to start, just to see if this kind of favorite long-shot bias actually appears in prediction markets, and it does. If you look at the difference between the biggest underdogs and the biggest favorites, the underdogs are way overpriced, so the returns are very, very negative." ⌚ 56:17 [transcript only: the chart]. What the exchange adds to the sportsbook data of \$19 is trading during the game: with sportsbooks "I get a closing price, and then I get no more information on prices until the game ends" ⌚ 56:41 , while on the exchange one can trade throughout the game — "in last night's baseball games, we probably make an average of about 90 to 100 bets per game, right? Pitch by pitch, you can make these bets" ⌚ 57:02 , and beyond betting, "you get information on the evolutionary probabilities" ⌚ 57:13 .

The finding: the favourite–long-shot bias, "preference for high-risk, low-payoff," 57:19 assets of a kind the recordings name differently (the Warning below), "exists throughout the game until the very end, and then something changes. I don't know what that is" 57:29 — perhaps "some combination of people's preferences and beliefs of different kinds of traders" 57:39 .

WARNING Said in class vs written — 57:19: the label on the overpriced assets

"preference for high-risk, low-payoff, or" — one recording hears "\$100-type", the other "underdog-type" — "assets". The note writes high-risk, low-payoff bets and adopts neither label.

The data are public: "On Polymarket, for instance, not only do you get to see every trade and every trading price, it's blockchain. You can identify who's trading." 57:47 A trader's identifier can be followed from trade to trade 57:56 ; the guest's joke on that point is left out. "So what's cool about that is you can do just a whole new area for science, of looking about how markets work when we map these things. And we're just starting to do that." 58:04 The chart was made "last week, we don't even have a paper yet, right? We're supposed to have one in two weeks." 58:14 .

Across all sports the picture is the same *[transcript only]*: a large long-shot bias through the game, and then, "At the very end, right before the game ends, something changes." 58:34 . The guest's hypothesis, offered as one: "I think what might be going on is just people unwinding their contracts. I suppose I bought it at 20 cents, and now it looks like they're going to win with 80% probability. I take the risk off the table. I don't know what's going on. That's a hypothesis." 58:37 . **EXPLANATION** A contract bought at 20 cents and now priced at 80 cents has quadrupled, $0.80/0.20 = 4$, a gain of 300% — a holder who sells it is locking in a win and removing the chance of a late reversal, which is what "take the risk off the table" 58:46 means. A sport-by-sport version was skipped.

WARNING Said in class vs written — 58:26: a clause both recordings carry but the transcript marks unclear

Both recordings render the bias as holding throughout the game except in the last 95 percent of the action — a phrase at odds with the clear sentence that follows it, that the change comes "At the very end, right before the game ends". The clause is marked unclear in the transcript and is not a figure or a finding; the note states only the clear sentence.

GAP Not recoverable — the prediction-market charts

The chart of underdog against favourite returns on the exchange's sports contracts, the in-game chart of the bias, and its sport-by-sport version are known only from the words above.

21.5 The talk's summing up, and why AQR is entering

"So, when I think about all these things, I try to think about, like, what things do we know from financial markets over the last 50 years? Beta works okay, things like value, momentum, carry, quality, defensive, they work even better at capturing what goes on in markets. And then they have analogs in all these new markets, which is interesting from a scientific perspective" ⌚ 58:54 . Why AQR is getting into these markets: "It's another diversifying asset class, right? We can do the same things we've always been doing in more places, just better Sharpe ratio." ⌚ 59:22 — §12.3's formula with one more low-correlation strategy added. "we're not the only ones. There's lots of other hedge funds that are looking into this" ⌚ 59:29 . The talk ends at ⌚ 59:42 with the offer of "at least 10 minutes for questions" ⌚ 59:37 .

Self-check. A "yes" contract trades at 30 cents and historically such contracts pay off 22% of the time (*illustrative numbers, the note's own, not the guest's*). In the note's symbols, what are q , π^* and p , and what does the gap between them say?

$q = 0.30$ is the price; $\pi^* = q(1 + R_f) \approx 0.30$ is the risk-neutral probability; $p = 0.22$ is the physical chance. $q > p$ means the contract is overpriced — a long shot that bettors pay too much for, the favourite-long-shot bias — and a seller of it earns about 8 cents per dollar of face value on average before fees.

Self-check. Five FOMC contracts trade at 5, 20, 55, 15 and 5 cents (*illustrative numbers, the note's own, not the guest's*). What should they add to, and what would it mean if they added to 110 cents?

About 100 cents, the price of a sure dollar over a short horizon (less a small discount). At 110 cents a trader could sell all five for \$1.10 and owe exactly \$1 whichever outcome occurs — a sure profit of 10 cents, an arbitrage, which competition removes; so prices that add to much more than a dollar signal either fees or a market nobody is yet arbitraging (§22.1).

22. Questions from the audience

⌚ 59:49

This section records the answers the guest gave in the ten minutes of questions, each standing on its own where the question itself was not captured.

GAP Not recoverable — four of the five questions

A second voice, not named by the recording, thanked the guest by first name — "Thanks, Toby."

⌚ 59:49 — and opened the questions; the rest of that sentence is rendered differently by the two recordings and is not a recap of anything, so nothing more of it is used. Four audience questions, at ⌚ 60:19, ⌚ 61:56, ⌚ 64:16 and ⌚ 66:13, were decoded by neither recording; each answer below stands without its question, and no question is invented. The guest's notice that AQR is hiring an undergraduate analyst for its prediction-market group is an announcement, not content.

22.1 Fees, and why arbitrage will follow them down

The answer at 60:28. The exchange's fees are "not 10%, but they're like, you know, on average about two and a half to three and a half percent. That's big, right?" ⌚ 60:28. For comparison, "AQR, we run \$260 billion, and we can trade in futures markets for like, you know, less than five" ⌚ 60:36 — a unit the recordings render differently (the Warning below). "So going from that to paying like 3% in fees, we can't put a lot of dollars to it, right?" ⌚ 60:50. Fees will fall: "We've got 12 exchanges now, all filing with the CFTC to compete." ⌚ 60:56. The US exchange wants institutions like AQR on board so that it is not treated as merely sports gambling, and for that "they need a number of things" ⌚ 61:14: a safe clearinghouse (which it has), lower fees, and leverage; it "has already filed with the CFTC to add leverage", and fees get negotiated down with volume "like you do with the CME" ⌚ 61:31. The economics in one sentence: "when fees are big, profit opportunities are big too, they gotta go hand in hand, as the fees come down, arbitrage will come in" ⌚ 61:47. The course's version is the Grossman–Stiglitz paradox — information is gathered only if its benefit exceeds its cost, so some mispricing must survive to pay for it (*05a p. 21*) — the costs its mutual-fund pages list (*05a p. 23*), which the note reads as one more limit on arbitrage (**EXPLANATION**), and Lamont and Thaler's account of what stops arbitrageurs (*LT03 pp. 10–11*).

WARNING Said in class vs written — 60:36: the unit of AQR's futures trading cost

"less than five" — one recording hears "days a month, like 0.5%, right?", the other "basis points, 0.05 of a percent, right?". Five basis points is 0.05% (*01a p. 8*), so the second recording's two numbers agree with each other and the first's do not; but internal consistency is not evidence of what was said, and the note states neither. The comparison with the exchange's 2.5–3.5% is therefore given per reading (**EXPLANATION**): against 0.05%, the exchange is $2.5/0.05 = 50$ to $3.5/0.05 = 70$ times dearer; against 0.5%, 5 to 7 times.

22.2 Price impact, market size, and the experiment approach

The answer at 62:20. What limits AQR is "what's called price impact. So if I try to put a billion dollars to work in a small market, the price might say I can trade at \$50 a share. As soon as I start trading, it's going to go up to like \$60." ⌚ 62:20 — a 20% move against the trader, $60/50 - 1 = 0.20$ **EXPLANATION**, which eats the edge. The course's definition is the third component of trading costs: price impact is the "Change in price caused by trade beyond the quoted depth", small in a deep market with many people trading and big in a thin one with few (*01b p. 11; the Lecture 2 note's §9.3*). On size: "\$3 billion was traded at the World Cup" ⌚ 62:40; on the NFL "you can trade a pretty decent amount of money, but not like" ⌚ 62:46 an amount the recordings disagree on.

WARNING Said in class vs written — 62:46 and 63:17: two scale figures

After "but not like", one recording hears "\$20 to \$60 million, of course," and the other "\$260 billion. Not even close,". And on how far the market could grow — "I think you can start trading tens of" — one hears "billions, maybe getting" and the other "millions, maybe even" — "closer to 100". The record settles neither; the note states only that NFL volume is limited and that the market could become very large — especially, the sentence ends, for small hedge funds in one rendering and small-sized markets in the other.

For now, "what we're doing and other hedge funds are doing is" to learn: "we have an account of, you know, we're talking about a few million dollars" — "nothing for an institution" — "really just seeing how they work, how the models work, how it clears" ⌚ 62:52. "At the rate it's growing, right, this thing could be very big" ⌚ 63:12 — in sports, the upcoming elections, and crypto. The approach is a big fund's standard one: "Citadel does the same thing", put "a little bit of dollars and a little bit of effort there to see where it goes, if it blows up, you're one of the first ones there, you make a lot of money. If it doesn't" ⌚ 63:34, it was an experiment "where you lost a few million bucks. Move on" ⌚ 63:50. Next year the guest "might say, hey, we shut it down, it didn't work" ⌚ 64:00; AQR took the same approach when it entered Chinese commodities and some emerging markets ⌚ 64:11.

22.3 Who makes the money, and is it inside information

The answer at 64:37, "No, I'll tell you why", takes two questions at once ⌚ 64:37. First, the informed participants who make most of the money are a small share — "it's a small percentage, but two or three percent. That percentage is actually somewhat larger than what we see in equity markets. So I think for mutual funds, about 2.6% actually have persistence, a really small number." ⌚ 64:52 [*the guest's figures; the clause before them is garbled on both recordings*]. The course's page on the same question, luck against skill among mutual-fund managers, prints no such percentage (*05a p. 25*). Second, "is that edge coming from illegal insider information?" ⌚ 65:14 — "insider information is a big word for manipulation of markets. Some of the contracts have more scope for manipulation. That's also true in traditional financial markets."

⌚ 65:18 A colleague's paper, presented the day before (Tice Jensen's, as heard), finds it "very small"

⌚ 65:34 : nobody has inside information about an FOMC announcement or last night's game, in the second recording's rendering, except perhaps a player's injury. The exchanges must remove it to survive, because a firm like AQR will not trade against someone holding material non-public information: "We're going to be on the wrong side of that all the time." ⌚ 66:01 . For AQR to enter, "markets just need to become more efficient" ⌚ 66:09 , in the guest's words; the clause that follows is garbled. The course's strong-form page says the same of stock markets: corporate insiders earn abnormal returns, "this is illegal, don't do it!" (05a p. 20); and the course's article on insider trading reports a case of it, a young investment-bank analyst charged by federal prosecutors and sued by the SEC for buying options on a company before its buyout was announced — a deal the analyst's own bank was financing — through a trading account kept hidden from the bank (*WSJins pp. 1–2*).

22.4 Adapting the models to short-dated, discrete contracts

The answer at 66:25, "that's a great question": "you have to modify the models somewhat, they're not exactly discrete because the prices are continuous, right, between zero and one, and like option contracts have a discrete payoff too, and certain bonds when you take them to maturity" ⌚ 66:25 — a price between 0 and 1 on a payoff of 0 or 1 is the Arrow–Debreu structure of §21.2. "the biggest thing that I've found is that that prediction market's a really short-term contract. Even the macro contracts, it's like what will the price of oil be like today? Our models aren't very good at that. Our models are good at figuring out what the price is, what could be a year from now, right? But trying to predict it on a daily basis is really tough."

⌚ 66:25 One of the exchange's most popular markets is "the 15-minute cryptocurrency market. I have no idea how to take that, to predict that" ⌚ 67:04 . So the models are hard to translate, and it is not a matter of just switching them on. The course's own statement of the horizon problem: returns are nearly unpredictable at short horizons and predictable only over longer ones (05a pp. 6–7).

22.5 What the guest would like to see: hedging, and "completing markets"

The one captured question, from a student: "What are some of the things that you would like to see in the future?" ⌚ 67:31 . The answer: this is what the US exchange would like to become — a competitor to the CME — and the example is "Liberation Day" ⌚ 67:44 : "we have massive positions in currencies and stock markets all over the world, we have no idea what our exposure is, right? Our job is not to bet on what Trump's going to say, because that's a fool's errand, but we know our models work on average." ⌚ 67:50 . "Unless we get crushed by something we can't measure or even know our risk, I would have loved to have hedged that." ⌚ 68:07 . The exchange now has a contract on whether tariffs on given countries rise by given amounts, and such contracts have two uses: "look at the pricing of those and measure our exposure" ⌚ 68:19 , and then decide "how much are we willing to pay to offset that exposure? Because that's not our business." ⌚ 68:26 . Those markets "are not liquid enough to hedge us all that much, but that's the dream" ⌚ 68:31 — to hedge regulation, elections, risks "that we can't currently hedge. We call it completing

markets in finance." ⌚ 68:36 . The course's definition belongs to a later block: markets are complete if the number of linearly independent assets equals the number of states (07b p. 8); a new contract on a state no existing asset spans is one more independent asset. "That's, I think, where these exchanges can add a lot of value to society." ⌚ 68:46 People may care enough about sports to hedge them too, "but you need speculators in order to hedge, so to me, that is a first use case for why we should have these markets that can do something that our current markets can't do. To me, that's really interesting." ⌚ 68:54 — the talk's last words, at ⌚ 69:06 .

Self-check. Why does the guest expect the exchange's fees to fall, and what does the course say must survive even then?

Twelve exchanges are filing with the CFTC to compete, and institutions will not trade at 2.5–3.5% when futures cost a small fraction of that; as fees fall, arbitrage capital arrives and prices tighten. The course's Grossman–Stiglitz paradox (05a p. 21) says some mispricing must survive to pay the arbitrageurs for gathering information — efficiency is partial, not complete.

Self-check. In what sense would a liquid tariff contract "complete" a market for a fund like AQR?

The fund's positions pay off differently in the state in which tariffs on country X rise by Y and in the state where they do not, but no existing asset pays on that distinction, so the exposure cannot be measured or offset. A contract that pays in that state is a new linearly independent asset; its price measures the exposure and buying it offsets the risk — 07b p. 8's complete markets, one state at a time.

Continues in

The course's next meeting is Tuesday 13 October, the Lecture 11 note (13 Oct, reconstructed): the market-efficiency deck (05a), whose pages this note cites as related, from its first page to p. 22 — the end evidenced by 15 October's opening, the start inferred. The deck's last pages, 23–27 on mutual funds, open the Lecture 12 note (15 Oct). This talk continues no deck of the course's and says nothing about either lecture. The course's own practice on market efficiency, the joint test and data mining — Block 5 of the practice workbook — is worked in the Lecture 11 note (problem block G below lists it), and both notes link back to the data-mining exercise of §13 rather than reworking it (Lecture 11 note §5.2; Lecture 12 note §2.4).

Worked problems

No workbook item is owned by this meeting: the course's practice on the topics the guest discussed belongs to the lectures that taught them, and is listed and linked at the end of this section. What this meeting owns is the five questions the guest put to the room and answered aloud — in-class exercises, worked here under the course's own basis for practice: "There are no graded homeworks, but you are expected to work through the concept questions and the practice problems every week" (*syl p. 3*). No official solution exists for any of them; each is derived from the course's own formulas and machine-checked. Four are worked below in the talk's order; the fifth, the buy-or-sell question at 14:05, is the course's own 04b p. 10 exercise and is worked in the Lecture 9 note, so the link block at the end restates its answer rather than reworking it. Every number the lecture body states about these problems is the problem's.

Lecture examples, and where the note works them

The guest's own numerical examples are worked where they fall in the lecture; this table says where.

The guest's example	Where it is worked
A long beta of 0.5 against a short beta of 1.2: "How do I make those even?"	§8.2, and problem C below (2.4 : 1)
About eight independent markets at a Sharpe ratio of 0.4–0.5: "more than double"	§8.4's derivation ($\sqrt{8} \times 0.4 = 1.13$, $\times 0.5 = 1.41$)
Thirteen factors at 0.63 against the market's 0.4: "more than 50% larger"	§12.2 ($0.63/0.4 = 1.575$)
Equal-weighted, a Sharpe ratio of 1.5: "two and a half times higher"	§12.2's Warning (2.38 or 3.75) and §12.3's derivation (implied $\bar{\rho} = 0.108$)
"cut that in half": 0.75–0.8	§12.4
A Sharpe ratio of three "should not lose money in 100 years"	§12.5's derivation ($\Phi(-3) = 0.00135$)
100 strategies at the 5% level: "five are gonna look good"	§13.3, and problem D below
A 58% favourite played 1,000 times	§17's Warning (580 as arithmetic)
Underdogs lose 22%: 10% plus 12%; favourites –10% plus 8–9%	§19.3
A contract at 56 cents "implies a 56% chance"	§21.2
A \$50 price that moves to \$60 as a billion dollars is traded	§22.2 (20%)

A. The tangency portfolio in equilibrium — the question at 04:27

The talk's first question to the room is the CAPM's one piece of economics: if every investor holds the same risky portfolio in some mix with the risk-free asset, what must that portfolio be? The room answered "the market portfolio", and §2.3 states why. The self-check below trains the derivation the answer rests on: adding up everyone's risky dollars asset by asset and setting the sum equal to what exists forces the common portfolio's weights to be market-capitalization weights, $w_M^i = p_i n_i / \sum_j p_j n_j$ (04a p. 6) — and it says why that lets a market index stand in for a tangency portfolio nobody can compute (§3.3). The material is the Lecture 8 note's; this meeting's contribution is the question in the guest's form.

Self-check — the guest's question at 04:27: what must the tangency portfolio be in equilibrium?

Source: a question the guest put to the room, 04:27 – 04:59 , answered aloud from the room and repeated by the guest at 04:50 . The course's own statement of the answer is 04a pp. 2, 4–7, taught in the Lecture 8 note (1 Oct). No written solution exists for the spoken question; the answer below is the course's argument, with one step the slides leave out written in full and labelled.

QUESTION

The guest had just drawn the efficient frontier of risky assets and the tangent line from the risk-free rate, and called the tangency portfolio "pure math": "There's nothing, there's no economics in this. That is pure math." (03:54). Then came the question:

"The economics is, what is the tangency portfolio in equilibrium? Economics is supply equals demand. We apply a little equilibrium concept. So what has to be the tangency portfolio? Don't be bashful. What does it have to be in equilibrium if we all hold the same portfolio in different combinations with a risk-free asset?" (04:27)

In the course's words: every investor splits wealth between the risk-free asset and one tangency portfolio T , the same for all. In equilibrium, which portfolio must T be, and with what weights?

ANSWER

Reveal the answer

The market portfolio — every risky asset, held in proportion to its market value

The room answered and the guest repeated it: "Thank you. The market portfolio. And the market portfolio is the market portfolio of everything weighted in proportion to how we value the economy." (🕒 04:50). The course says the same in its own words: "the tangency portfolio must be the portfolio of all existing risky assets, the 'market portfolio'" (04a p. 5), with weights equal to market capitalizations (04a p. 6).

Why

Portfolio theory alone gives only the **demand** for securities (04a p. 2). If all investors "perceive the same means, variances, and covariances for returns" (04a p. 4, assumption 6), they all compute the same tangency portfolio and differ only in how much of their wealth they put in it (04a p. 5: "The tangency portfolio is the same for all investors"; "Key assumption: homogeneous expectations"). Equilibrium adds the **supply** side, "Supply = Demand" (04a p. 2): the supply of a risky asset is its shares outstanding, and the demand is the tangency portfolio (04a p. 5). The only way the same portfolio, held by everyone, can absorb exactly the shares that exist is if it holds every asset in proportion to the value of its shares. The slides state the conclusion; the aggregation that delivers it is below.

DERIVATION Full derivation (not done in class) — supply = demand forces $w_T^i = p_i n_i / \sum_j p_j n_j = w_M^i$

Write W_k for investor k 's wealth, y_k for the share of it put in T (the rest is in the risk-free asset, positive or negative), w_T^i for T 's weight on risky asset i , p_i for its share price and n_i for its shares outstanding (04a p. 6's symbols). Write $D = \sum_k y_k W_k$ for the total dollars all investors put in T .

$$\sum_k y_k W_k w_T^i = w_T^i \sum_k y_k W_k = w_T^i D$$

Why this step follows: investor k holds $y_k W_k w_T^i$ dollars of asset i ; summing over investors, the common factor w_T^i comes out of the sum because T is the same for everyone.

$$w_T^i D = p_i n_i \quad \text{for every risky asset } i$$

Why this step follows: market clearing, supply = demand asset by asset (04a pp. 2, 5); the supply of asset i in dollars is the value of its shares outstanding.

$$D \sum_i w_T^i = \sum_i p_i n_i$$

Why this step follows: add the clearing condition over all risky assets i .

$$D = \sum_i p_i n_i$$

Why this step follows: the weights of a portfolio sum to one, $\sum_i w_T^i = 1$.

$$w_T^i = \frac{p_i n_i}{\sum_j p_j n_j} = w_M^i$$

Why this step follows: divide the clearing condition for asset i by D and substitute the line above; the result is 04a p. 6's market portfolio weight, a security's market capitalization over the total market capitalization.

Two things fall out. The tangency portfolio's weights are the market's, whatever the investors' wealth and risk aversion. And the total held in risky assets equals the value of all risky assets, $D = \sum_i p_i n_i$: taken together, the investors hold the whole market.

A numerical check with 04a p. 6's example. Two risky assets, $p_A = 2$, $n_A = 25$, $p_B = 4$, $n_B = 50$:

$$p_A n_A = 2 \times 25 = 50, \quad p_B n_B = 4 \times 50 = 200$$

Why this step follows: a market capitalization is price per share times shares outstanding (04a p. 6).

$$w_M^A = \frac{50}{50 + 200} = \frac{50}{250} = 20\%, \quad w_M^B = \frac{200}{250} = 80\%$$

Why this step follows: each capitalization over the total, 250; the two weights sum to 100%.

So in equilibrium every investor's tangency portfolio holds 20% in A and 80% in B, and investors together put \$250 into it, exactly the value of the shares that exist. (04a p. 6 prints the first weight as w_A^i ; by its own notation it is w_M^A . The numbers are right.)

How prices get there (04a p. 7). If investors' desired weights put relatively more on i than the market does, $w_M^i / w_M^j > (p_i n_i) / (p_j n_j)$, then "more dollars are chasing i than the value of shares of i ". The price of i is pushed up, which lowers its expected return and the weight investors want in it, until the ratio of desired weights equals the ratio of market values. The Lecture 8 note's 04a p. 7 exercise works this adjustment through.

Why the guest asked. The answer turns a statistical problem into an economic shortcut. The tangency portfolio needs every mean, variance and covariance to compute, and those are what an investor wants it for: "It circumvents that circular problem" (06:43). If equilibrium says the tangency portfolio is the market, "now if I have a proxy for the market, I can back out all these things that are really difficult to estimate" (06:56). The course puts it as "No need to calculate the tangency: use the market portfolio!" (04a p. 9).

What the answer is not (*the note's own*)

- **Not each investor's own favourite mix.** With the same inputs, the best risky mix is the same for everyone (04a p. 5); investors differ only in how much they hold in it and how much in the risk-free asset.
- **Not an equally weighted portfolio.** Supply is measured in value: a company whose shares are worth four times as much must be held four times as much, as B is against A in the example above.
- **Not the global minimum variance portfolio.** That is the leftmost point of the frontier (03d p. 3). The tangency portfolio is "the portfolio with the highest Sharpe ratio" (03d p. 4), where the line from the risk-free rate touches the frontier. Equilibrium does not move investors to a different point of the frontier; prices adjust until the market portfolio is that point.

- The argument as Lecture 8 taught it, with the market portfolio's weights and the adjustment: the Lecture 8 note (1 Oct), §9.
- What follows from it, the capital market line and "use the market portfolio": the Lecture 8 note, §10.

B. Why a portfolio's average is usable and a stock's is not — the question at 09:28

The second question is the one the early tests turn on (§5.2): why is it "okay to take a historical average of a portfolio of stocks, but it's not really okay to do it for an individual stock?" The guest's answer, "not just risk, but also estimation error", is made exact here with the course's own volatilities — 44% for one stock, 12% for 500 (*03d p. 14*) — over the five-year window the course's beta regressions use (*04b p. 6*): the standard error of a mean, $\sigma/\sqrt{Y}$, is 19.68% for the stock and 5.37% for the portfolio, and uncorrelated errors average toward zero exactly as idiosyncratic shocks do (*03d p. 8's algebra*). The self-check also asks what does *not* average away and how many years a given precision would need — the numbers behind §3.2's "absolute garbage".

Self-check — the guest's question at 09:28: why is a portfolio's historical average a usable mean when a single stock's is not?

Source: a question the guest put to the room,  09:28 –  09:50, while describing the early tests of the CAPM; the guest answered it after a reply from the room that the recording does not carry. The course's written sources for the answer are *04b p. 8*, *03d pp. 8, 14–15* and *03S p. 12*. No written solution exists for the spoken question. The numbers below are the note's own illustration with the course's figures, and are labelled.

QUESTION

The early tests plotted portfolios, not single stocks: "On the y-axis, you have the actual average excess return on these portfolios, averaged over time." ( 09:20). The guest then asked why:

"So averaging an individual stock return is not a very good measure of its mean. For a portfolio, it's much better. Why is that? Why is it okay to take a historical average of a portfolio of stocks, but it's not really okay to do it for an individual stock? It's not a trick question. What do you think? What does a portfolio do for you?" ( 09:28)

ANSWER

Reveal the answer

Diversification works on estimation error as well as on risk — a portfolio's average return is far less noisy, and the stocks' errors, if uncorrelated, average away

The guest's answer, building on a reply from the room: "Yeah, not just risk, but also estimation error, right? So, you know, we always think in terms of risk, because that's the way we write it down in terms of theory, but in reality, in the world we live in, where I'm trying to estimate this in AQR, it's also about estimation error, which also diversifies away, right? It should be uncorrelated to each other." (🕒 09:50). Earlier the guest had put the problem bluntly: "if you take the historical average return of a single stock, it is absolute garbage" (🕒 05:19), because "signal-to-noise ratios in financial markets are tiny" (🕒 05:28) and the average is "mostly just an average of noise" (🕒 05:34).

Why, step 1: a historical average is noisy in proportion to the volatility (*the note's own illustration*)

Assume each year's return is an independent draw with the same mean and volatility σ ; the guest did not state this (EXPLANATION). The average of $\bar{Y}$ years then has a standard error of

$$SE(\bar{R}) = \frac{\sigma}{\sqrt{Y}}$$

Why this step follows: the average of $\bar{Y}$ independent draws has variance $Y\sigma^2/Y^2 = \sigma^2/Y$, the equally weighted variance of 03d p. 8 with years in place of stocks; the standard error is its square root.

For a single stock, the course's volatility is 44% (03d p. 14: "Volatility of holding a single stock: 44%"). Over five years, the window the course uses for betas (04b p. 6: "e.g., 5 years/60 months"):

$$SE = \frac{44\%}{\sqrt{5}} = \frac{44\%}{2.2361} = 19.68\%$$

Why this step follows: substitute $\sigma = 44\%$ and $\bar{Y} = 5$; $\sqrt{5} = 2.2361$.

That is 3.3 to 4.9 times the whole market risk premium, "4% to 6% in practice" (04a p. 14): $19.68/6 = 3.28$ and $19.68/4 = 4.92$. Five years of one stock's returns cannot tell a stock that deserves a 4% premium from one that deserves none. For a portfolio of 500 stocks the course's volatility is 12% (03d p. 14):

$$SE = \frac{12\%}{\sqrt{5}} = 5.37\%$$

Why this step follows: the same formula with $\sigma = 12\%$.

The portfolio's average is $44/12 = 3.67$ times more precise, because diversification has removed most of the noise in its returns.

Why, step 2: the stocks' estimation errors average out, if they are uncorrelated (*the note's own algebra, on the course's formula*)

Write each stock's historical average as its true mean plus an error e_i with variance ν_i^2 **EXPLANATION**. The average of an equally weighted portfolio of N stocks is the average of their averages, so its error is $\frac{1}{N} \sum_i e_i$. If the errors are uncorrelated, the guest's condition,

$$\text{Var} \left[\frac{1}{N} \sum_{i=1}^N e_i \right] = \frac{1}{N^2} \sum_{i=1}^N \nu_i^2 = \frac{1}{N} \left[\frac{\sum_{i=1}^N \nu_i^2}{N} \right]$$

Why this step follows: 03d p. 8's variance of an equally weighted portfolio with zero correlations, applied to the errors instead of the returns: each squared weight is $1/N^2$ and every covariance term is zero; then factor out $1/N$ to leave the average error variance.

As N grows, this goes to zero: the law of large numbers, "'Randomness disappears' by averaging" (03S p. 12). The course makes the same argument for betas, and states the guest's condition: in the second step of the test "one forms portfolios of stocks, because betas are estimated more precisely for portfolios", and "This make sense if measurement noise is uncorrelated" (04b p. 8, as printed). The guest applies the same logic to average returns.

What does not average out (*the note's own caveat*)

An error that every stock shares, such as the whole market's good or bad luck over the sample years, does not shrink with N , for the same reason systematic risk does not: with correlated terms the limit is the average covariance, not zero (03d p. 11). That is why the guest's condition matters: the errors "should be uncorrelated to each other" (🕒 09:50).

How long a sample would be needed (*the note's own illustration*)

Turning the standard error around, $Y = (\sigma/\text{SE})^2$. To bring the error down to 2%, half of a 4% premium:

$$Y_{\text{one stock}} = \left(\frac{44}{2} \right)^2 = 484 \text{ years}, \quad Y_{500 \text{ stocks}} = \left(\frac{12}{2} \right)^2 = 36 \text{ years}$$

Why this step follows: solve $\text{SE} = \sigma/\sqrt{Y}$ for Y by squaring both sides and rearranging, then substitute each volatility with $\text{SE} = 2\%$.

Even a diversified portfolio needs decades of data, which is the course's first problem with mean-variance optimization in practice: "Estimates of means are imprecise over short periods", "Garbage in-garbage out" (03d p. 15).

Tempting answers that are wrong (*the note's own*)

- **A portfolio earns more.** No: an equally weighted portfolio's expected return is the average of its stocks' (formula sheet p. 1 item 7). Diversification shrinks the noise, not the mean.
- **A portfolio's errors always cancel.** Only errors that are uncorrelated across stocks; a shared error stays, as above.
- **A portfolio's beta is 1, so its mean is known.** No: the test sorts stocks into portfolios with deliberately different betas, from the lowest-beta tenth to the highest (04b p. 8), and estimates each portfolio's average return.

LINKS

- The equally weighted variance with independent returns, the formula used in step 2: the Lecture 7 note (29 Sep), §11.2.
- The law of large numbers: the Lecture 5 note (22 Sep), §4.2.
- The 44% and 12% volatilities, and why estimated means are "garbage in": the Lecture 8 note (1 Oct), §3 and §4.
- The two-step test and portfolio sorting by beta: the Lecture 9 note (6 Oct).

C. Making the two legs of Betting Against Beta even — the question at 16:49

The one numerical exercise that tests material only this meeting taught. Given the guest's betas, 0.5 on the portfolio bought and 1.2 on the portfolio shorted, the problem derives the dollar ratio that makes the position's beta zero (2.4 : 1), the borrowing it needs (\$1.40 per dollar shorted), the long leg's loan-to-value, margin and leverage (03c p. 10), the same proportion by scaling each leg to a beta of one, and the expected payoff $2.4\alpha_L - \alpha_H$ — zero under the CAPM, and the alphas otherwise. It uses only the formula sheet's covariance rule and beta (formula sheet p. 1, items 6 and 10) and the reading's portfolio-beta footnote (FF04 p. 8), and it ends with §7's point that zero beta is neither zero risk nor an arbitrage. §8.2 states its results; the derivation lives here.

The guest's question at 16:49 — making the two legs of Betting Against Beta even

Source: a question the guest put to the room, (🕒 16:49), answered by a student at (🕒 17:19) and by the guest at (🕒 17:24). The betas are the guest's. No written solution exists: the answer is derived from the course's own formulas (formula sheet p. 1 items 6 and 10; 03c pp. 10–11; 01b p. 13) and machine-checked.

The setting is the strategy the guest had just introduced, from a paper by Andrea Frazzini and Lasse Pedersen: "if I buy the low beta stocks and short the high beta stocks" (🕒 15:48) and take the same amount of risk on each side, "I can make what looks like really good money on a zero investment strategy" (🕒 16:18). The question was how to take the same amount of risk on each side:

"I buy a portfolio of stocks whose on average their beta is about 0.5, and I'm shorting a portfolio of stocks whose beta on average let's say is 1.2. How do I make those even?" (🕒 17:03)

A student answered: "You buy more of the lower beta stocks relative to the company you're shorting." (🕒 17:19). The guest agreed: "That's right. I buy more of the low-beta stocks in exact proportion to match that 1.2 beta, so I'm evenly matched. And how am I going to do that if I don't have a lot of money? I'm going to borrow it and use leverage." (🕒 17:24).

SETUP

How many dollars of the low-beta portfolio must be bought for each dollar of the high-beta portfolio sold short so that the position's beta is zero, how is the purchase financed, and how much leverage does that take?

GIVEN

Input	Value	Units	Source
Beta of the portfolio bought (low-beta stocks), β_L	about 0.5	—	the guest, ⌚ 16:49
Beta of the portfolio shorted (high-beta stocks), β_H	about 1.2	—	the guest, ⌚ 16:49
Amount shorted	\$1 of the high-beta portfolio	dollars	the note's normalisation EXPLANATION : every amount below is per dollar shorted
Beta	$\beta_i = \text{Cov}[R_i, R_M] / \sigma_M^2$	—	formula sheet p. 1 item 10; 04a p. 12
Covariance of a weighted sum	linear in each weight; a constant has no covariance	—	formula sheet p. 1 item 6 (03S p. 10); FF04 p. 6 fn 3
Borrowing and lending	at the risk-free rate R_f , a constant	per year	the course model's assumption (03c p. 11); the guest did not state the rate EXPLANATION , and questions the assumption at ⌚ 12:10
Leverage measures	$LTV = B/V$, margin = $(V - B)/V$, leverage = $V/(V - B)$	—	03c p. 10
A short sale	"sell something you don't own"; it pays minus the return of what is shorted	—	01b p. 13
A short sale's requirements (in practice)	"Short sale proceeds must remain with the broker"; "The short seller must deposit collateral (post margin)"	—	01b p. 13
Own money put up	\$1, the long leg's down payment	dollars	the note's normalisation EXPLANATION : the short sale's \$1 stays with the broker, so the long

Input	Value	Units	Source
			leg is paid for with own money and borrowing
What the proceeds held by the broker earn	R_f	per year	the note's assumption EXPLANATION : 01b p. 13 does not say; WHAT VARIES gives the case where they earn nothing

Here R_L , R_H and R_M are the returns of the low-beta portfolio, the high-beta portfolio and the market over the period; x_L is the dollar amount of the low-beta portfolio bought; Π is the position's payoff in dollars over the period, per dollar shorted, in excess of what the investor's own \$1 would have earned at R_f ; β_{Π} is its beta; B and V are the amount borrowed and the value bought; α_L and α_H are the two portfolios' alphas.

STRATEGY

The position's beta is the dollar-weighted sum of its legs' betas, with the short leg entering with a minus sign and borrowed money contributing nothing; set that sum to zero and solve for the amount bought. Then read the long leg as a levered purchase, paid for with \$1 of own money and borrowing since the short sale's proceeds stay with the broker, with 03c p. 10's three measures, and check the answer the other way the guest describes, by scaling each leg to a beta of one.

SOLUTION

Step 1. The position's payoff. Put up \$1 of own money, borrow the rest of the long leg's cost at R_f , buy x_L dollars of the low-beta portfolio, and short \$1 of the high-beta portfolio, whose \$1 of proceeds stays with the broker (01b p. 13):

$$\Pi = x_L R_L - R_H - (x_L - 1) R_f$$

Why this step follows: the long leg earns $x_L R_L$; $x_L - 1$ dollars of it are borrowed, which costs $(x_L - 1) R_f$; a short sale pays minus the return of what was sold (01b p. 13), $-R_H$; and the \$1 of proceeds held by the broker earns R_f (the note's assumption, in GIVEN), which exactly offsets the R_f the investor's own \$1 would have earned elsewhere, so the two do not appear.

$$\Pi = x_L (R_L - R_f) - (R_H - R_f)$$

Why this step follows: expand $-(x_L - 1) R_f = -x_L R_f + R_f$ and group each leg with its own R_f .

Step 2. The position's beta.

$$\text{Cov}[\Pi, R_M] = x_L \text{Cov}[R_L, R_M] - \text{Cov}[R_H, R_M]$$

Why this step follows: covariance is linear in each weight, and the constant R_f terms have no covariance with R_M (formula sheet p. 1 item 6's rule; FF04 p. 6 footnote 3 takes the same step for the market portfolio).

$$\beta_{\Pi} = \frac{\text{Cov}[\Pi, R_M]}{\sigma_M^2} = x_L \beta_L - \beta_H$$

Why this step follows: divide both covariances by σ_M^2 and use the definition of beta (formula sheet p. 1 item 10). It is a portfolio's beta as a weighted sum of betas (FF04 p. 8 footnote 4), with weight x_L on the long leg and -1 on the short leg.

Step 3. Why equal dollars are not enough. With \$1 bought and \$1 shorted, $x_L = 1$:

$$\beta_{\Pi} = 1 \times 0.5 - 1.2 = -0.7$$

Why this step follows: Step 2 with $x_L = 1$. That position is net short the market. Under the CAPM, beta alone would make it lose 0.7 times the market premium on average, $0.7 \times 4\% = 2.8\%$ to $0.7 \times 6\% = 4.2\%$ a year per dollar at 04a p. 14's "4% to 6% in practice", whatever the two portfolios' mispricing. This is the risk the guest wanted to balance.

Step 4. Set the beta to zero.

$$x_L \times 0.5 - 1.2 = 0$$

Why this step follows: "evenly matched" means the two legs' market exposures cancel, Step 2's beta at zero, with the guest's two betas.

$$x_L = \frac{1.2}{0.5} = 2.4$$

Why this step follows: add 1.2 to both sides, then divide by 0.5.

So the position buys **\$2.40 of the low-beta portfolio for every \$1 of the high-beta portfolio sold short**: more of the low-beta stocks, as the student said, and, in the guest's words, "in exact proportion to match that 1.2 beta" (🕒 17:24). The long leg's market exposure is now $2.4 \times 0.5 = 1.2$, the same as the short leg's.

Step 5. The financing.

$$B = x_L - 1 = 2.4 - 1 = 1.4$$

Why this step follows: what is bought, \$2.40, less the investor's own \$1.

\$1.40 is borrowed per dollar shorted, on top of \$1 of the investor's own money: "I'm going to borrow it and use leverage." The \$1 the short sale raises pays for none of it, because the course's practical requirements keep it with the broker and make the short seller post collateral as well (01b p. 13); that collateral is a further sum the page does not size, so it is left out of the count (EXPLANATION).

Step 6. The long leg as a levered purchase. Treat the investor's own \$1 as the down payment on $V = 2.4$ of the low-beta portfolio, with $B = 1.4$ borrowed:

$$LTV = \frac{B}{V} = \frac{1.4}{2.4} = 0.5833$$

Why this step follows: 03c p. 10's loan-to-value, the borrowed share of what is bought.

$$\text{margin} = 1 - LTV = \frac{2.4 - 1.4}{2.4} = 0.4167$$

Why this step follows: 03c p. 10's margin, the down payment's share.

$$\text{leverage} = \frac{1}{\text{margin}} = \frac{2.4}{2.4 - 1.4} = 2.4$$

Why this step follows: 03c p. 10's leverage ratio; it equals the weight on the risky asset per dollar of own money, as every row of 03c p. 11's table shows ($w = \text{leverage}$).

Leverage scales risk as well as return: per dollar of down payment the long leg's excess return and its volatility are both 2.4 times the low-beta portfolio's, $\sigma_p = w\sigma_i$ (03c p. 11). Its beta becomes $2.4 \times 0.5 = 1.2$: leveraging the low-beta portfolio gives it the high-beta portfolio's market risk. That is the sense in which matching betas matches risks; in the guest's words, "beta, by the way, predicts risk really well. It just doesn't predict returns all that well." (16:47).

Step 7. The same proportion, by scaling each leg to a beta of one. The guest described the paper's test as netting out the beta: "I neutralize it by either leveraging or de-levering" (18:01). Hold $1/\beta_L$ of the low-beta portfolio and $1/\beta_H$ of the high-beta one:

$$\frac{1}{0.5} = 2.0, \quad \frac{1}{1.2} = 0.8333$$

Why this step follows: a position of $1/\beta$ dollars in an asset with beta β has beta $(1/\beta) \times \beta = 1$ (Step 2's rule with one leg); the low-beta leg is levered up, the high-beta leg scaled down.

$$2.0 \times 0.5 - 0.8333 \times 1.2 = 1 - 1 = 0, \quad \frac{2.0}{0.8333} = 2.4$$

Why this step follows: Step 2's beta with these two amounts; then the ratio of the amounts, $\beta_H/\beta_L = 1.2/0.5$.

The ratio is Step 4's 2.4 : 1. Only the scale differs: per dollar shorted, the two answers are the same position.

Step 8. What the position earns. Write each leg's expected excess return with an alpha (04b p. 4):

$$\mathbb{E}[R_i] - R_f = \alpha_i + \beta_i(\mathbb{E}[R_M] - R_f)$$

Why this step follows: 04b p. 4's form of the security market line, the one the tests estimate; the CAPM says every α_i is zero.

$$\mathbb{E}[\Pi] = x_L \alpha_L - \alpha_H + (x_L \beta_L - \beta_H)(\mathbb{E}[R_M] - R_f)$$

Why this step follows: take the expectation of Step 1's second line, substitute the line above for each leg, and collect the two beta terms.

$$\mathbb{E}[\Pi] = 2.4 \alpha_L - \alpha_H$$

Why this step follows: Step 4 made the bracket zero, with $x_L = 2.4$.

With the betas matched, market movements cancel and the position collects the alphas: 2.4 times the low-beta portfolio's, plus the high-beta portfolio's with its sign reversed. If the CAPM held, both alphas would be zero and so would the expected payoff. The guest's evidence is that the alpha is highest for low-beta stocks and lower for high-beta ones (🕒 18:07); the two recordings describe the high-beta end differently, and the guest gave no alpha for these two portfolios, so no number is computed here. Zero beta is not zero risk: each leg's own shocks remain and the alphas are averages, so the position can lose money, and it is not an arbitrage (the Lecture 9 note's WB 4.1 Q16).

ANSWER

Buy \$2.40 of the $\beta = 0.5$ portfolio per \$1 of the $\beta = 1.2$ portfolio shorted (2.4 : 1)
Net beta: $2.4 \times 0.5 - 1.2 = 0$
Financing: \$1 own money + \$1.40 borrowed; the short sale's \$1 stays with the broker
The long leg is levered 2.4 times: LTV 58.33%, margin 41.67%
The same proportion as 2.0 long and 0.8333 short (each leg at $\beta = 1$)
Expected payoff: $\mathbb{E}[\Pi] = 2.4 \alpha_L - \alpha_H$

The answers given in class — the student's, to buy more of the lower-beta stocks, and the guest's, "in exact proportion" (🕒 17:24) — are the first line.

SANITY CHECK

The long leg is larger than the short leg, 2.4 against 1, as it must be when its beta is lower; the larger it is relative to the short, the more must be borrowed, so a low-beta-long, high-beta-short position needs

leverage whenever $\beta_L < \beta_H$. If the two betas were equal the ratio would be 1 and nothing would be borrowed. Scaling each leg to a beta of one gives the same ratio. With every alpha zero, Step 8's expected payoff is zero: under the CAPM a zero-beta position earns, on average, only the risk-free return on the investor's own \$1. A leverage of 2.4 is modest beside the 30 to 1 that 03c p. 11 reports for LTCM at its peak.

WHAT VARIES

Input	Direction	Size of the effect
The long leg's beta β_L (decisive)	a lower β_L needs more bought and more borrowed: $x_L = \beta_H / \beta_L$	inversely proportional; at the guest's values $\partial x_L / \partial \beta_L = -\beta_H / \beta_L^2 = -4.8$ per unit of beta
The short leg's beta β_H	a higher β_H needs more bought	proportional; $\partial x_L / \partial \beta_H = 1 / \beta_L = 2$ per unit of beta
The cost of borrowing	does not change x_L ; if the \$1.40 costs $R_f + \delta$ instead of R_f , the expected payoff falls by 1.4δ	the guest's wedge between the risk-free rate and what investors can actually borrow at (12:10) is the guest's explanation of why the opportunity exists
What the broker pays on the short sale's proceeds	does not change x_L or the beta; if the \$1 held by the broker earns nothing instead of R_f , the payoff falls	by R_f per dollar shorted, the interest the \$1 no longer earns (EXPLANATION)
Whether the investor can borrow at all	without borrowing, the investor's own \$1 buys only \$1 of the long leg per dollar shorted	beta -0.7 (Step 3): the position cannot be balanced; the guest's examples of investors constrained by leverage are individuals and many mutual funds (20:57)

D. A hundred strategies at the 5% level — the question at 38:33

The data-mining arithmetic of §13.3 in full: the expected number of false positives among 100 strategies tested at the 5% level (5, with no independence needed), the chance of at least one (99.41%, with independence assumed and labelled), the spread of the count, and what an out-of-sample test does — a second independent 5% test cuts the expected number of survivors to 0.25. The problem also scales the count to the guest's figures for the literature (157 factors; 300–400 papers) as arithmetic about a search of that size, not a claim about those factors. The course's page prints the headline count (05a p. 25); the

market-efficiency notes link here rather than reworking it (Lecture 11 note §5.2; Lecture 12 note §2.4, where 05a p. 25 was taught).

The guest's question at 38:33 — a hundred strategies tested at 5%: how many look good by chance, and what is the defence?

Source: the guest's example and question, 38:33 – 39:30 ; the room's answer was repeated by the guest. The course prints the same count on 05a p. 25 and the defence on 05a pp. 16–17; the reading FF04 p. 14 gives the standard response. No written solution exists for the spoken question: the arithmetic below is derived from the premise and machine-checked.

The guest's three schools of thought on what drives the anomalies ended with the third:

"And then the last thing you do have to worry about is data mining. [...] Those patterns may not actually be real, right? If I look at 100 different stock strategies and I have a 95% significance level that I'm testing, just by random chance, five are gonna look good, right? That doesn't mean anything, it's just random, right? So I have to, what's my best weapon against data mining? How do I know I've found something real?" (38:33)

SETUP

If 100 strategies with no real edge are each tested at the 5% significance level, how many should look significant by chance, how likely is it that at least one does, and how does testing out of sample separate a real pattern from a lucky one?

GIVEN

Input	Value	Units	Source
Strategies tested	100	strategies	the guest, ⌚ 38:33 (the same in both recordings)
Significance level	5%: a strategy with no edge passes the test with probability 0.05	probability	the guest's "a 95% significance level" is a test at the 5% level, that is with 95% confidence; 05a p. 25 says "significant at the 5% level"
The strategies' true edge	none: any success is "just by random chance"	—	the guest's premise
Independence of the 100 tests	assumed for Steps 3 to 6; neither the guest nor 05a p. 25 states it EXPLANATION	—	the note
The course's statement	"5 out of 100 will perform well enough to be significant at the 5% level"	—	05a p. 25

I_k is 1 if strategy k looks significant and 0 if not; K is the number that look significant.

STRATEGY

Each test is a yes-or-no event with probability 0.05, so the number of false discoveries is a sum of 100 such events. Its expected value needs only the rule that expectations add; the chance that at least one appears needs independence. Then put the survivors through a second, independent test, to see what an out-of-sample check does to the count.

SOLUTION

Step 1. One strategy.

$$\Pr(I_k = 1) = 0.05$$

Why this step follows: testing at the 5% level means that a strategy with no edge produces a result past the cutoff 5% of the time, by luck alone.

Step 2. The expected count.

$$\mathbb{E}[K] = \mathbb{E}\left[\sum_{k=1}^{100} I_k\right] = \sum_{k=1}^{100} \mathbb{E}[I_k]$$

Why this step follows: the expectation of a sum is the sum of the expectations (03S p. 10's rule for means, extended to 100 terms); no independence is needed.

$$\mathbb{E}[K] = 100 \times 0.05 = 5$$

Why this step follows: a 0-or-1 variable's expectation is its probability of being 1, 0.05 for each of the 100 strategies.

Five strategies are expected to look good by chance, the guest's "five are gonna look good" and 05a p. 25's "5 out of 100".

Step 3. The chance that none looks good.

$$\Pr(K = 0) = 0.95^{100} = 0.005921$$

Why this step follows: with independent tests, the probability that all 100 fail to pass is the product of 100 probabilities of $1 - 0.05 = 0.95$.

Step 4. The chance that at least one looks good.

$$\Pr(K \geq 1) = 1 - 0.005921 = 0.9941$$

Why this step follows: at least one strategy passing is the complement of none passing, so subtract Step 3's probability from 1.

A search through 100 worthless strategies turns up at least one that passes the test 99.41% of the time.

Step 5. How much the count itself varies.

$$\text{Var}[K] = 100 \times 0.05 \times 0.95 = 4.75, \quad \sqrt{4.75} = 2.18$$

Why this step follows: a 0-or-1 variable with probability 0.05 has variance 0.05×0.95 ; with independent tests every covariance is zero, so the variances of the 100 terms add (03S p. 10's variance rule); the standard deviation is the square root.

Five is an average, and a single search can land well away from it:

$$\Pr(3 \leq K \leq 7) = \sum_{k=3}^7 \binom{100}{k} 0.05^k 0.95^{100-k} = 0.754$$

Why this step follows: with independent tests, K counts the successes in 100 independent tries with probability 0.05 each, so a count of exactly k has the binomial probability shown (the number of ways to pick which k strategies pass, times the probability of that pattern); add the five terms (the note's own illustration).

About one search in four turns up fewer than three or more than seven lucky strategies.

Step 6. The best weapon: test again out of sample. The room's answer, as the guest repeated it: "Out-of-sample testing, right? Because if it's just noise, it won't repeat. And out-of-sample can take many forms. Different markets, different asset classes, different time periods, right?" (39:11). A strategy with no edge passes a fresh, independent test with probability 0.05 again, so it passes both with probability

$$0.05 \times 0.05 = 0.0025$$

Why this step follows: the probability that two independent events both happen is the product of their probabilities.

$$\mathbb{E}[\text{strategies passing both tests}] = 100 \times 0.0025 = 0.25$$

Why this step follows: Step 2's argument with the probability 0.0025 in place of 0.05.

$$\Pr(\text{at least one passes both}) = 1 - (1 - 0.0025)^{100} = 1 - 0.7786 = 0.2214$$

Why this step follows: Steps 3 and 4 with 0.0025 in place of 0.05.

The second test cuts the expected number of false discoveries from 5 to 0.25, twenty times fewer, because luck has to strike twice. A real effect is not cut this way: it shows up again in the new sample. That is the course's point that "In-sample predictability need not imply out-of-sample predictability" (05a p. 16), and its figure of a fifth-order polynomial that fits the points it was estimated on and fails beyond them, "Parsimonious models often perform better out-of-sample" (05a p. 17). The reading gives the same response: "A standard response to this concern is to test for similar findings in other samples" (FF04 p. 14), and reports the book-to-market effect found again in Japanese stocks and in four European stock markets, and the price ratios that trouble the CAPM in U.S. data showing up in the same way in twelve non-U.S. major markets.

ANSWER

Expected false discoveries: $100 \times 0.05 = 5$ (no independence needed)

Chance of at least one, with independent tests: $1 - 0.95^{100} = 99.41\%$

The defence is out-of-sample testing: a lucky strategy passes a second test only 5% of the time, so the expected number passing both falls to $100 \times 0.05^2 = 0.25$

The guest's "five" (🕒 38:51) and "Out-of-sample testing" (🕒 39:11) are the first and third lines; 05a p. 25 prints the first.

SANITY CHECK

Five is 5% of 100, as 05a p. 25 says ✓. The chance of at least one false discovery is close to one because there are 100 chances at 5% each: 0.95^{100} is tiny ✓. With a single strategy the same formula gives $1 - 0.95 = 0.05$, the level itself ✓. Out of sample, the expected count is the in-sample count times 0.05, smaller by the same factor that made each strategy unlikely to pass in the first place ✓.

WHAT VARIES

Input	Direction	Size of the effect
The number of strategies tried (decisive)	the expected count of false discoveries rises in proportion	for a search the size of the guest's database of 157 replicated factors (🕒 32:47), $157 \times 0.05 = 7.85$ if none had an edge; for the "three or four hundred" papers on new signals (🕒 30:53), 15 to 20. This is the arithmetic for a search of that size, not a claim about those factors
The significance level	a stricter level cuts false discoveries in proportion	the course's rule of thumb " $t > 2$ means coefficient statistically significant from zero" (04a p. 24) is about a 4.55% two-sided level under a normal distribution, $\Pr(Z > 2) = 0.0455$ for a standard normal Z : 4.55 expected out of 100 (<i>the note's own bridge</i>)
Correlation between the tests	leaves the expected count at 5 (Step 2 needs no independence); positively correlated tests lower the chance of at least one	at the extreme of 100 identical tests the chance is just 0.05
Each further out-of-sample test	multiplies the expected count of false discoveries by 0.05	5, then 0.25, then 0.0125

E. Which assets to buy and which to short — the question at 14:05 — and the course's practice on the guest's topics

The guest's third question, which assets on the too-flat chart look overpriced and which underpriced, is the course's own question under its decile chart — "Which stocks do you want to buy? And to sell?" (04b p. 10) — and is worked once, in the Lecture 9 note; the first block below restates its answer with the guest's at 14:05. The blocks after it list the course's own practice on everything the guest discussed, each with its owner's answer and a link: the too-flat line and the long-low-beta, short-high-beta trade; alpha positions as mispricings, not arbitrages; size, value and the three-factor benchmark, including the adapted manager-ranking problems; expected return linear in covariance with the tangency portfolio; diversification and estimation; and market efficiency, data mining and the joint test, whose workbook block is worked in the Lecture 11 note (13 Oct). Items marked *Adapted* are given with changed numbers or a changed claim, so that the originals stay fresh for later practice.

Practice on the guest's topics, worked in other notes

Source: the guest's question at 14:05, worked in the Lecture 9 note (6 Oct) as the 04b p. 10 exercise; and 41 workbook, slide, handout and section items on the topics the guest discussed, each registered to the meeting that teaches what it tests.

The talk touched almost every part of the course's treatment of the CAPM, and the course's practice items on those parts are worked in the notes of the meetings that taught them. Each block below states the problem, gives the answer as the owning note works it, links to the full working, and says where the guest's talk meets it. Nothing here is re-worked. The fourteen market-efficiency items of the workbook's Block 5 belong to the note on market efficiency, which has not been written yet; they are listed by topic only.

A. THIS MEETING'S QUESTION, WORKED IN THE LECTURE 9 NOTE

The guest's question at 14:05 — which assets look cheap, which look dear, and what to do about it = the 04b p. 10 exercise, reading the beta-sorted evidence: six questions · worked in the Lecture 9 note (6 Oct).

The question as asked. Going back to the early tests' picture, whose points lie along a line flatter than the CAPM's 45-degree line (10:12): "suppose you believe in the CAPM, right, and you believe everything should fall on the 45 degree line, but it doesn't, how would you make it? [...] Which of these assets look overpriced and underpriced to you? Which ones look like they're offering better returns than they should? Those are stocks or portfolios I want to buy or sell. Any ideas?" (14:19).

The guest's answer, taken from the room. "Yeah, you want to buy. Be confident. [...] If they're offering better returns than the risk, they look like a good deal to me. I'll buy those all day long. But better than that, at the same time, stocks or portfolios of stocks that are offering much lower returns than they should, those

prices are too high. I want to short those guys as well." (🕒 14:51) Done "in the right combination", the guest said, "I make money, and I can actually balance the risks so that I'm net risk neutral" (🕒 15:35). "Net risk neutral" (🕒 15:35) here means a net beta of zero, as the question at 16:49 makes precise: the position still carries each stock's own risk, and the phrase has nothing to do with the risk-neutral probabilities of the course's later arbitrage block.

The answer in the Lecture 9 note. The 04b p. 10 exercise asks the same question of the course's own chart, ten beta-sorted portfolios from 1927 to 2016 against the security market line, a different chart from the guest's: buy the low-beta portfolios, which lie above the line with positive alphas and are cheap relative to the CAPM, and sell or short the high-beta portfolios, which lie below it. With 04b p. 11's printed alphas, +0.3% for the lowest-beta portfolio and -7% for the highest, \$1 long the first and \$1 short the second has an alpha of $0.3\% - (-7\%) = 7.3\%$ a year. It is not an arbitrage: the two portfolios' betas differ, and even with matched betas the position is risky. Matching the betas is the next question the guest asked, worked in this note.

B. A TOO-FLAT SML, AND THE TRADE IT IMPLIES

Where the guest meets these: the early tests' line "is too flat" (🕒 10:25), and the strategy built on it, long low-beta and short high-beta stocks (🕒 15:48 – 🕒 16:47).

WB 4.4 Q4 — which line does the Fama–MacBeth test find too flat?

= **Workbook Block 4.4 Q4, p. 82** (the workbook labels it a past exam question, Midterm, Fall 2020, Short Question 9) · worked in the Lecture 9 note.

The problem. True or false: the Fama–MacBeth test finds the security characteristic line too flat, so one should go long low-beta stocks and short high-beta stocks.

The answer there. False as worded: the line found too flat is the security market line of the second step, not the characteristic line of the first; the trade it states is the one a flat security market line implies.

Adapted self-check X36 — the high-beta fund earned more: did it do better?

= **adapted from a past exam; claim changed** · worked in the Lecture 9 note.

The problem. A fund holding the highest-beta portfolio of 04b p. 11 averaged 12.5% a year and a fund holding the lowest-beta portfolio 9.5%. True or false: the high-beta fund did better on a risk-adjusted basis, so an investor following the evidence should tilt toward high-beta stocks.

The answer there. False: the high-beta fund earned 3 points more before adjusting for risk but 7.3 points less after, with alphas of -7% against +0.3%; the evidence favours low-beta stocks.

Adapted self-check X44 (also X99) — reading the second step's intercept and slope

= adapted from a past exam; claim changed · the same check serves two records, X44 and X99 · worked in the Lecture 9 note.

The problem. True or false: in the second Fama–MacBeth step the CAPM predicts an intercept equal to the risk-free rate and a slope equal to the market's average return; the slope comes out far smaller, which means low-beta stocks are overpriced, so sell them and buy high-beta stocks.

The answer there. False: in the excess-return regression the CAPM predicts $\gamma_0 \rightarrow 0$ and $\gamma_1 \rightarrow \bar{R}_M - \bar{R}_f$; the too-small slope belongs to the security market line, and it makes low-beta stocks cheap, so buy low-beta and sell high-beta.

C. BUY POSITIVE ALPHA, SHORT NEGATIVE ALPHA: A MISPRICING, NOT AN ARBITRAGE

Where the guest meets these: the prescription at  14:05 –  15:47, section A above, and the hedge-fund strategy it describes.

Self-check — WB 4.1 Q15: the profit opportunity in a positive and a negative alpha

= *Workbook Block 4.1 Q15, p. 68* · worked in the Lecture 9 note.

The problem. Stock A has a positive alpha and stock B a negative alpha of the same size: what is the profit opportunity?

The answer there. (a), buy A and short-sell B: A is cheap and B expensive relative to the security market line, and equal amounts long A and short B have an expected abnormal return of $2\alpha_A$.

Self-check — WB 4.1 Q16: is the same pair an arbitrage?

= *Workbook Block 4.1 Q16, p. 69* · worked in the Lecture 9 note.

The problem. Is the position of WB 4.1 Q15 an arbitrage?

The answer there. (d), none of the above: there is no arbitrage, only a mispricing. The pair's excess return carries market risk unless the betas are equal, and the two stocks' own shocks, which never cancel in general.

Adapted self-check X47 — a sure 3% a year for nothing?

= adapted from a past exam; claim changed · worked in the Lecture 9 note.

The problem. Stock C has an alpha of +2% a year and a beta of 1.3; stock D has an alpha of –1% and a beta of 0.6. True or false: buying \$1 of C and shorting \$1 of D costs nothing and earns a sure 3% a year, an

arbitrage.

The answer there. False: it is a positive-alpha position worth taking, with an alpha of 3% and an expected 7.9% a year in that note's numbers, but it is risky, with a net beta of 0.7 and two stocks' own shocks; it can lose money, so it is not an arbitrage. Its equal-dollar legs with unequal betas are the problem the guest's question at 16:49 solves by matching betas.

The 04a p. 27 exercise — "Is this a good manager?"

= 04a p. 27 · worked in the Lecture 9 note.

The problem. With a risk-free rate of 2%, a risk premium of 6% and a fund beta of 0.5, a manager returned 6%: a good manager?

The answer there. The CAPM benchmark is $2\% + 0.5 \times 6\% = 5\%$, so the alpha is $6\% - 5\% = 1\% > 0$: a good manager relative to the CAPM, as the slide says.

The 04a p. 28 exercise — "Does a positive or negative alpha imply that there is an arbitrage?"

= 04a p. 28 · worked in the Lecture 9 note.

The problem. The slide's chart puts security A above the security market line ("Underpriced (cheap)") and B below it ("Overpriced (expensive)"), and asks whether a nonzero alpha implies an arbitrage.

The answer there. No: a nonzero alpha is an abnormal expected return on a risky position, a mispricing, not a riskless profit.

The 04b p. 4 exercise — a positive alpha: what would the CAPM say, and is it an arbitrage?

= 04b p. 4 · worked in the Lecture 9 note.

The problem. With the security market line written with an alpha, what does a positive alpha mean, and is it an arbitrage?

The answer there. A positive alpha is "Abnormal compensation", pay beyond the reward for beta, which the CAPM predicts is zero; it is not an arbitrage, since collecting it carries risk.

D. SIZE, VALUE AND THE MULTIFACTOR MODELS

Where the guest meets these: the CAPM "not very good at explaining differences within an asset class, like why small stocks have better returns than large stocks, or value stocks do better than growth stocks" (

⌚ 08:33

); value, measured by book-to-market, and Fama and French (

⌚ 24:39

, ⌚ 24:52

); multiple factors

as new sources of risk, "I don't know if you've covered that yet" (🕒 36:29); and the factor regression of Buffett's returns (🕒 40:17 – 🕒 41:37).

Self-check — WB 4.1 Q17: what the size and value effects show

= **Workbook Block 4.1 Q17, p. 69 · worked in the Lecture 9 note.**

The problem. What do the size and value effects show about the CAPM?

The answer there. (d), they are not explained by it: high book-to-market firms earn more, 16.41% a year against 10.16% for growth firms (04b p. 12), without having higher betas.

Self-check — WB 4.1 Q18: what the Fama–French three-factor model says

= **Workbook Block 4.1 Q18, p. 69 · worked in the Lecture 9 note.**

The problem. What does the Fama–French three-factor model claim?

The answer there. (b): once a stock's betas on the market, on HML and on SMB are accounted for, no alpha is left, $\mathbb{E}[R_i] - R_f = \beta_{iM}(\mathbb{E}[R_M] - R_f) + \beta_{is}\mathbb{E}[R_{SMB}] + \beta_{ih}\mathbb{E}[R_{HML}]$ (04b p. 16).

Adapted self-check X39 — small stocks earned more: is the CAPM refuted?

= **adapted from a past exam; claim changed · worked in the Lecture 9 note.**

The problem. With a risk-free rate of 2.5% and a market premium of 6%, small stocks with a beta of 1.35 averaged 10.6% a year and large stocks with a beta of 0.95 averaged 8.2%. True or false: the 2.4-point gap is enough to conclude that the CAPM fails and a size factor is needed.

The answer there. False: here the 2.4-point spread is exactly $0.4 \times 6\%$, the reward for the higher beta, and both alphas are zero; only a nonzero alpha counts against the CAPM, and even that does not by itself require the three-factor model.

Adapted problem X89 (also X98) — two investors against the three-factor benchmark, one with a growth tilt

= **adapted from a past exam; numbers changed · the same problem serves two records, X89 and X98 · worked in the Lecture 9 note.**

The problem. Investor C earned 12% and investor D 11% last year; with their market, size and value betas and the factor premiums that note gives, which investor beat the three-factor benchmark by more?

The answer there. Benchmarks of 8.55% for C and 9.25% for D, so alphas of 3.45% and 1.75%: C outperformed by more.

Adapted problem X33 — the manager with the lower return is the better one

= adapted from a past exam; numbers changed · worked in the Lecture 9 note.

The problem. Manager E earned 14.8% and manager F 10.5%; against the three-factor benchmark, with that note's loadings and premiums, who is the better manager?

The answer there. Benchmarks of 15.15% for E and 6.85% for F, so alphas of -0.35% and 3.65% : F is the better manager despite the lower return.

Adapted problem X46 — two investors, and a ranking that turns on the size premium

= adapted from a past exam; numbers changed · worked in the Lecture 9 note.

The problem. Investor G earned 13% and investor H 12.5%; against the three-factor benchmark, who outperformed by more, and at what size premium would the answer change?

The answer there. Benchmarks of 11.45% for G and 12.4% for H, so alphas of 1.55% and 0.1% : G outperformed by more, as long as the size premium is below 2.95% .

The 04b p. 12 exercise — "Could these results be rationalized with the CAPM?"

= 04b p. 12 · worked in the Lecture 9 note.

The problem. Value stocks out-earned growth stocks by 6.25% a year and small stocks out-earned large ones by 7.10% (04b p. 12): could the CAPM explain it?

The answer there. Not from the spreads alone, "Perhaps, we have said nothing about betas!" (04b p. 12): at a 6% premium the CAPM would need beta gaps of about 1.04 for value and 1.18 for size, and the book-to-market deciles' betas read from 04b p. 14 are nearly equal.

The 04b p. 13 exercise — the value anomaly: what would a hedge fund manager do?

= 04b p. 13 · worked in the Lecture 9 note.

The problem. With the value and growth deciles' alphas, what would a manager judged against the S&P 500 do, and why can the CAPM not explain it?

The answer there. Long value, short growth, for an alpha of 2.90% a year over 1926–2010 and 6.77% over 1963–2010. The slide answers whether the CAPM can explain it with "No. Why?" (04b p. 13) and leaves the why open; that note's own reason is that alphas are already adjusted for beta and the model says they are all zero.

The 04b p. 15 exercise — the size anomaly: what would a hedge fund manager do?

= 04b p. 15 · worked in the Lecture 9 note.

The problem. The same question for the size deciles.

The answer there. Long small, short large, for an alpha of 3.23% a year over 1926–2010 and 3.50% over 1963–2010.

WB 4.5 — the Block 4 key concepts the Lecture 9 note covers

= *Workbook Block 4.5, pp. 89–90* · in the **Lecture 9 note**.

What it lists. The rest of the Block 4 checklist: estimating beta and alpha; stock selection and active management, and capital budgeting; tests of the CAPM; the size and value effects; and the Fama–French three-factor model. Each is stated there with the formula or sentence that makes it usable and the items that practise it.

E. THE LINEAR RELATION BETWEEN EXPECTED RETURN AND COVARIANCE

Where the guest meets it: "there's actually a linear combination between the expected return on every asset in the economy and its covariance with the tangency portfolio" (⌚ 05:10).

Handout 04 p. 1 — the ratio is 4 for Apple and 2 for Microsoft: what would you do?

= *handout 04 p. 1* · worked in the **Lecture 9 note**.

The problem. For an efficient portfolio, an asset's marginal effect on the mean over its marginal effect on the variance must be the same for every asset; if the ratio were 4 for Apple and 2 for Microsoft, what would you do?

The answer there. Shift weight from Microsoft to Apple, keeping the mean unchanged and lowering the variance, and repeat until the two ratios are equal. A ratio equal across assets is the guest's linear relation: expected excess return proportional to covariance with the tangency portfolio.

F. DIVERSIFICATION, ESTIMATION, AND THE KEY CONCEPTS OF PORTFOLIO CHOICE

Where the guest meets these: single-stock averages as noise, the parameter count and the market as a proxy (⌚ 05:19 – ⌚ 07:03); diversifying across roughly independent strategies in eight markets (⌚ 19:48 – ⌚ 20:38); the Sharpe ratio of the average against the average of the Sharpe ratios (⌚ 33:37); only covariance with the economy is compensated (⌚ 43:17).

WB 3.5 — the Block 3 key concepts the Lecture 8 note covers

= *Workbook Block 3.5, p. 64, bullets 32–35* · in the **Lecture 8 note (1 Oct)**.

What it lists. Systematic and idiosyncratic risk; risk reduction through diversification; the pitfalls of mean-variance optimization in practice, among them "Estimates of means are imprecise over short periods" and the $2N + N(N - 1)/2$ parameters it needs (03d p. 15); and the single factor model.

WB 4.5 — the Block 4 key concepts the Lecture 8 note covers

= *Workbook Block 4.5, pp. 89–90, bullets 1–12 and the first two parts of bullet 14* · in the Lecture 8 note.

What it lists. The CAPM's equilibrium and assumptions, market capitalization and the market portfolio's weights, the tangency portfolio as the market, the capital market line, the price and quantity of risk, the security market line, beta, systematic and idiosyncratic risk, the variance decomposition and the security characteristic line: the model the guest recalled at the start of the talk (⌚ 03:09 — ⌚ 08:42).

WB 3.5 — the Block 3 key concepts the Lecture 7 note covers

= *Workbook Block 3.5, pp. 63–64, bullet 11's later parts and bullets 27–31* · in the Lecture 7 note (29 Sep).

What it lists. The optimal weight with one risky asset, the tangency portfolio and many risky assets; leverage and margin; two-fund separation; and the insurance principle, that averaging many independent risks removes them, which is the guest's case for spreading one strategy over many roughly independent markets.

WB 3.4 Q8 — three uncorrelated assets: a 50/30/20 mix, and a mix that beats the safest asset

= *Workbook Block 3.4 Q8, pp. 58–59* · worked in the Lecture 5 note (22 Sep).

The problem. Three uncorrelated assets: find the mean and volatility of a 50/30/20 mix, and find a portfolio with a higher mean and a lower volatility than asset C, the safest of the three.

The answer there. (a) An expected return of 14.6% with a volatility of 3.26%; (b) for example 10% in A and 90% in C, with an expected return of 10.8%, above 10%, and a volatility of 1.90%, below 2%. Uncorrelated returns diversify, the property the guest relies on when combining independent strategies.

G. MARKET EFFICIENCY: THE WORKBOOK'S BLOCK 5, FOR THE NOTE ON MARKET EFFICIENCY

Where the guest meets these: the joint hypothesis problem (⌚ 35:52); data mining and out-of-sample testing (⌚ 38:33), worked in this note as the question at 38:33; the closing price as the best forecast of a game's outcome (⌚ 46:04); who gathers information at a cost (⌚ 61:47); luck against skill among investors (⌚ 64:52); and trading on inside information (⌚ 65:14 — ⌚ 65:49).

= **Workbook Block 5, pp. 91–93** · fourteen items, all in the **Lecture 11 note (13 Oct)** except the last key concept of **WB 5.3, mutual funds, which is the Lecture 12 note's (15 Oct)**. Those notes work them; none is worked or answered here.

- **WB 5.1's concept questions** (five, in the Lecture 11 note's Self-check): which statement is consistent with the efficient markets hypothesis; what follows if prices reflect all public information; abnormal returns before acquisition announcements, as evidence against which form of efficiency; which timing of an abnormal rise after a positive earnings surprise is inconsistent with semi-strong efficiency; what a random walk implies for returns.
- **WB 5.2's labelled past exam questions** (three, worked in the Lecture 11 note's groups A, D and F): a manager who beats the market trading on earnings news in the paper, weak or semi-strong efficiency (Midterm, Fall 2020, Short Question 12); a low-earnings surprise with abnormal price falls a day before, at and a day after the announcement (Midterm, Fall 2021, Short Question 11); strong-form efficiency, with evidence (Midterm, Fall 2021, Short Question 12).
- **WB 5.3** — the Block 5 key concepts: the efficient markets hypothesis and its weak, semi-strong and strong forms; the random walk; long-run predictability; event studies; anomalies; the joint test; data mining; the Grossman–Stiglitz paradox; mutual funds. The first eight are in the Lecture 11 note's Key takeaways, mutual funds in the Lecture 12 note's.
- **Adapted self-checks X88, X50, X51, X52 and X53** — adapted from a past exam; in the Lecture 11 note's groups B (X51), D (X52), E (X50, also X88) and F (X53).

Common mistakes

The CAPM and its tests

- **Calling the mean–standard-deviation frontier a parabola.** It is a hyperbola; the parabola is the mean–*variance* picture (§2.2). The guest said both, correctly conditioned; the course's figures plot standard deviation.
- **Treating the tangency portfolio is the market as a theorem of mathematics.** It is an equilibrium statement: it needs homogeneous expectations and market clearing (04a pp. 4–5). The mathematics alone gives only each investor's demand (§2.3).
- **Estimating a stock's expected return from its own history.** A five-year average of a 44%-volatility stock has a standard error of about 20 points; the whole point of the CAPM for an investor is that covariance with the tangency portfolio is estimable and the mean is not (§3.2).
- **Reading the SML is too flat as beta does not matter.** The slope is positive — a qualitative success — but smaller than the market premium — a quantitative failure (04b p. 11). Low-beta portfolios earn more than predicted and high-beta ones less; that gap is the trade (§§5, 8).

- **Running the test on single stocks.** Portfolios are used because measurement errors, if uncorrelated, average away (04b p. 8); a decile sort keeps the spread in beta while shrinking the noise (§5.2).
- **Transferring numbers between charts.** The course's +0.3% and −7% belong to its 1927–2016 chart (04b p. 11); the guest's chart covers other years and is not in the build. The note never puts one chart's numbers on the other (§5.3, §8.3).

Betting Against Beta and the factors

- **Sizing the legs with equal dollars.** Equal dollars long and short leave a net beta of $0.5 - 1.2 = -0.7$; matching requires $1.2/0.5 = 2.4$ dollars long per dollar short, and so borrowing (§8.2, problem C).
- **Hearing "net risk neutral" as riskless, or as an arbitrage.** It means zero net beta. Idiosyncratic risk and alpha risk remain, and the course's own question — does a nonzero alpha imply an arbitrage? — is answered no (§7's Warning).
- **Averaging Sharpe ratios.** The Sharpe ratio of an equal-weighted combination is $\bar{s}\sqrt{N}/\sqrt{1 + (N-1)\bar{\rho}}$, not $\bar{s}$: the numerator averages, the denominator shrinks (§12.3). Thirteen factors at 0.63 give 1.5 only because their average correlation is about 0.11, not zero — independence would give 2.27.
- **Forgetting that leverage leaves a Sharpe ratio unchanged.** Rescaling factors to a common volatility is leverage; it changes the mean and the volatility in proportion (03c p. 11), so an apples-to-apples comparison is one of Sharpe ratios, not returns (§12.1).
- **Taking "two and a half times" as a fact with one base.** Against 0.63 it is 2.38; against 0.4 it is 3.75; the guest did not say which (§12.2's Warning).
- **Inferring wealth from a Sharpe ratio.** A Sharpe ratio fixes nothing about dollars without the return, the volatility, the leverage and the capital (§12.5's Gap).
- **Giving every factor the course's symbols.** Only R_{SMB} , R_{HML} and, by name, momentum are on course pages; momentum, low-risk and quality factors written as symbols in §14 are the note's own labels.

Efficiency, betting markets, prediction markets

- **Treating a 5% false-positive rate as proof that any particular finding is noise.** Five in a hundred are expected to look good by chance; which five, only an out-of-sample test can say (§13.3, problem D).
- **Explaining a loss on long shots as a risk premium.** A bet on tonight's game has zero covariance with everything the economy cares about, so its beta is zero on every factor and it is priced at R_f ; a systematic loss is a mispricing or a fee (§15).

- **Mistaking a reversal for information.** If the open-to-close move reverses on average, the move was an over-reaction; if it continues, an under-reaction; under efficiency the move predicts nothing (§17).
- **Comparing risk across deciles with the money line.** Its risk per dollar rises toward the underdogs, as $\sqrt{(1-p)/p}$ does; only the spread, which forces $p \approx \frac{1}{2}$, holds risk fixed (§19.2).
- **Reading a contract price as the physical probability.** It is a state price, a risk-neutral probability once scaled (07b pp. 6, 17): 56 cents implying 56% describes the market's price, and the favourite–long-shot bias is exactly the gap between that and the true chance (§21.2).
- **Stating a disputed figure.** Every value the two recordings render differently — the high-beta alpha, the years of Buffett's 0.6, the favourite's payoff, the count of wins in 1,000, the unit of AQR's trading cost, the NFL volume — is carried with both readings and never as fact.

Key takeaways

1. **The CAPM's gift to an investor is estimability.** Expected return is linear in covariance with the tangency portfolio, and covariances can be estimated where means cannot; economics breaks the chicken-and-egg problem by naming the tangency portfolio — the market — so a proxy index does the work (§3).
2. **The CAPM explains across asset classes, not within.** Stocks against bonds, yes; small against large and value against growth, no — spreads of 7.10% and 6.25% a year on the course's own numbers (§4).
3. **The fitted SML is flatter than the CAPM's, and getting flatter.** Low-beta portfolios earn more than beta predicts, high-beta ones less, on the course's 1927–2016 chart and on the guest's update to 2026 (§5).
4. **A borrowing wedge flattens the line.** With borrowing at $R_f + \delta$, the pricing line's intercept rises by δ and its slope falls by δ , through the same market point; investors who cannot lever buy high beta instead, and investors with cheap leverage buy low beta and lever (§§6, 9).
5. **Betting Against Beta is the too-flat line as a trade.** Long 0.5-beta stocks, short 1.2-beta stocks, \$2.40 long per \$1 short, paid for with \$1 of own money and \$1.40 borrowed (the short sale's proceeds stay with the broker), leverage 2.4; zero net beta, not zero risk; alpha highest for low-beta deciles and monotone across them (§8).
6. **Diversification applies to strategies.** Eight roughly independent markets at a Sharpe ratio of 0.4–0.5 give $\sqrt{8}$ times as much; thirteen factors averaging 0.63 give 1.5 at an average correlation of about 0.11; costs, leverage limits and overfitting halve it in practice (§§8.4, 12).
7. **The factors are old economics with new measurement.** Value, momentum, quality, carry, trend, accruals, sentiment: "the concept's the same, but the way you measure it, it evolves" (§11). The course's deck calls their theoretical motivation absent; the guest says there are good theories behind each — both are recorded (§11.6).

8. **Three schools explain anomalies: risk, behaviour, data mining.** The joint-hypothesis problem means a CAPM failure is either inefficiency or the wrong model; the live debate is risk against behaviour; the defence against data mining is out-of-sample testing (§13).
9. **Sports betting removes the risk school.** Bets have zero beta and short terminal dates, and every contest ends in an outcome, so a systematic loss on underdogs (22%) cannot be a risk premium; the favourite–long-shot bias is the low-beta effect, and open-to-close reversals are over-reaction (§§15–19).
10. **A prediction-market price is a state price.** A 56-cent contract's 56% is a risk-neutral probability, not the physical chance; a complete set of outcome contracts gives a distribution; the bias persists through a game until its very end; the markets' promise, in the guest's view, is hedging what cannot now be hedged — completing markets (§§21–22).

Connections

Backward. The frontier, the tangency portfolio and two-fund separation are the Lecture 7 note's (29 Sep), where Dávila said of the tangency portfolio that "Toby comes in at least, he may use those words" (9/29 [27:04]) — the guest did, in §2. The capital allocation line and the Sharpe ratio are the Lecture 6 note's (24 Sep), where Dávila predicted "he may talk about Sharpe ratios" (9/24 [65:47]) — §§8.4 and 12 are that talk. The equally weighted portfolio's variance, independent and correlated, which §§8.4 and 12.3 turn into Sharpe-ratio arithmetic, is the Lecture 7 note's §§11–12. The CAPM as equilibrium, the market portfolio, the SML and beta, the variance split and the SCL are the Lecture 8 note's (1 Oct), which announced this guest lecture without a date; the date is now settled. Alpha, the Fama–MacBeth test, the too-flat chart, the size and value anomalies, the three-factor model and the Fama–French reading are the Lecture 9 note's (6 Oct, reconstructed), which this talk neither recaps nor mentions; its workbook items on the long-low-beta, short-high-beta trade and on alpha positions stay there, linked from problem E. The law of large numbers and value at risk are the Lecture 5 note's (22 Sep); short sales and price impact the Lecture 2 note's (8 Sep); basis points and the course's plan for guest lectures the Lecture 1 note's (3 Sep), where Dávila said the guest would speak "sometime in October" (9/3 [01:54]) — Thursday 8 October, as it turned out.

Forward. The market-efficiency deck, 05a, is cited throughout this note as related and not yet taught on 8 October — the efficient market hypothesis and the random walk (§17), anomalies and the joint-test problem (§13), data mining and out-of-sample tests (§13.3), strong-form efficiency, the Grossman–Stiglitz paradox and mutual-fund costs (§22) — and the next two notes teach it. The Lecture 11 note (13 Oct, reconstructed) has pp. 1–22: the hypothesis and its three forms (§1), the random walk (§2), event studies and post-earnings drift (§4), anomalies, the joint test and data mining (§5), strong form and the Grossman–Stiglitz paradox (§6), with the insider-trading, Shiller and Lamont–Thaler readings (§6.3, §8, §9) and the workbook's Block 5. The Lecture 12 note (15 Oct) has pp. 23–27, mutual funds, their costs, and luck against skill. State prices, risk-neutral probabilities and complete markets (§§21–22) are the arbitrage block's, 07b, with the Nobel readings

on carry, momentum and limits to arbitrage (§§11, 13); the volatility smile (§20) is the options block's, 09d. Neither block has a note yet; this note cites their pages and draws nothing from them beyond a definition.

Sideways. The guest's firm, strategies and figures — AQR's fifteen years of Betting Against Beta, the thirteen themes, the \$1 trillion of prediction-market volume — are the talk's own and have no course source; where the course prints a number of the same kind (the market's Sharpe ratio 0.44, the value and size spreads, the +0.3% and −7% alphas), the note gives the course's beside the guest's and never substitutes one for the other.

Self-check

The two concept exercises from the guest's questions — the tangency portfolio in equilibrium and why portfolio averages are usable — are worked as self-checks under *Worked problems* (A and B). The questions below test the talk's own arguments; each answer is in the sections named.

Self-check. The guest called the CAPM "great from a pedagogical standpoint" but said AQR looks for "a better proxy for that object T". What is T, why does an investor need a proxy for it at all, and what does a better proxy buy?

T is the tangency portfolio, the risky portfolio every investor holds under homogeneous expectations. Computing it needs every mean, variance and covariance — 125,750 numbers for 500 stocks, with the means unestimable — so one needs a model that says what T is without estimating it. The CAPM says T is the market; a better proxy is any portfolio whose covariances line up better with average returns, which is what §3.1's linear relation then turns into expected returns for every asset (§3).

Self-check. State the borrowing-wedge result in one sentence and give its numerical check.

With lending at R_f and borrowing at $R_f + \delta$, the pricing line becomes $\mathbb{E}(R_i) = (R_f + \delta) + [(\mathbb{E}(R_M) - R_f) - \delta]\beta_i$: intercept up by δ , slope down by δ , through the market point. With $R_f = 5\%$ and $\mathbb{E}(R_M) = 12\%$ (04a p. 9) and $\delta = 2\%$ (*illustrative, the note's own*), the line $5\% + 7\%\beta$ becomes $7\% + 5\%\beta$: at $\beta = 0.5$, 9.5% instead of 8.5%; at $\beta = 1.5$, 14.5% instead of 15.5% (§6.3).

Self-check. A fund holds \\$3 of a 0.4-beta portfolio and is short \\$1 of a 1.5-beta portfolio (*illustrative numbers, the note's own*). What is its net beta, is it balanced, and how much is borrowed if the fund puts up \\$1 of its own money?

Net beta $3 \times 0.4 - 1.5 = -0.3$: not balanced, still net short the market. Balance needs $1.5/0.4 = 3.75$ dollars long per dollar short. The \$3 long leg less the \$1 of own money means \$2 borrowed; at \$3.75 it would be \$2.75. The short sale's \$1 pays for none of it, since the proceeds stay with the broker (01b p. 13; §8.2, problem C's method).

Self-check. Why is the Sharpe ratio of the average larger than the average of the Sharpe ratios, and when are they equal?

The combination's mean is the average of the means, but its volatility is below the common volatility whenever the strategies are not perfectly correlated, so the ratio rises: $SR_N = \bar{s}\sqrt{N} / \sqrt{1 + (N-1)\bar{\rho}}$. They are equal only at $\bar{\rho} = 1$, when combining adds nothing (§12.3).

Self-check. Give the risk-school, the behaviour-school and the data-mining reading of the value premium, and the one test that separates the third from the first two.

Risk: value stocks covary with a priced source of risk the CAPM omits, so the premium is compensation (04b p. 16's R_{HML}). Behaviour: investors underprice unglamorous firms and limits to arbitrage keep them cheap. Data mining: with hundreds of signals tried, some look good by chance. The separating test is out-of-sample evidence — other markets, asset classes and periods — which noise fails and both real explanations pass (§13).

Self-check. In a sports market, what does the guest's test of betting against risk compare, and what result did it give?

It compares returns across deciles of games sorted from the biggest underdogs to the biggest favourites, using the money line, whose risk per dollar rises toward the underdogs. Underdog bets lose about 22% on average and favourite bets lose about 1–2% net of the bookmaker's 10%: the high-risk bets earn less, which is the low-beta effect in a market where no risk premium can exist (§19).

Self-check. Why can a set of FOMC contracts give "the distribution of potential outcomes" when a futures contract on the rate cannot?

Each contract pays \$1 in one outcome only, so its price is that outcome's risk-neutral probability discounted; an exhaustive set of such prices adds to about a dollar and reads as a distribution. A single futures price is one number, an average over the outcomes, from which the distribution cannot be recovered (§21.3).

What the materials don't settle

Questions this note cannot answer from the recording or the course's files, for office hours or the next meeting:

1. **What happened in the roughly five minutes before the audio** — the introduction of the guest, any course business, any remark about the midterm. Nothing on file covers them, and no later meeting's recording has yet described them (§1).
2. **What the guest's slides showed.** The 2026 update of the early-tests chart, the Betting Against Beta decile chart, the thirteen-theme chart, the Buffett charts, the spread-against-money-line chart, the options chart and the prediction-market charts are known only from their descriptions. The guest offered to make the slides available  40:39 ; if they are posted, most of item 3 settles with them.
3. **Which rendering is right for each disputed value:** the size of the high-beta deciles' alpha (§8.3), whether the strategy's 0.4–0.5 Sharpe ratio is net of costs (§8.4), the clause after "shorting high-risk assets" (§11.2), the word for the company in the maxim's second half (§11.3), the sentence about value's last ten years (§11.6), the years of Buffett's 0.6 Sharpe ratio (§12.5), the start year of the Buffett chart and the two reversed phrases (§14), the count of wins in 1,000 plays (§17), the favourite's payoff per dollar (§19.1), the phrase on favourites' returns (§19.3), the label on the overpriced assets and the guest's own view of the 56-cent price (§21), and the unit of AQR's trading cost and the two scale figures (§22).
4. **Whether the guest lecture's content is examinable.** The course says exams use "material explicitly covered in class, handouts, and notebooks" (*syl p. 5*); nothing said on 8 October bears on it either way (the Exam signal at the top).
5. **The four audience questions** neither recording decoded (§22), and what the second voice said after "Thanks, Toby."
6. **What "the last 95% of the action" meant** in the guest's description of the in-game bias (§21.4).
7. **Who wrote the quality paper** the guest cited (§11.3), and the name of the US prediction-market exchange the guest works with — neither is in the course's files.

8. **Whether the course had covered multifactor models before 8 October.** The guest did not know ("I don't know if you've covered that yet",  36:29); the Lecture 9 note's range is inferred, and only a later meeting's recap can settle it.
9. **Why the in-game bias changes at the very end of a game.** The guest's hypothesis — holders unwinding winning positions — was offered only as a hypothesis (§21.4).

Coverage audit

P9 (NOTE_SPEC Stage 4), fin-2026-10-08-L10 (the guest lecture, recorded, stitched), 2026-10-13, batch fin-midterm, full P9 in a fresh prep-model session (the write session had stopped; this session held no note text and no P8b conversation). Verdict: FAIL — not done. Complete: the beat sheet (254 of 254 placed, 2 thin, 0 MISSING), the math inventory (M01–M55, 55 of 55), the problem map (5 records in 5 files, each included once; 41 links), the stitched-recording header, every disputed value of the gate's §10a, the Exam-signal callout, figures (5 redraws, 4 course pages, 0 KaTeX errors) and the pointer scan (0). `tools\p9_quotes.py 1.8` finds 0 hits. Failing: the math check (one written equation in §12.3 off at the third decimal), the symbol table, one quotation given to the wrong speaker, a systemic citation-time defect (most transcript quotations carry the paragraph's stamp instead of their own segment's start), one second reading and two unclear spans settled in the note's voice, two cited-but-not-restated readings, and the no-sealed-content row (non-visible workbook numbers for targets; one adapted-items sentence). Fix list: A11.

Audited file: `notes/fin/fin-2026-10-08-L10.md`, assembled 20:43:04 from parts 00–10 and 5 includes, re-assembled with this part (`--no-manifest`). Inputs: the beat sheet (254 beats), the map's math inventory and figure list, the gate `work/_batches/fin-midterm/gate-fin-2026-10-08-L10.md` (§§2–4, 6, 9, 10), `_index.md` and all five draft files, the clean transcript and the stitched record, `variants/targets.json` and `display-ids.json`, the workbook conversion (originals for the sealed check, never the papers), and the visual reads of 03c, 03d, 04a, 04b, 05a, FF04 and the formula sheet. Scripts in the P9 scratch folder `p9_fin-2026-10-08-L10`.

A1. Beat audit: 254 placed (2 thin), 0 MISSING; logistics L1–L8 excluded by the beat sheet

Beats	Heading
Missing opening	§1, the opening Gap callout (span, both ends, nothing rebuilt)
1	§1 Gap callout (first words, never the opening)
2–5	§1, The framing
6–9	§1, Prediction markets, in one paragraph of context
10	§1, Why it should carry over (the course offer named as an announcement)
11–17	§2 intro, §2.1 (figures F1, 03d p. 3, 04a p. 9), §2.2 (derivation, Warning W10)
18–20	§2.3; Worked problems A (draft t0427)
21, 25	§3.1
22–24	§3.2 (Gap: second moments)
26–31	§3.3
32–36	§4
37–40	§5.1 (F2 schematic) — 39 thin : the aside that Jensen did not get a Nobel Prize is dropped
41–43	§5.2; Worked problems B (draft t0928)
44–47	§5.3 (Gap, 04b p. 10 chart)
48	§5.4
49–50	§6.1
51–54	§6.2 (Gap: reconstructed drawing; F3 with slider)
55	§6.3 (derivation)
56	§6.4 (Gap)
57–62	§7 (Warning W3; 04a p. 28 figure); Worked problems E (link block A)
63–66	§8.1
67–70	§8.2; Worked problems C (draft t1649)
71–74	§8.3 (Warning D1, Gap, F4)

Beats	Heading
75–81	§8.4 (Warning S1, derivation)
82–88	§9 (derivation)
89–93	§10
94	§11 intro
95–97	§11.1
98–102	§11.2 (Warning D2; horizons by source)
103–106	§11.3 (Warning S2)
107–111	§11.4 (derivation)
112–115	§11.5
116–121	§11.6 (Warnings W4, D3)
122–127	§12.1 (Gap)
128–129	§12.2 (Warning W1)
130–131	§12.3 (derivation)
132–133	§12.4
134–139	§12.5 (Warning D4, Gap Gp4, derivation)
140–145	§13, §13.1
146–150	§13.2
151–155	§13.3; Worked problems D (draft t3833)
156	§13.4
157–169	§14 (Warnings S3/S4, D5; Gap)
170–173	§15 (derivation)
174–178	§16
179–183	§17 (Warning D6, derivation)
184–188	§18 (Gap)

Beats	Heading
189–193	§19.1 (Warning D7)
194–196	§19.2 (derivation, F7)
197–199	§19.3 (Warning S5)
200–201	§20
202–207	§21.1
208–211	§21.2 (Warnings S6, W9)
212–214	§21.3 (derivation)
215–222	§21.4 (Warnings D8, X279; Gap)
223–225	§21.5
226	§22 Gap (V1: only the thanks)
227–232	§22.1 (Warning D9/D10)
233–239	§22.2 (Warning D11/D13)
240–244	§22.3 — 244 thin : the guest's point that the markets need to become more efficient is dropped
245–247	§22.4
248–254	§22.5 (the talk's last words)

A2. Readings

None assigned (map: 0 reading beats; no work\readings\fin-2026-10-08-L10\ , so no textbook gap list). Touch points: FF04 (pp. 2, 5, 6, 8–10, 13, 14, 15–16, 17, 20), LT03 (p. 10), Shiller03 (pp. 11–12, 14, 19), Nobel13 (pp. 22, 30, 43), Nobel13pop (p. 4) are each restated in a sentence or two. **WSJins** is named (§22.3) with no sentence of its content, and **Thorp06 p. 26** is listed under Sources used but never cited or restated in the body (A11 items 13–14).

A3. Problems

Record	Where	Form	Checks
t0427 tangency in equilibrium	Worked problems A	Self-check	answer matches index; cites [04:22] for the question, gate says [04:27] (A11 item 6)
t0928 portfolio averages	Worked problems B	Self-check	19.68%, 5.37%, 3.67, 484, 36 match index
t1649 BAB legs	Worked problems C	seven headings in order	2.4, 1.40, 58.33%, 41.67%, 2.0/0.8333 match index; ANSWER's financing line contradicts its own Step 5 (A11 item 7)
t3833 data mining	Worked problems D	seven headings in order	5, 0.9941, 4.75, 0.754, 0.25, 0.2214, 7.85 match index
t1405 + 41 links	Worked problems E (<code>_LINKED.md</code>)	link blocks	$23 + 2 + 1 + 1 + 14 = 41$; targets by display id with the generic label; section L sync passes

Basis cited (syl p. 3); no exam question worked. Index: t0427's source stamp [04:22] and the X46 answer line (1% / 2.9% against the link file's 1.55% / 2.95%) are stale — the X46 difference is the expected re-sync after 10/6's change, not a writer defect.

A4. Math check

Script	Result
<code>fin-2026-10-08-L10__p8a.py</code>	415 checks, 0 failing
<code>fin-2026-10-08-L10__p8b_body.py</code>	441 checks, 0 failing
<code>fin-2026-10-08-L10__p9_note.py</code> (new: a prose-and-math equation evaluator over all parts and drafts, the header's clock and coverage arithmetic, worded arithmetic, course numbers against the visual reads)	54 checks, 1 failing : §12.3 writes $0.63 \times 3.606 = 2.271$; with the rounded root the product is 2.272 ($0.63\sqrt{13} = 2.2715$). Plants caught: 4 of 4 wrong equations, 1 of 1 altered read value
runner <code>fin-2026-10-08-L10.py</code>	3 parts, FAIL

A5. Quotations

p9_quotes.py 1.8: source self-test 551/551 (179 correct pass, 372 plants caught; 94/94 sources); note: 741 quotations, 716 cite a time or page, 716 ok, **0 hits**, 0 unpaired marks, 25 uncited (0 not found); planted self-test 10/10. Beyond the tool: (a) §7 gives the guest's words at 14:51 to the room; (b) **324 transcript quotations are not cited at their own segment's start** (312 at a P4 paragraph stamp; 170 more than 30 s early, 59 more than 60 s, at most 129 s), which the gate's cite rule forbids; the tool passes them because it matches ± 30 s of the paragraph span; the re-stamp list is in the P9 record; (c) no quotation sits inside a garbled [unclear] span; (d) no other-recording reading is put inside quotation marks except the disputed marks, which the tool reads both ways.

A6. Symbols, derivations, figures, preview

Symbols: missing from the table or clashing — $\underline{a}$, $\underline{b}$, $\underline{c}$ of §2.2 (clash with the bet's $\underline{b}$ and the deck's $\underline{c}$); R_1 , R_2 ; σ_ε (§8.1); μ as a mean excess return (§§8.4, 12.3; the table gives only the random-walk drift) and μ_i ; s_i (a Sharpe ratio in §12.3, an error's standard deviation in draft B); $\mathbb{E}_t$ (§17); §14's R_B , α_B , β_M , β_s , β_h , β_{mom} , β_{bab} , β_q , $\varepsilon_B(t)$; §21's $\underline{q_k}$, $\underline{d_k}$, $\mu(s)$, E^* , q_s , π_s^* , S ; SD (§19); the drafts' $\underline{W_k}$, $\underline{y_k}$, $\underline{D}$, $\underline{w_T^i}$ (A), $\underline{I_k}$, $\underline{K}$, $\underline{Z}$ (D), which the index lists for the table; the table says V is not reused but its HPR row writes $V(T)/V(0)$; the SR_N row points to a §8.5 that does not exist (§8.4). Derivations: every displayed step carries a reason; no jump of more than one manipulation found. Figures: F1–F4, F7 redrawn (labels 11.5–15 units), 03d p. 3, 04a pp. 9, 28 and 04b p. 10 embedded, each captioned with a how-to-read line; Gap callouts for the early-tests update, the bond chart, the BAB deciles, the factor chart, the Buffett charts, the sports-momentum chart and the prediction-market charts; **no Gap line for the guest's spread-against-money-line chart** (§19.2). Preview: 705 KaTeX spans, 89 displays, **0** math-error, 0 display annotations starting with a greater-than sign. Links: 115 cross-note and internal links resolve; the one dead link is the provisional Lecture 11 note; 333 jump links match their mm:ss.

A7. Guest-lecture checks (fin-midterm-p9, 10/8)

Header: recorded, stitched from two recordings with the second in two parts, no hole, τ to file once, the ≈ 5 minutes not on file with both ends — pass. The note cites τ only — pass. Disputed values D1–D13: each in a Warning with both readings, none as fact — pass. Second readings: S1, S3, S4, S6 pass; **S2 fails** (§11.3 states the poor-company half as said and the Warning adopts that sense); S5 leans (the better roughly breaks even, §19.3 and the self-check). Nothing on the guest's slides presented as a course slide — pass. Formulas fixed by course sources — pass, except 07b p. 17's formula, which no visual read covers. **Readings of unclear spans in the guest's mouth: 2** (§20, priced more reasonably, from the garbled 51:57 clause; §15 and takeaway 9, known outcomes, the other recording's word for the garbled 43:46 one, unlabelled). The one Exam signal quotes the course's sentences — pass, but its formula-sheet clause lacks 9/10 [66:16]. Nothing labelled examinable — pass. Names and pronouns:

guest named from syl p. 6 / 00 p. 12 / 10/1; no word given to Dávila; 0 gendered pronouns outside quotations; no banned names.

A8. No sealed content (targets carried or linked: X33, X36, X39, X44, X46, X47, X89, X98, X99; X50–X53, X88)

Reader-visible: (i) generic labels only — pass; (ii) workbook numbers, papers or years for a target: 0; (iii) number windows: 1 scanner hit (Worked problems' examples table), judged a coincidence with the guest's own figures; six-word runs: 7 hits of two generic phrases (the too-flat line, 2 of them inside the guest's quotation; low-beta stocks earning more than the CAPM predicts); (iv) past papers: the generic labels, 2020/2021 labels for non-targets (allowed), and two extra sentences (Worked problems E lead-in, which also invents a course policy; the link file's opening paragraph). Non-visible: the link file's header comment maps all 14 targets to workbook numbers; `__p8a.py` names the sealed years and 9 target workbook numbers in its guard pattern and uses 8 target registry ids as sync keys.

A9. Reasons sweep (NOTE_SPEC §7)

Defects 3: Worked problems E lead-in (a course workbook policy that does not exist); §22 Gap (questions too faint for either recording — the cause is not established); §12.5 (03S p. 13's peso problem as the point behind Madoff's fabrication, unlabelled). Mild 2: §22.1 (05a p. 23's fund costs as limits to arbitrage); §8.1 (the variance split as why beta predicts risk, unlabelled). Inherited from 10/6's owner drafts (re-sync after its fixes): the link file's 04b p. 12 and p. 13 blocks.

A10. NOTE_SPEC §8

Row	Result
Every beat maps to a section	pass (254/254; 39 and 244 thin)
Every symbol defined and in the symbol table	fail (A6)
Every derivation step has a justification	pass
No algebra jump of more than one manipulation	pass
Every lecture figure present, captioned, explained	pass (redraws and Gap lines; one Gap line missing, counted below)
Every worked problem follows the template	pass
Slide-only content included and flagged	pass (no deck taught)
Gaps flagged, not filled with invention	fail (S2 settled; two unclear spans read; no Gap line for one lost chart; one cause asserted)
Every math-inventory item present and fully derived	pass (55/55)
Every spoken-vs-written conflict and erratum flagged	pass (W1–W12; no new erratum)
The math check passes	fail (1 equation)
Every problem-map item present with its source	pass
Every assigned question worked, basis cited	pass
No exam question worked ahead of its exam	pass
Every quotation matches, at the right time and speaker	fail (1 speaker; 324 times not at the quoted segment)
No point repeated; no padding	pass
Core and optional readings covered in full; gap list	pass (none assigned)

Row	Result
No sentence sends the reader to a source; every cited source restated	fail (WSJins, Thorp06)
Readable start to finish without another file	pass
No sealed content (batch fin-midterm)	fail (A8 non-visible items; two sentences)

A11. Fix list for P8b

1. **Math** 04-anomalies-sharpe.md §12.3 derivation (line beginning The guest's numbers): $0.63 \times 3.606 = 2.271$ — write $0.63 \times 3.6056 = 2.2715 \approx 2.271$, or 2.272.
2. **Speaker** 02-tests-wedge.md §7 (Theories prescribe paragraph): the words you want to buy are the guest's (clean transcript 14:51, unlabelled turn), not the room's — give them to the guest, as the link file's block A does.
3. **Cite times** (all parts and drafts): re-stamp each 10/8 quotation to its own segment's start (gate §3c cite codes, §3d, §6); 324 entries in the P9 record's `cite_restamp` list; largest in §14 (40:39 for words at 42:43–42:48), §21.3 (54:19 for 56:15), §10 (22:53 for 24:21).
4. **S2** 04-anomalies-sharpe.md §11.3: drop the sentence stating the mirror image as said and the Warning's which is how the note reads the maxim; give the second half with both words, neither as fact (gate §10b S2). S5: §19.3 and 10-selfcheck (sports self-check) say lose about 1–2% net rather than roughly break even.
5. **Unclear spans** 06-sports.md §20: priced more reasonably reads the garbled 51:57 clause — state only the deep-out-of-the-money half and the course's smile (gate §4c 51:57). 05-schools-markets.md §15 and 09-closing.md takeaway 9: known outcomes is the other recording's word for a garbled one — say so or keep only the short terminal date (gate §4c 43:46).
6. **t0427** draft heading, Source line, both quotation cites and `_index.md` (lines 23, 34): [04:27]–[04:59] for the question and [04:50] for the reply (gate §3a item 2, Q18); 08-worked-problems.md already says 04:27.
7. **Financing** fin-ex-l10-t1649.md ANSWER line 3 and Step 6, 03-bab-leverage.md §8.2 and its first self-check, 09-closing.md takeaway 5, 10-selfcheck.md BAB self-check: the 1 dollar from the short sale cannot fund the long leg (01b p. 13, which Step 5 itself quotes) — frame as 1 dollar of own capital plus 1.40 borrowed (gate M13), or state the assumption once and flag it.
8. **Symbols** 00-header.md symbol table: add or rename every symbol in A6; fix the V row, the §8.5 pointer (§8.4), and the s_i , μ , b/c clashes; add a term (or a note) for §14's unrecoverable factor, whose self-check counts six factors.

9. **Gap line** 06-sports.md §19.2: one Gap callout for the guest's spread-against-money-line chart (gate Gp2).
10. **Exam signal** 00-header.md callout: cite 9/10 [66:16] with formula sheet p. 1 (fin-midterm-p9 formula-sheet rule).
11. **Buffett** 03-bab-leverage.md §9 (the course names Buffett once) and 05-schools-markets.md §14 (the course's one line on Buffett): Thorp06 (Example 7.3, Berkshire) and the MarketWatch diversification reading also name Buffett — narrow to the decks.
12. **10-selfcheck.md** closing list item 3: add S1 (net of cost, \$8.4) and S2 (the maxim's adjective, \$11.3) (gate §4d seed 3).
13. **Thorp06** 00-header.md Sources used: cite and restate it where the talk meets it (the Buffett passage) or drop it from the list.
14. **WSJins** 07-prediction-qa.md §22.3: one sentence of the article's content, or drop the cite.
15. **Reasons** 08-worked-problems.md E lead-in (no course workbook policy; keep only the generic label); 07-prediction-qa.md §22 Gap (neither recording decoded, not too faint); 04-anomalies-sharpe.md §12.5 (label the peso-problem link as explanation); mild: §22.1's 05a p. 23, §8.1's variance-split why.
16. **Sealed** non-visible: `_LINKED.md` header comment lines 12–20 and `__p8a.py` (guard pattern, sync keys) — display ids or a ruling (the L9 and S4 precedents); visible: the two past-exam sentences (08-worked-problems.md E lead-in; `_LINKED.md` opening paragraph) to the generic label; the two generic runs in note prose (02 §5 heading, 09 takeaway 3, `_LINKED.md` B heading; 02 §6.3 and 05 §13 self-check) — reword or rule.
17. **07b** 07-prediction-qa.md §21.2: 07b p. 17's g_k formula has no visual read (the read covers pp. 1, 3, 9, 13, 18, 23–25) — read the page or cite the formula sheet item 15 only.
18. **Thin beats** 39 (§5.1: Jensen had no Nobel Prize) and 244 (§22.3: the markets need to become more efficient), one clause each.
19. **Minor** W8: FF04 p. 17's three to twelve months is in the gate's list and not in §11.2; the symbol table lists m and ε_{it} , which no formula uses.
20. **Not writer defects (re-sync)**: X46 in `_LINKED.md` matches 10/6's draft of 20:30 (section L passes); `_index.md` line 73's X46 answer is stale; the link file's 04b p. 12 and p. 13 blocks inherit 10/6's reason defects (L9 audit item 14); the Lecture 11 link is provisional.

Advisory, for a ruling: the self-checks in §17 (50 to 60 cents, 55%), §21 (30 cents and 22%; 5, 20, 55, 15 and 5 cents) and the closing self-check (3 dollars, 0.4, 1.5) use number sets no course source gives (rules §0: every number from a course source).

Confirming re-check — batch fin-midterm round 1, 2026-10-13

Fresh prep-model session after the fixer's round (log work/_batches/fin-midterm/fixlog-fin-2026-10-08-L10.md); nothing in the note was fixed here. Held to A11 above, the gate and the orchestrator's rulings in the dispatch. **Verdict: FAIL — item 3 is not closed:** two quotations are still cited before their own segment, two course-page quotations carry a 10/8 time, and the re-stamping left 18 paragraph stamps behind, two of them on disputed values. Every other item is closed.

Checks.

- **Math.** Runner work/mathcheck/fin-2026-10-08-L10.py : 3 parts, 1,088 checks, PASS (__p8a 436/0, __p8b_body 598/0, __p9_note 54/0; P9's part is unchanged since P9 and now evaluates 81 written equations). Planted runs all exit 1: __p8a --selftest (2 caught), __p8b_body --plant , __p9_note --plant (4 of 4 equations, 1 read value).
- **Quotations.** p9_quotes 1.8 source self-test 551/551 (179 correct pass, 372 plants caught; 94/94 sources); the note: 745 quotations, 720 cite a time or page, 720 ok, 0 hits, 0 unpaired marks, 25 uncited (0 not found); planted self-test 10/10. No quotes.own.yaml . Beyond the tool, an independent segment-start check of every quotation followed by a 10/8 time (stitched segments, floored): 423 at their own segment's start; 3 benign (01 l. 20 cites its quotation just before it; the F2b-only passage at 17:19, stamped so by the clean transcript, in 03 l. 13 and draft C l. 41); 2 off their segment and 2 course-page quotations with a 10/8 time (item 3). Scan for quoted words under an [unclear] or [low confidence] mark: 30 quotations share a four-word run with a marked sentence; each stops before the garbled words, or is the closing list's mention of the 58:26 clause (gate §4d seed 6): 0 defects.
- **Assembler, pointers, render.** assemble_note.py --check --no-manifest : 12 parts, 5 includes, 50,174 words. Pointer scan: 0 (every hit on the word see is a verb, in a quotation or in prose). render_note.py : 836 math spans, 1 dead note link (the provisional Lecture 11 note). Headless Edge DOM dump (through cmd /c , scratch user-data-dir, 2.28 MB): 0 math-error ids, 836 KaTeX spans, 89 displays, 0 annotations starting with a greater-than sign.
- **Reasons sweep** over every edited passage (the parts at the fixlog's lines, draft C ll. 55–188, the link file's edited lines): 46 candidates, 0 defects; one mild, 03 l. 66's clause on why the guest calls it a leverage trade, which rests on the guest's own words at 17:24.
- **No sealed content.** leak_scan.py imported for its workbook half only (units: the 14 carried or linked targets as the workbook conversion prints them; nothing private or sealed opened). Six-word runs: 1 (02 l. 45, inside the guest's own 10:12 quotation, kept as said). Number windows: the ruled one (Worked problems' examples table), its math-check twin (__p8b_body l. 202) and A11's advisory line above, which lists the self-checks' illustrative numbers together — in all three every matched value is the guest's figure or the note's labelled illustrative number. Target registry ids, target workbook numbers, sealed paper names or years in the parts, drafts, index, assembled note, preview and the four check scripts: 0. Generic labels only; one neutral sentence on why

adapted items change (08 l. 49); no other past-paper mention beyond the allowed 2020/2021 labels. Gendered pronouns outside quotations: 0.

#	Item	Result	Where; what remains
1	§12.3 product	closed	04 I. 115; __p9_note A1 passes
2	speaker, 14:51	closed	02 I. 152
3	cite times	not closed	(a) 04 I. 26: the heard word Dimension is cited 26:20; its segment starts 27:08. (b) Draft D (t3833) I. 128: the guest's word five is cited 38:33; its segment starts 38:51. (c) 03 I. 114: 03c p. 4's words on John Bogle carry the 10/8 time 22:53; they are the deck's, not said (the guest's Bogle is at 24:02). (d) 07 I. 90: 01b p. 11's definition of price impact carries 62:20; the deck's words. (e) 18 stale P4 paragraph stamps left after the inserted cites, each now after a paraphrase said later: 01 I. 22 ×2 (00:24; said 00:34 and 00:49), 03 I. 114 ×2 (22:53; said 23:15 and 23:39), 04 I. 20 (25:49; 26:16), 04 I. 58 (29:23; 29:41–29:52), 04 I. 81 (30:53; 31:08), 04 I. 83 after the crowdsourced-additions clause (31:42; after 32:26), 05 I. 74 (40:39; 42:08), 05 I. 91 (42:56; 43:41), 06 I. 30 (45:29; 46:00), 06 I. 32 (45:29 on the disputed count; gate D6 cites 46:19) , 06 I. 64 (47:02; 47:56), 07 I. 9 (53:26; 53:35–53:41), 07 I. 44 (57:44; 57:56), 07 I. 90 (62:20 on the disputed amount; gate D11 cites 62:46) , 07 I. 95 (63:56; 64:11), 07 I. 99 (64:37; 65:34–65:44)
4	S2, S5	closed	04 II. 26, 29; 06 I. 122; 10 I. 43
5	unclear spans	closed	06 I. 142; 05 I. 100; 09 I. 41
6	t0427 stamps	closed	draft A II. 11, 22, 24, 32, 45; index II. 23, 34 (the registry's source field is the orchestrator's)
7	financing	closed	draft C, GIVEN to WHAT VARIES (seven headings in order; Step 1's identity re-derived); 03 II. 19, 21–22, 66; 09 I. 37; 10 II. 20–22; index; 01b p. 13's words confirmed
8	symbol table	closed	00 II. 48–95; draft B's error variance renamed; §14's six factors stated (05 I. 65)
9	Gap line	closed	06 II. 86–87

#	Item	Result	Where; what remains
10	Exam signal	closed	00 l. 44
11	Buffett	closed	03 l. 83; 05 l. 76
12	closing list	closed	10 l. 60
13	Thorp06	closed	02 l. 86; agrees with Thorp06 p. 26
14	WSJins	closed	07 l. 99; agrees with WSJins pp. 1–2
15	reasons	closed	08 l. 49; 07 ll. 79, 83; 04 ll. 126, 153; 03 l. 9
16	sealed	closed	as above. Advisory, non-blocking: the link file's block G lists WB 5.1 Q1 and Q3–Q6, so the one gap in that run still shows where a target sits
17	07b p. 17	closed	the new visual read agrees with the 150-dpi render; 07 l. 23; 00 l. 52
18	thin beats	closed	02 l. 23 (beat 39); 07 l. 99 (beat 244; 66:09 is a segment start)
19	minor	closed	04 l. 22 (FF04 p. 17 confirmed); the table's rows
20	re-sync	closed	index l. 73; the link file's 04b p. 12 and p. 13 blocks
Adv.	illustrative values	open (one)	the seven self-checks are each labelled the note's own; not yet: the 2-point wedge in §6.3's numerical check (02 l. 130, labelled course numbers, which is true of the 5% and 12% only) and its repeat in the closing self-check's answer (10 l. 15, no label)

NOTE_SPEC §8, re-decided: symbols pass; gaps flagged, not filled with invention pass; the math check passes; cited sources restated pass; no sealed content pass; **every quotation matches, at the right time and speaker: fail** (item 3). The other rows stand as passed.

Fix list for the next round. (1) 04 l. 26: Dimension → 27:08. (2) Draft D l. 128: five → 38:51. (3) 03 l. 114 and 07 l. 90: drop the 10/8 time after the 03c p. 4 and 01b p. 11 quotations. (4) Remove or retime the 18 stale stamps in item 3(e); 06 l. 32's goes to 46:19, and 07 l. 90's trailing one is simply dropped (62:46 already precedes it). (5) Label the 2-point wedge as the note's own illustrative value in 02 l. 130 and 10 l. 15.

Confirming re-check — batch fin-midterm round 2, 2026-10-13

After the fixer's round 2 (appended to the same log). **Verdict: PASS.** Item 3 and the round-1 fix list are closed; nothing else regressed.

- **Item 3, re-checked independently.** Every quotation followed by a 10/8 time: 424 at their own segment's start, 0 off (2 benign: the F2b-only passage at 17:19, stamped so by the clean transcript, in 03 l. 13 and draft C l. 41). The four round-1 cases: the heard word Dimension now 27:08 and the word five 38:51 (both segment starts); 03c p. 4's and 01b p. 11's words now carry only their slide cites (the guest's Bogle cited separately at 24:02). Every 10/8 jump link in the parts and drafts: 469 segment starts, 20 one-file passages, 4 by design (the undecoded questions 61:56 and 64:16; the link file's range ends 15:47 and 08:42). The 18 stale stamps and the 3 more the fixer found are re-timed to the segment that says them or dropped; the disputed count is at 46:19 (gate D6) and the trailing stamp on the disputed amount is gone (gate D11, 62:46). What remains of the backward-step scan is the header's and §1's pointers, a quotation in parentheses, and the contracts' lead-in at 54:19.
- **Wedge.** 02 l. 130 and 10 l. 15 now label the 2-point wedge as illustrative, the note's own, and the two rates as the course's (04a p. 9).
- **Block G** (ruled): no Block 5 item number remains; four bullets, the adapted items last.
- **Math.** Runner 3 parts, 1,082 checks, PASS (__p8a 437/0, __p8b_body 591/0, __p9_note 54/0). The 6 fewer are legitimate: __p8b_body.py is byte-for-byte the round-1 file, and its loss of 7 is its data-driven L1 check, one per jump link in the parts (404 now; the round dropped 9 stamps and added 2); __p8a.py gains 1 (block G names no item number) and its bullet count now tests 4. Planted runs still exit 1 (__p8a --selftest , __p9_note --plant).
- **Quotations.** p9_quotes 1.8 source self-test 551/551; the note: 744 quotations, 719 cite a time or page, 719 ok, 0 hits, 0 unpaired, 25 uncited (0 not found); planted self-test 10/10. Quoted words under an [unclear] mark: the same 30 overlaps, 0 defects.
- **Assembler** --check --no-manifest : 12 parts, 5 includes. Pointer scan 0. Gendered pronouns outside quotations 0. Reasons sweep over the round-2 lines: 0.
- **No sealed content.** The same scoped scan: 1 six-word run (inside the guest's 10:12 quotation); the three X89 windows of round 1, each matched value the guest's figure or the note's labelled illustrative number; 0 target ids, workbook numbers, sealed papers or years.

NOTE_SPEC §8: every row passes; the quotation row now passes.