

Stitched (two recordings, the second saved in three parts, joined on their overlapping words; the whole meeting is on file, no hole) · **Midterm 20 Oct:** in — The review itself. 05a pp. 23–27 are “the last thing you need to know” (10/15 τ 00:30) · **P9:** Passing — last confirming re-check 15 Oct.

Financial Economics — Lecture 12: The Midterm Review, after the Last Market-Efficiency Slides

RECORDED. Thursday 15 October 2026: Eduardo Dávila's review session for the midterm of Tuesday 20 October, named on the review deck's title slide ("Midterm Review", "Eduardo Dávila", "Updated: October 15, 2026", 21 p. 1). The note is written from **two recordings of the meeting, the second saved in three parts, stitched on their overlapping words into one timeline; together they leave no hole.** Every time below is a time on that stitched timeline, written τ. **The whole meeting is on file:** the record opens with the meeting's first words, "All righty, let's get started." at ⌚ 00:22 (about 1:06 pm by one phone's clock), and ends with the closing sentence at ⌚ 66:37 — ⌚ 66:45 (about 2:13 pm), seven minutes before the scheduled 2:20 — the lecturer had said the session would end "a few minutes early today" ⌚ 19:41. From ⌚ 40:28 only recording 1 runs, so everything after that moment — both CAPM questions, all fourteen written comments, thirteen of the thirty-eight exam statements — rests on one recording. The meeting first finishes the market-efficiency deck (05a pp. 22–27, to ⌚ 11:43), then reviews the midterm on the review deck, **every page from p. 2 to p. 21 in order**; the deck is the spine of this note.

Thursday 15 Oct 2026 · 66:46 recorded (speech from τ 00:22) · Eduardo Dávila · ECON 2251

To find a time in the original files, once: recording 2, part 1 is at the same time as τ (it covers τ 00:00–21:01). Recording 1 is at τ minus 12:34.3 (it covers τ 12:34–66:46). Recording 2, part 2 is at τ minus 21:14.0 (τ 21:14–29:29). Recording 2, part 3 is at τ minus 29:36.2 (τ 29:36–40:28). The text follows part 1 to ⌚ 20:57, recording 1 to ⌚ 31:04, part 3 to ⌚ 37:05, and recording 1 after that. Where the two recordings render a number or a phrase differently and no written source settles it, the note gives both renderings in a Warning and states neither as fact.

Sources used. Each is restated below where the meeting used it; the codes are how the note cites them.

- 21 p. N — the review deck, *Midterm Review* (21 pp., made the morning of the meeting). **It is the lecturer's written statement of what the midterm covers: where it and the recording disagree on scope, the deck is the record and the recording the gloss.** pp. 8 and 18 print the answers to its two practice pages; they were worked aloud but never displayed or named.
- 05a p. N — the deck *Market Efficiency* (28 pp.): pp. 22–27 finished here (p. 22 as the opening recap; pp. 23–27 taught); p. 19 recalled once. The rest of the deck is the Lecture 11 note (13 Oct, reconstructed).
- formula sheet p. N — the course's formula sheet, three pages; p. 1 is headed *Midterm* and lists eleven equations. **21 p. 4 reprints that list word for word.**

- 00 p. N, syl p. N, XR p. 1 — the course overview deck, the syllabus, and the lecturer's one-page list of extra readings.
- handout 02 p. N, handout 04 p. N — the present-value handout (pointed to at 31:50) and the CAPM handout (pointed to at 65:45); each is digested in full in its home note and restated here in a sentence or two.
- 01a, 01b, 02a, 02b, 03a, 03b, 03c, 03d, 04a, 04b p. N — the course decks a review page restates; deck 21 reprints four of their figures (03a p. 18 on 21 p. 10; 03c p. 17 on p. 11; 03d p. 3 on p. 12; 04a p. 17 on p. 15). 03S p. N is the statistics review deck. Plain 03a/03b/03c codes mean the export posted 13 October; the pages cited here number the same in both versions.
- WB x.y Qn, p. N — the practice workbook with solutions. Items named here are owned by earlier notes and linked, never re-worked; a workbook item adapted from a past exam is named only by its display id, *Adapted problem/self-check Xnn*.
- The recording, cited by time on the stitched timeline: 17:20 . A time from another meeting carries its date: 9/10 [66:16], 9/18 section [02:39], 10/8 [τ 15:48].
- Problems: the five calculations the lecturer demonstrated (21 pp. 7–8 examples 1–2; the 21 p. 12 limit; 21 p. 17 questions 1–2) and the two printed questions of 05a pp. 23 and 25 that were put to the room are this meeting's exercises, worked below under the course's own basis for practice: "There are no graded homeworks, but you are expected to work through the concept questions and the practice problems every week" (*syl p. 3*). The midterm itself and the final are never worked.

At a glance

Where this sits. Eleven meetings in, the course has covered its first five blocks — markets and instruments, present value and returns, portfolio choice, the CAPM, and market efficiency — and this is the last meeting before the midterm. It does two things. First it finishes Block 5: do mutual funds beat the market? (passive index funds win after costs; active managers show some stock-picking skill before costs; a few winners prove little, because luck alone makes five of a hundred look significant). Then it reviews the midterm from a deck made that morning: what the paper looks like, which equations are printed on it, the five blocks and what matters in each, two discounting examples and two CAPM questions worked with the room, and fourteen questions students had sent in, answered one by one. Nothing here is new to the course except the review's own emphases and routes; every topic links to the note that taught it in full.

After this meeting you must be able to:

- **State** how the midterm runs and what earns credit — eight numerical items and twelve short questions at five points each, 75 minutes, a calculator and nothing else, an explanation required for every short question — from §1.

- **Say** which eleven equations are printed on the paper and which familiar results are not, and **derive** the ones that are not (the equal-weight variance split) from the ones that are.
- **Date** a present-value formula correctly: a perpetuity or annuity formula values its stream one period before the first payment, on the clock the rate is quoted in, and a one-year move at the annual rate brings it to today — the two examples of 21 pp. 7–8.
- **Draw** the two-asset opportunity set at correlations 1, 0.27, –0.27 and –1, the capital allocation lines to the tangency portfolio, and the CML beside the SML, and **read** each slope as a price of risk.
- **Solve** two SML equations for the risk-free rate and the market risk premium, and **argue** three true/false claims about the CAPM in one to three lines each, with a diagram.
- **Answer** the fourteen student questions of 21 pp. 19–21 in the course's own words: the MVP in general, the correlation cases, why factor models multiply, efficient against optimal against feasible against desirable, the four lines, negative beta, the two tests of the CAPM, the CAL against the frontier, low price and high return, and what "the CAPM holds" means.

Prerequisites. Everything: this is a review. Each section links the note that teaches its page. The time value of money is the Lecture 3 note (10 Sep), the Section 2 note (18 Sep) and the Lecture 4 note (17 Sep); statistics and two-asset portfolios the Lecture 5 note (22 Sep); preferences and the capital allocation line the Lecture 6 note (24 Sep); the tangency portfolio, two-fund separation and many assets the Lecture 7 note (29 Sep); the CAPM the Lecture 8 note (1 Oct); its applications and tests the Lecture 9 note (6 Oct, reconstructed) and the Lecture 10 note (8 Oct, the guest lecture); market efficiency the Lecture 11 note (13 Oct, reconstructed); market making, spreads, orders and short sales the Lecture 2 note (8 Sep); the course's rules the Lecture 1 note (3 Sep).

EXAM SIGNAL What this note labels, and what it does not

Every statement the lecturer made about the midterm is in §1, quoted word for word with its time, and repeated as an Exam signal at the page it concerns. The deck's record stands beside each one: "everything that is in the slides is fair game for the exam" ⌚ 17:32, and 21 p. 3 prints, in bold, "Every question can be answered with the material covered in class". **The note labels nothing examinable or unexaminable in its own voice.** Where the lecturer said a thing will not be asked, the note says *said it will not be asked*, with the time; where the deck lists a block, the note says the deck lists it.

Symbol table

Notation is normalized across the meeting. The deck writes expectation, variance and covariance in blackboard bold ($\mathbb{E}$, Var , Cov); this note keeps $\mathbb{E}[\cdot]$ and writes $\text{Var}[\cdot]$ and $\text{Cov}[\cdot, \cdot]$, as the Lecture 5–10 notes do. The two-year rate is the deck's R_b ("bi-annual" means once every two years, as on 02b p. 10); the

lecturer said it as *r-bar*; the Lecture 4 note writes $R_{2\text{yr}}$ and the section notes r_2 . One letter R is the annual rate; the lecturer's spoken "r" is the same thing. The market risk premium is written out as $\mathbb{E}[R_M] - R_f$ (21 p. 18 abbreviates it "MRP"; the Lecture 8 note's symbol table records π for two workbook items). The deck writes the security characteristic line twice, with a time-varying $R_f(t)$ on p. 4 and a constant R_f on p. 16: two course forms, both kept. A few letters are local to one passage and the table says where.

Symbol	Meaning	Units	First used
R	the annual net interest or discount rate, compounded annually (21 pp. 4, 7–8); the lecturer's spoken "r"	per year, decimal	§1.1
R_b	the rate over one two-year period, $1 + R_b = (1 + R)^2$ — 21 p. 8's letter; said as <i>r-bar</i> ; L4 writes $R_{2\text{yr}}$, S1/S2 r_2	per two years, decimal	§5
C	a level payment (coupon)	dollars per payment	§1.1
T	the number of periods, in the formulas of 21 p. 4	periods	§1.1
PV, FV ; PV_t	present and future value; a value dated at year t (PV_8 , PV_1 , PV_0 on 21 p. 8)	dollars at the date named	§1.1
V_{-1}	the value, dated at year -1 , of a stream the note constructs as a check (21 p. 7 example 2's worked problem only)	dollars at year -1	Worked problems
q	two years of discounting, $(1 + R)^{-2}$, in the sum check of 21 p. 7 example 2 (the worked problem only)	pure number	Worked problems
m, APR , EAR	compounding periods a year; the quoted annual percentage rate; the effective annual rate, $1 + EAR = (1 + APR/m)^m$	per year; per year, decimal	§1.1
HPR, IRR	the holding-period return; the internal rate of return (named on 21 p. 7, not worked)	per period	§5
$\mathbb{E}[\cdot]$, $\text{Var}[\cdot]$, $\text{Cov}[\cdot, \cdot]$	expectation, variance, covariance — <i>the deck writes</i> $\mathbb{V}ar$, $\mathbb{C}ov$	units of the argument; (per period) ²	§1.1
$a, \dots, f$; X , Y	the constants and the two random variables of the covariance rule, 21 p. 4 item 6	—	§1.1
R_i , R_p , R_M , R_f	the return on asset i ; on a portfolio; on the market portfolio; the risk-free rate	per period, decimal	§1.1
$R_i(t)$, $R_M(t)$, $R_f(t)$, $\varepsilon_i(t)$	the same returns observed at date t , and the regression residual, in the security characteristic line (21 p. 4 item 11 writes $R_f(t)$; 21 p. 16 a constant R_f)	per period	§1.1

Symbol	Meaning	Units	First used
α_i, β_i	the intercept and slope of security i 's characteristic line; $\beta_i = \text{Cov}[R_i, R_M] / \sigma_M^2$	per period; pure number	§1.1
w_i, N	the weight of asset i in a portfolio, $\sum_i w_i = 1$; the number of assets	fraction of wealth; count	§1.1
w_M^i, p_i, n_i	asset i 's weight in the market portfolio, its share price and its shares outstanding: $w_M^i = p_i n_i / \sum_j p_j n_j$ — 21 p. 13 writes the sum's index as i ; read j	fraction; dollars; count	§10
ω_i, R_m, σ_m	security i 's weight in the market portfolio, in the derivative $\partial \sigma_m / \partial \omega_i$ (how much the market's volatility changes when that weight changes) — handout 04 p. 2 and 04a p. 12 write ω_i ; the handout writes a lower-case m for the market, so its R_m, σ_m are this note's R_M, σ_M (not the compounding count m)	fraction of wealth	§17
$\sigma_i, \sigma_p, \sigma_M, \sigma_i^2, \sigma_p^2$	the volatility (standard deviation) of asset i , a portfolio, the market; their variances	per period; (per period) ²	§1.1
σ_ε^2	stock i 's idiosyncratic variance in the split $\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma_\varepsilon^2$ — 21 p. 13 and 04a p. 19 print it without the i	(per period) ²	§10
$\rho_{ij}, \rho_{12}, \rho_{iM}; \rho$	the correlation of two assets' returns; of asset i with the market; the US/JP correlation in the legend of 21 p. 10	pure number	§1.1
σ_{12}	the covariance of the two risky assets, $\rho_{12} \sigma_1 \sigma_2$, in 03a p. 17's MVP formula	(per period) ²	§15
w^G, w_1^G, w_2^G	the minimum-variance portfolio's weights (03a p. 17; the Lecture 5 note's letters)	fraction of wealth	§15
$\overline{\sigma^2}, \overline{\text{Cov}}$	the average variance and the average pairwise covariance of N assets, 21 p. 12's two brackets	(per period) ²	§9
SR_i, SR_M	the Sharpe ratio of asset i , $(\mathbb{E}[R_i] - R_f) / \sigma_i$, and of the market — 21 pp. 13, 16 write them in words	pure number	§8
$\mathbb{E}[R_M] - R_f$	the market risk premium, the SML's slope — 21 p. 18 writes "MRP"; the Lecture 8 note's π	per period, decimal	§1.1

Symbol	Meaning	Units	First used
T (tangency), MVE, MVP	the tangency portfolio (21 p. 11's label MVE, mean-variance efficient; the Lecture 7 note's T); the minimum-variance portfolio (21 p. 11's MVP; w^G above) — T is also the period count in the formulas; the context says which	—	§8
A, B; Q, K	two efficient risky portfolios on 21 p. 11's chart; a portfolio and a security with equal expected returns on 21 p. 15's figure	figure labels	§8, §12
Δ	a small extra holding of stock i added to the market, in claim 1 of 21 p. 17 question 2 (the self-check only)	fraction of wealth	Self-check
$\gamma_0, \gamma_1; \bar{R}_i, \bar{R}_f, \bar{R}_M$	the intercept and slope of the Fama–MacBeth second-step regression of average excess returns on betas; the sample averages it uses (04b p. 7)	per period; per period per unit of beta	§16
P, X	the price today and the fixed future payoff of an asset, in the note's derivation of "low price, high return" (§17 only; this X is not the covariance rule's)	dollars	§17
US, JP	the two risky assets of 03a p. 8 (US 13.6% / 15.4%; JP 15.0% / 23.0%) behind 21 pp. 10–11's charts	—	§7

Names. Eduardo Dávila (the written sources; never said on the recording); Toby Moskowitz, the 8 October guest, referred to as "Toby" four times, on three occasions; Warren Buffett, David Swensen, Robert J. Shiller, Burton G. Malkiel, Gregory Zuckerman (the three book covers of 05a p. 27); Fidelity Magellan, Vanguard, the Wilshire 5000, Renaissance, AQR (05a pp. 23–24; ⌚ 02:40); Wermers (05a p. 25, named on the slide only); Fama, MacBeth, French, Carhart (21 p. 14; 04b). Four names the recording garbles are never written as names: a second fund manager named beside Buffett at ⌚ 05:45, the accommodations office (the syllabus's "Resource Office on Disabilities", *syl* p. 4), the model of graphing calculator a student asked about at ⌚ 19:09, and the book recommended at ⌚ 62:23 (§16). No student is named.

Picking up from the 13 October meeting, and from the guest lecture

The meeting opens by finishing the market-efficiency deck. The previous lecture (Tuesday 13 October, not recorded; its note is reconstructed) had ended on 05a p. 22's summary: the lecturer's first words of content

are "I just want to finish these few slides that have been in, and then that will be the last thing you need to know from the exam" ⌚ 00:30, and the recap that follows calls p. 22 "kind of a good summary slide" ⌚ 01:21. That recap, in full, is §2.1; it is the only evidence on file of what the 13 October meeting reached, and it names no day and no page. The 8 October guest lecture by Toby Moskowitz is recalled three times as material already covered — a hedge fund as an example of an active manager ⌚ 02:40, sports as a place where skill is easy to see ⌚ 07:17, and the flat SML and betting against beta ⌚ 41:30 — each in its place below.

1. What Dávila said and printed about the midterm

⌚ 11:59 · *every statement about the midterm from this meeting, word for word and timed, with the review deck's page beside it; the formula sheet first*

This section establishes, in one place, what the midterm of Tuesday 20 October looks like, which equations are printed on it, what it covers, how it is marked, and how the lecturer said to prepare. The written record is the review deck's pp. 2–5 (21 pp. 2–5), restated in full below, and the formula sheet's first page; the syllabus adds one rule the meeting did not repeat. Everything else rests on what was said in the room, and each statement below is quoted at the moment it was said. Where the two recordings render a sentence differently in a way that matters, both renderings are given and neither is treated as fact. Each statement that concerns one review page is repeated as an Exam signal at that page, later in the note; this section is where they all are.

1.1 The formula sheet: eleven equations, printed on the paper

EXAM SIGNAL The equations the exam prints — and the only three statements about the list

21 p. 4 is headed "Equations in the Exam" and lists eleven formulas. They are, word for word and in the same order, the formula sheet's *Midterm list* (formula sheet p. 1; the two text layers were machine-compared and differ only in the bullet glyph). On the recording: "In terms of equations, these are the equations that you see in the exam." ⌚ 17:20; "you have it in your cheat sheet" ⌚ 32:29 (both recordings). **The words *formula sheet* and *provided* were not said on 15 October and are not printed on the deck;** the page's title is its only statement. The earlier record is consistent: on 10 September the lecturer said "I don't know if I've said this, but I'll give you formulas for the exam, for everything." and "all the formulas are posted online, you can take a look" (9/10 [66:16]), and the 18 September section leader said "I think Eduardo will give you these formulas in the exam, so you don't need to memorize it, you only need to know how to use it." (9/18 section [02:39]).

Equations in the Exam

- ▶ $FV = PV(1 + R)^T$
- ▶ $FV = PVe^{RT}$
- ▶ $PV_{\text{annuity}} = \frac{C}{R} \left(1 - (1 + R)^{-T}\right)$
- ▶ $PV_{\text{perpetuity}} = \frac{C}{R}$
- ▶ $1 + EAR = \left(1 + \frac{APR}{m}\right)^m$
- ▶ $\text{Cov}[aX + bY + c, dX + eY + f] = (ae + bd) \text{Cov}[X, Y] + ad \text{Var}[X] + be \text{Var}[Y]$
- ▶ $\mathbb{E}[R_p] = \sum_{i=1}^N w_i \mathbb{E}[R_i]$
- ▶ $\text{Var}[R_p] = \sum_{i=1}^N w_i^2 \sigma_i^2 + 2 \sum_{i=1}^N \sum_{j>i}^N w_i w_j \rho_{ij} \sigma_i \sigma_j = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \rho_{12} \sigma_1 \sigma_2$
- ▶ $\mathbb{E}[R_p] = R_f + \frac{\mathbb{E}[R_i] - R_f}{\sigma_i} \sigma_p$
- ▶ $\mathbb{E}[R_i] = R_f + \beta_i (\mathbb{E}[R_M] - R_f)$ and $\beta_i = \frac{\text{Cov}[R_i, R_M]}{\sigma_M^2}$
- ▶ $R_i(t) - R_f(t) = \alpha_i + \beta_i (R_M(t) - R_f(t)) + \varepsilon_i(t)$

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21 p. 4 — original slide: "Equations in the Exam", the eleven formulas

How to read this: eleven bullets, no words. Items 1–5 are Block 2 (time value of money), item 6 is the statistics review's covariance rule, items 7–9 are Block 3 (portfolio choice), items 10–11 are Block 4 (the CAPM). Nothing from Blocks 5–10 is on it.

The eleven, as printed, with what each is and the note that teaches it:

#	As printed on 21 p. 4 (= formula sheet p. 1)	What it is	Taught in
1	$FV = PV(1 + R)^T$	compound growth of one sum over T periods at R a period	Lecture 3, §2.3
2	$FV = PVe^{RT}$	the same under continuous compounding	Lecture 4, §3.2
3	$PV_{\text{annuity}} = \frac{C}{R} \left(1 - (1 + R)^{-T}\right)$	T level payments, the first one period after the valuation date	Lecture 3, §10.2; Section 2, §6
4	$PV_{\text{perpetuity}} = \frac{C}{R}$	level payments for ever, the first one period after the valuation date	Lecture 3, §11.1
5	$1 + EAR = \left(1 + \frac{APR}{m}\right)^m$	the effective annual rate from a quoted rate compounded m times a year	Lecture 4, §2.4
6	$\text{Cov}[aX + bY + c, dX + eY + f] = (ae + bd) \text{Cov}[X, Y] + ad \text{Var}[X] + be \text{Var}[Y]$	the covariance of two linear combinations — "my covariance operator thing"  18:20	Lecture 5, §3.3
7	$\mathbb{E}[R_p] = \sum_{i=1}^N w_i \mathbb{E}[R_i]$	a portfolio's expected return	Lecture 5, §7.1
8	$\text{Var}[R_p] = \sum_{i=1}^N w_i^2 \sigma_i^2 + 2 \sum_{i=1}^N \sum_{j>i}^N w_i w_j \rho_{ij} \sigma_i \sigma_j = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \rho_{12} \sigma_1 \sigma_2$	a portfolio's variance, for N assets and for two — "the nasty looking one for the variances"  18:02	Lecture 5, §7.2; Lecture 7, §11.1
9	$\mathbb{E}[R_p] = R_f + \frac{\mathbb{E}[R_i] - R_f}{\sigma_i} \sigma_p$	the capital allocation line through asset i , for a long position in it	Lecture 6, §10.3
10	$\mathbb{E}[R_i] = R_f + \beta_i (\mathbb{E}[R_M] - R_f)$ and $\beta_i = \frac{\text{Cov}[R_i, R_M]}{\sigma_M^2}$	the security market line, and beta	Lecture 8, §11.3

#	As printed on 21 p. 4 (= formula sheet p. 1)	What it is	Taught in
11	$R_i(t) - R_f(t) = \alpha_i + \beta_i (R_M(t) - R_f(t)) + \varepsilon_i(t)$	the security characteristic line, the regression that estimates beta	Lecture 8, §15.1; Lecture 9, §5.4

F1 — what was said about the list ⌚ 17:20, both recordings. "In terms of equations, these are the equations that you see in the exam. Somebody was asking me, hey, there are other equations that you use that are not here, do I need to know them? Kind of, okay, because like everything that is in the slides is fair game for the exam, I put these ones because I thought they were the most natural ones to have, but if you think there's something that you wouldn't know how to derive out of this, for example, or that you really need to know, just learn it by heart," — then a clause the two recordings render differently (the Warning below) — "okay, I think this is plenty, okay, but again, you don't kind of tell me, oh, it was an equation that was I'm putting what I think is useful to know. I put the nasty looking one for the variances and so on. So I think this is all you kind of need to know. But again, if there's something else you think you should learn, it won't be too much. Okay? Does that make sense? I think most of what you need to know is here. You can sort of derive many things here. I even put my covariance operator thing, which we use all the time in this course, which I think is super, super important." (The other recording renders "fair game" as *per game, for example*, and "I even put my covariance operator thing" as *I don't put*; 21 p. 4 item 6 prints the covariance rule, so it is on the list.)

WARNING Said in class vs written — 17:32: how many equations beyond the list

After "just learn it by heart", one recording renders the next clause as "it would be maybe two, three, or four" and the other as "It will be in the report." The record does not settle which was said, and no written source counts the equations that are not on the list. **What stands:** results not on 21 p. 4 that you cannot derive from it are to be learned by heart; **no number of them enters this note as fact.** This is the first item of the closing list, *What the materials don't settle*.

F2 — the two portfolio formulas ⌚ 32:21, both recordings. "Portfolio choice. Next, we got these type of equations. Okay, the first one, how you compute expected returns. The second one, how you compute the variance. Again, it's nasty, but you have it in your cheat sheet, so you know, you don't have to worry, because you have it there. All you have to do is know how to interpret all those Greek letters we got over there, okay? Rows and sigmas and whatnot, okay?" — items 7 and 8, restated on 21 p. 9 (§6).

F3 — what is *not* on the list: the equal-weight variance split of 21 p. 12 ⌚ 37:49, both recordings to ⌚ 38:27. Someone asked whether 21 p. 12's two equations are "in the paper or not" (the recording does not show whether the question came from the room or from the lecturer checking the list; it names no

speaker). The answer, in the body's words: "They're not? Well, it doesn't make me want to keep them in mind. Again, it's not that hard. You know these things." — then a sentence the recordings render differently (below) — "from the equation over there. If you do one over n , you get exactly" the same equation. Then the rule: "I think the rule that I followed was,"  38:19 — and what follows the recordings render differently (the same Warning). **Settled by the written sources:** 21 p. 4 does not carry 21 p. 12's split; you derive it from item 8 by setting every weight to $1/N$, which §9 and the worked problem there do.

WARNING Said in class vs written — 38:06 and 38:19: the rule for what went on the list

One recording renders the reason as "I think the reason why you put them is because you derive them yourself", the other as "I think the reason why I didn't put them is because you can just derive them yourself"; and the rule as "I only put stuff that was really hard to derive, or anything that can be derived from this" against "I only put stuff that was a little bit hard to derive, everything that can be derived from this, you can't even do yourselves". The two agree on the fact — the equal-weight equations are not on the sheet and follow from item 8 with $w_i = 1/N$ — and on the words "from the equation over there. If you do one over n , you get exactly". **The wording of the rule is not settled**, and the note states only the fact.

1.2 Date, place, length, form and points (21 p. 2)

Midterm

Tuesday, October 20th, in class

- ▶ Structure
 - ▶ 8 Numerical Questions (5 points each)
 - ▶ 12 Short Questions (5 points each)
 - ▶ You need to explain your answer (very important)
- ▶ Maximum score is $20 \times 5 = 100$ points
- ▶ Calculator required
 - ▶ No Cell Phones/Tablets/Computers
- ▶ 75 minutes
- ▶ No cheating policy strictly enforced

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21 p. 2 — original slide: "Midterm", the date, the structure, the points, the calculator, the length and the honesty policy

How to read this: the page is the written record of the exam's format; every line of it was also said aloud between 11:59 and 15:59, in the page's order except the length.

What 21 p. 2 prints, line by line: "Tuesday, October 20th, in class" (bold, centred); "Structure" — "8 Numerical Questions (5 points each)", "12 Short Questions (5 points each)" and, indented under the short questions, "You need to explain your answer (very important)"; "Maximum score is $20 \times 5 = 100$ points"; "Calculator required" (the word *required* underlined), with "No Cell Phones/Tablets/Computers" beneath it; "75 minutes"; "No cheating policy strictly enforced". The split of the points is the note's arithmetic, not printed: $8 \times 5 = 40$ points for the numerical items and $12 \times 5 = 60$ for the short questions, and $8 + 12 = 20$ questions. The page does not say a clock time, a room, whether notes or books are allowed, or how the grade is weighted; "You need to explain your answer" is printed under the short questions, and whether it also covers the numerical items the page does not say.

M1 — the date, the place, the format  12:03, one recording (recording 1 starts at τ 12:34). "It's on Tuesday, it's here, and you have, I think I posted the last two that I did, like there were two years I didn't

teach the class, but I think 22 and 25, those are the last two, format will be exactly the same, eight numerical questions with five points per question, and again, there may be, typically it's like, I've written half the exam right now, but it's gonna be like three or four, like, questions, and then ABC, okay, but it's only" — the sentence breaks off there.

WARNING Said in class vs written — 12:03: "three or four, like, questions, and then ABC"

Heard (one recording): "it's gonna be like three or four, like, questions, and then ABC". **Written:** "8 Numerical Questions (5 points each)" (21 p. 2); and at 12:32 the lecturer corrects to "eight items, I should say". The deck is the record: eight numerical items at five points each. How those eight items are grouped into questions with lettered parts is not settled by anything on file, and the note does not interpret the heard phrase; it is item 3 of the closing list.

M2 — eight items, partial credit, a blank scores zero 12:32, both recordings. "So it will be like eight items, I should say, okay, eight items, and each one is worth five points. We'll give some partial credit, okay, so I know this course is hard and everything, but you need to make life easy for us, okay? Like if you don't write anything, you get a zero. If you write something that makes sense," — then a clause the recordings render differently (below) — and, after a garbled few words, "So just make it easy for us." 12:54. **Partial credit is given** (both recordings) and is on no deck page.

WARNING Said in class vs written — 12:47: how partial credit is scored

After "If you write something that makes sense," one recording renders "you may get seven" and the other "we'll make it simple". No written source mentions partial credit or a score of seven; the record does not settle which was said. **Partial credit exists; no figure for it enters this note.** Item 1 of the closing list.

M3 — twelve short questions, and what earns credit on them 12:57, both recordings. "And then we have like 12 short questions, okay? So the short questions, I'm gonna go over a few of them" (the other recording renders *a couple of short questions*; 21 p. 2 prints 12). "But, you know, I imagine all of you are not super familiar with the review workbook that I gave you, so it's going to be similar to that, I think a lot of the ones I have there are multiple choice, in the exam they are the short question form, so it will be like short question, there's some true, false, just true or false comes zero, it's all about the explanation, but I think you know me by now, there's always a very clear answer to my questions that is like pretty obvious and self-evident, and if it's not and you say something clever, that's fine, I'm happy to also give you credit for that, but it has to be like a clear, coherent idea, okay?" 13:03. "It cannot be like, sometimes you hear these answers that are like, fishing for, you know, you mentioned all the buzzwords in the area, that's not going to fly, okay?" 13:36. "So there's typically a very clear, sharp answer, and again, you don't have to write like

novels, okay?" ⌚ 13:44 . "Typically, one line, two lines, three lines, that's enough, okay?" ⌚ 13:51 . The deck's version is five words: "You need to explain your answer (very important)" (21 p. 2).

M4 — 75 minutes ⌚ 13:55 , both recordings. "Let me say something about the time. It's only 75 minutes, so you're going to be, I want to say, pressed by time, This is not an exam where you get a coffee and chill. This is an exam where you have to sit down and get some flow, so please make sure you are ready for that." (The other recording renders the length later as *seven to five minutes*; 21 p. 2 prints "75 minutes".) "I don't think it's impossible. Every year I have people doing super well, I've had perfect scores many years, but it's not an exam where you can just be chilling. You need to be focused." ⌚ 14:16 . "The final, for what it's worth, will be longer, so there will be many more questions and a much longer exam, so there I think there will be more time to maybe enjoy the experience. Here it's more like you gotta get going." ⌚ 14:29 . "I'm telling you this so that you're aware that 75 minutes is short. In my experience it's just not a lot of time and you need to get focused really quickly." ⌚ 14:42 .

M5 — the maximum ⌚ 14:54 , one recording. "Okay, so the most you can get is 100." — points, not dollars: 21 p. 2's "Maximum score is $20 \times 5 = 100$ points".

M7 — how a numerical question is put ⌚ 46:48 , one recording, on 21 p. 17's question 1: "Yeah, I'm just saying, well, how do you tackle this question?" — the question is worked in §14.

1.3 The level of the questions, and how to prepare (21 p. 3)

What 21 p. 3 prints: "Short questions — similar to the concept questions"; "Numerical questions — similar to the exercises made in class and to the practice problems"; and, bold and centred, "**Every question can be answered with the material covered in class**". The bold line was not read out SLIDE ONLY; it was said in other words at ⌚ 51:44 (S13 below). The title was read: "Level of the questions, I think I just, let me say it again." ⌚ 16:02 .

M6 — the workbook is the preparation ⌚ 16:02 , both recordings. "Short questions similar to the concept questions you have in the workbook, and numerical questions similar to the ones in class, and to all these practice problems that you have. Okay, so you should be, I imagine this weekend, And finally, the workbook will be your best friend. You're going to be working like crazy on the prompts." "Sometimes it will tell me, oh, I don't have enough time to finish them all. Then maybe find more time. Sometimes it will tell me, I finished them all. Do you have more? I don't have more. There's a lot of them. Probably in 2026, you can ask your favorite AI thing to do more questions. But I think there's plenty of material. If you work out every question in the practice workbook and you fully understand, you comfortably can answer it, you're going to do well in the exam," ⌚ 16:23 — a garbled phrase — "okay? So that's for sure. If you barely understand what's going on or you haven't worked through everything, then it's not going to be that good, okay? So I think it's super important. I don't want to hear people, you know, like, we don't have homework in the class this year, so I didn't do anything." ⌚ 16:44 . "I've been telling you for weeks that you must be working on

the homework, and I know some of you are, and then that's going to show up on your performance next week." ⌚ 17:09 .

S8 — after the discounting examples ⌚ 31:32 , both recordings. "Very good, I think these are good, a lot of them in the workbook problems just works to them, and there's honestly a finite number of things I can do, okay, so, yeah, it's what it is."

S13 — not trying to trick anyone ⌚ 51:36 , one recording, at the end of 21 p. 17's question 2. "I'm not trying to trick anyone, this is just your understanding of the framework." "So again, one, don't worry too much." "I'll try to ask questions that either I spend time on, or it can be easily derived with a little bit of logic from things that we have worked on." ⌚ 51:44 . "It should be easy for you to understand." ⌚ 51:56 . "I have a solution. Again, draw a diagram, draw a diagram. Diagrams are super helpful to make this work." ⌚ 51:59 . (Just before these, the lecturer rated the third claim's difficulty in a sentence that ends in a garbled passage, at ⌚ 51:30 ; the clear words that follow are the ones quoted. The rating is not quoted as a verdict.)

S20 — the workbook is harder than class ⌚ 66:14 , one recording, the last written comment. "I found some of the practice problems more difficult compared to the questions we did in class. Absolutely, right?" "The workbook is a little bit harder because if I spent, I don't know, 20 minutes here doing a complicated problem in class, they wouldn't have time to do anything." ⌚ 66:21 . "But because you've been working on them slowly for the last six weeks, you're confident now and you're going to do pretty well." ⌚ 66:30 .

1.4 Rules: calculator, devices, honesty, accommodations

R1 — bring a calculator, maybe two ⌚ 14:58 , both recordings. "You need a calculator, so bring a calculator, please. Every year somebody forgets a calculator, maybe bring two if you're worried about the batteries. Okay? It doesn't have to be graphical or anything," — a remark about a graphing calculator follows, garbled — "Okay, but all you need is a normal calculator, up to whatever, division or something, or exponentials, that's, like, easy, I think, to find." ⌚ 15:24 . The written rule is the syllabus's: "a scientific calculator that can compute exponentials and logarithms" (*syl p. 3*); 21 p. 2 prints "Calculator required".

WARNING Said in class vs written — 15:24: the end of the calculator sentence

The other recording renders the end of the calculator sentence as *a division or something exponential that you can put in your computer*. That rendering is never used: the body's recording has no computer in it, and 21 p. 2 prints "No Cell Phones/Tablets/Computers". What you may bring is a calculator.

R2 — no phones, tablets or computers; no cheating ⌚ 15:30 , both recordings. "No cell phones, no tablets, no computers. We've had, every year, there's an issue with this, and I think it's just, like, awful, so please don't, you know, just put everything in your bag, and then we get done." "And again, please don't try to cheat, because again, every, over the years, these situations have gotten worse and worse, so it's just not

worth for anybody to waste time on these issues, okay?" ⌚ 15:41 . "Okay, so we have a bit of a compact class. We're gonna be close to each other. I'll be here, some teaching fellows will be here. But yeah, it's just not worth it, all right?" ⌚ 15:50 . 21 p. 2: "No cheating policy strictly enforced".

R3 — which calculators ⌚ 18:57 , both recordings. A student asked: "Are there any calculators we're not allowed to have?" ⌚ 19:03 , and, asked what was meant, named a graphing calculator used in high school (the model's name is garbled on both recordings and is not written here).

The lecturer's reply: "That seems like a tricky question. What do you have in mind?" ⌚ 19:07 ; then "I think that should be fine. You're not supposed to put text on it, okay? Even if you have text out. Honestly, you won't have time to look at the text, even if you, it's just, it's fine. Okay? Yeah. Okay, if you have it, you're not supposed to use it, okay? We'll take it out." ⌚ 19:13 — the rest is garbled. **The rule as said: a graphing calculator is fine; text stored on it is not to be used.**

L1 — accommodations ⌚ 18:28 , both recordings. "Yes, this is about the exam." Anyone with an accommodation through the disabilities office (the office's name is garbled on both recordings; the syllabus's "Resource Office on Disabilities", *syl p. 4*) — "I think you've been in touch with me, but make sure you set up the exam in the center, and they'll have the exam there. I think I've told this to anybody who sent me emails, but if I haven't, I'll say it here."

Closed book — not said on 15 October, not on the deck. The syllabus: "Both exams are closed-book." (*syl p. 4*).

1.5 What the exam covers

EXAM SIGNAL Scope: the deck is the record

21 p. 5, "Blocks of content so far", lists Block 1 Financial Instruments and Markets, Block 2 Present Value and Returns, Block 3 Portfolio Choice, Block 4 Capital Asset Pricing Model (CAPM), Block 5 Market Efficiency, and pp. 6–16 give each of them one or more summary pages; 21 p. 3 prints "Every question can be answered with the material covered in class"; the eleven equations of p. 4 all come from Blocks 2–4. **No page of the deck puts any block, deck or topic out.** By name, the deck does not mention the equity-valuation deck (06a), the arbitrage decks (07a, 07b), the fixed-income decks (08a–08d), the options decks (09a–09d), the financial-intermediation deck (10) or any of the optional decks (z03a, z03b, z04, z06, z08, z09a, z09b): they are simply absent from it. The recording narrows nothing the deck lists, except by the four statements the table below lists as said not to be asked, which the deck does not contradict (arbitrage as a direct question, the time-series test, deriving the SML, and 21 p. 17's first claim, with the remark that follows it). This note labels nothing in or out in its own voice.

S1 — Block 5 to its end; equity valuation by video ⌚ 00:30, one recording (recording 1 had not started). "Welcome everyone, it's Thursday, which means our exam is on Tuesday, and we're going to talk about it in 10 minutes, but I just want to finish these few slides that have been in, and then that will be the last thing you need to know from the exam, okay? So remember the equity valuation block, I'll just send you a video for you to watch," — the rest of the sentence is garbled — "And again, in a few minutes, I'll ask all your questions about the midterm, but let me finish what we're doing here." ⌚ 00:56. The slides finished are 05a pp. 23–27 (§2).

GAP Not recoverable — 00:45

"So remember the equity valuation block, I'll just send you a video for you to watch, and that will be part of the time, but for next Tuesday we're going to go into the end of the spot" — the end is garbled, and only one recording has it. What is clear: the equity-valuation block (deck 06a) will be delivered as a video. **Whether any of it is on the 20 October paper is not settled by anything on file:** deck 21 names no Block 6 and lists Blocks 1–5. Item 5 of the closing list.

S3 — the core ⌚ 20:51, both recordings, identical. "And then really the core of this first half of the semester is sort of blocks three, four, and five."

S16 — arbitrage: not asked explicitly ⌚ 58:31, one recording, answering a written comment. "Can we clarify the meaning of arbitrage? As I've said many times, arbitrage won't show up explicitly in the midterms." "I will not ask a question like, oh, is this an arbitrage or not?" ⌚ 58:38. A sentence about the final follows, garbled, and is not quoted. "But remember, arbitrage is the idea that you turn a profit, you make a profit, without putting too much effort into it." ⌚ 58:45; then 22.7 seconds the recording does not decode. "We're going to spend a lot of time talking about it after the break, but again, that is the notion." ⌚ 59:13. The course's written definition is tighter than the spoken one; the Warning is at §16.

S17 — only the cross-sectional test; time series not asked ⌚ 60:24, one recording. "Can you review the two approaches to the CAPM?" "Let's be honest, we only reviewed the cross-sectional approach, which is the Fama-MacBeth approach." ⌚ 60:26. "So that's cross-sectional. We never do time series. I'm not going to ask about time series." ⌚ 61:28. "That would be a time-series test, which we don't do here because it requires some of the econometrics," ⌚ 62:12.

S19 — no derivation of the SML ⌚ 65:35, one recording. "Well, the CML, I think, is very straightforward. The SML, as I said, I didn't give you the full proof," — a clause about "the question yesterday" is garbled — "so don't worry, I'm not going to ask you to derive the SML, okay?" "But the idea is, as before, you've got to be indifferent between holding everything, and when you have that, go through some of the handouts where I have the more formal proofs, then you end up getting the SML. That's where the equations are coming from." ⌚ 65:45. "but again, I care more to understand the economics than the exact mathematics.

To do it well, I need to spend a little bit more time doing some linear algebra, things like this. I do this in some other sort of higher-level courses." ⌚ 65:55 .

Scope in one table. Said, printed, or neither:

	Where
Listed by the deck	Blocks 1–5 (<i>21 p. 5</i>), one or more pages each (<i>21 pp. 6–16</i>); the eleven equations (<i>21 p. 4</i>)
Said to be asked, or always asked	discounting, "I always ask some discounting questions" ⌚ 23:34 ; the two-asset case, "conceptually, there are always questions on this" ⌚ 32:43 ; the MVP in the general case, "in principle yes" ⌚ 52:20 ; the correlation cases ⌚ 52:54 ; 4- and 5-factor models, "I could ask you this question" ⌚ 53:16 ; Block 1's ideas, which the exams "typically" carry ⌚ 22:30 (the verb is garbled)
Said not to be asked	an arbitrage question "like, oh, is this an arbitrage or not?" ⌚ 58:38 ; the time-series test ⌚ 61:28 ; a derivation of the SML ⌚ 65:35 ; 21 p. 17's first claim, "I'm not going to ask this one over" ⌚ 47:51 — followed by "maybe like one or two questions of this" ⌚ 48:57 (S11 below)
Never named, in or out	decks 06a–10 and the optional decks (absent from deck 21; the equity-valuation video is Gap 00:45); the 8 October guest lecture, recalled three times as covered material, with nothing said about its place on the paper
Written only	closed book (<i>syl p. 4</i>); "Every question can be answered with the material covered in class" (<i>21 p. 3, bold, not read out</i>)

What the earlier notes had assumed about the paper — Blocks 1–5, the formula sheet's p. 1 provided, the later decks not yet on it — is contradicted by nothing said or printed on 15 October.

1.6 Topic remarks, page by page

Each of these is repeated as an Exam signal at its page.

S4 — Block 1 ⌚ 22:30 , both recordings (*21 p. 6*). After an opening whose verb is garbled ("Ideas that are important, that I typically ... on my exams"): "make sure you understand what market making is, how it works." "There's a, what used to be a homework question, on market making with negative inventory, which is I think the first question in the workbook, that people sometimes get confused, so I worked that one out" ⌚ 22:38 — WB 1.2 Q1, worked in the Lecture 2 note. "ask and bid prices, what determines the bid-ask spread, volume, volatility of price, competition, these ideas of compensation for risk that came up later in the context of the CAPM, you can also see how to use them there" ⌚ 22:45 . "short selling, remember, at the

beginning, like half the class, we didn't really know about short selling, that was a very natural thing, and after the break, it became really second nature because we're going to be doing short selling even more." 🕒 23:06 (the second half of that sentence is carried by one recording). The names of the order types are garbled; then "types of orders, how they work, and simple examples, and a lot of these questions in the practice workbook." 🕒 23:14. "These are the key main things from the first couple of days." 🕒 23:28 (the other recording renders "lectures").

S5 — discounting is always asked; timing is the issue 🕒 23:34, both recordings (21 p. 7). "Then, discounting. The midterm will have some discounting problems, because I always ask some discounting questions. Maybe a longer one, maybe a shorter one, that varies every year. But again, you need to remember, The main issue always is how to clear the timings, like when is the value computed." (The middle sentence, "Maybe a longer one, maybe a shorter one, that varies every year.", is carried by one recording only, at 🕒 23:42.)

S6 — no tricks with timelines 🕒 26:31, both recordings (21 p. 7, example 2). "I'm not going to try to trick you with a timeline. If there's any ambiguity, you can always ask and I'll address it, okay? But I put it there, in the 3, 5, 7, 9. Okay."

S7 — summing each payment by hand 🕒 30:50, both recordings (21 pp. 7–8, example 2). A student asked a question twice, which the two recordings render differently (the Warning in §5.2).

The lecturer's answer, on both recordings: "Probably not, because it's right, but I would probably make it more complicated. So you wouldn't be able to do it." 🕒 30:57. "Or if I do it with numbers or something, I can do 15 cash flows, and then you're going to waste an hour, so it will not be worth your time, I think, to do it." 🕒 31:04. After a garbled clause the lecturer added that summing the payments was not the point of the question.

S9 — the review list's topics 🕒 32:15, both recordings (21 p. 7). "Okay, so I think these are kind of" — then a clause the recordings render differently — "because again, these are the key ideas."

WARNING Said in class vs written — 32:15: whether the review list was said to be on the exam

One recording renders the middle clause "all of these will be in the exam", the other "all of this will be in the expand". The topics are 21 p. 7's review list — compounding at different frequencies, EAR against APR, HPR, IRR — and stand on the page either way; whether the sentence said they will be on the exam is not settled.

S10 — the two-asset case 🕒 32:43, both recordings (21 p. 10). "Okay, conceptually, there are always questions on this, okay?" — the other recording renders the last words as *on the list*. Said between the two portfolio formulas of p. 9 and "Here we have the two asset case"; the note attaches it to the two-asset material and adds no referent.

S11 — 21 p. 17's first claim ⌚ 47:34, one recording. "So it's kind of hard, I would say, because I didn't spend a lot of time on this, so somewhere buried in the notes." "Again, this would be, I think, unduly hard, because I haven't spent a lot of time on this, so I'm not going to ask this one over," ⌚ 47:51. "not the most exciting question, okay, but it will be something, this is kind of buried in some of the slides and some of the content of the workbook, so keep an eye out for it, the other two are more natural" ⌚ 48:15. "All stock, again, this will be, I will consider this a kind of very hard question, okay? That would, if you have worked out all the workbook, you will know the answer, but I think it'll be pretty hard, because I really barely spend more time on this." ⌚ 48:43. "Okay, so maybe like one or two questions of this, of this." ⌚ 48:57.

GAP Not recoverable — 48:57: what "one or two questions of this" refers to

"I'm not going to ask this one over" and "maybe like one or two questions of this" are twenty-five seconds apart, and the recording does not make clear what "this" is — the first claim's kind of argument, or true-or-false questions like these three. Neither reading is stated; item 4 of the closing list.

S12 — the second claim, and diagrams ⌚ 49:01, one recording. "Let's do the following. All stocks will have Sharpe ratios that are less than or equal to that of the market portfolio." "This is just very standard, like very easy in a sense for somebody who is here." ⌚ 49:09. "You can do pictures, by the way. So like if you answer with a diagram." ⌚ 49:25. "I love diagrams, as you may have noticed." ⌚ 49:29.

S14 — the MVP in general, and the correlation cases ⌚ 52:15, one recording (21 p. 19). "So, do we need to know how to calculate minimum variance portfolio in a general case?" "Okay, I'll say it is in the slides, we work it out, so in principle yes." ⌚ 52:20. "Okay, I don't know if I'm going to ask for now, but I mean, I don't think, it would not be unfair if I ask you something about" — the end is garbled. The second comment (memorise the formulas, or learn to optimise in the various correlation cases?) was read in a garbled sentence; the answer: "If you don't memorize anything, remember, it's all about thinking here. But you should understand how things work when you have different correlations." ⌚ 52:43. "So, yeah, it would be good to know what happens if positive one, negative one, etc. So, maybe spend a bit of time on that slide again." ⌚ 52:54. "I have asked questions about these in the past. I cannot tell you what's happening exactly this year, but it's happening." ⌚ 53:01.

S15 — four- and five-factor models ⌚ 53:08, one recording (21 p. 19). "Question 3. I don't understand what made four-factor or five-factor models when the Fama-French model demonstrates that alpha is equal to zero. Can you please explain this? Very good question. I could ask you this question. Anybody wants to help me answer the question? That would be a great-sounding question. I should ask it somewhere."

S18 — what "the CAPM holds" means in a question ⌚ 64:58, one recording (21 p. 21). "What is the big picture with CAPM? What does it mean? Well, when you get the CAPM holds, it means that all the equations are true. You may have noticed that I use it always in my question, because, again, we know that CAPM

doesn't really hold in practice, but we, you know, when I ask you questions, I say, assume the CAPM holds, it means the CML is true, the SML is true, that the framework we are considering is true. The assumptions are true, the predictions are true, that's the way I typically ask the questions, okay?"

1.7 The session's plan, and its close

S2 ⌚ 11:43, one recording: "But I'm done with the content, so let's get into the midterm review, unless you have any particular questions. No questions. And yeah, please take notes after, don't forget that." **L2** ⌚ 19:41, both recordings: "All right, so let me just spend the last," — about 35 minutes, "a few minutes early today" (garbled) — "so you can go work out the workbook to tell you a bit of what we've done and I, you know, some people ask me in the form a few questions and" — a clause the recordings render differently — "so I have a few ones at the end that may be helpful. So let's go for it, okay?" **L3** ⌚ 31:50, both recordings, inside the review list: "The interest rate is 6% compounded monthly, so you have to do a lot of that, and there's a very nice handout that it took me a lot of time to write years ago that goes through all of this in great detail, I hope you guys have read it," — the handout is the present-value handout, whose compounding section works that very example (§5.3). **L4** ⌚ 46:13, one recording: "I'm done with the review. What I want to do now in the last 20-ish minutes is answer a few questions, and then I'm going to leave you two minutes at the end and I'll take your questions and I'll have you answer any. And if not, you can just go ahead and extend." **L5** ⌚ 66:37, one recording: "Alright, so let me close it there." — a garbled clause — "get a great sleep, and see you next time."

WARNING Said in class vs written — 19:43: where the written comments come from

One recording renders "I gather questions over the years", the other "I've gathered questions already here". Both agree that "some people ask me in the form a few questions" (the syllabus's anonymous feedback form, *syl* p. 4), and 21 pp. 19–21 print them as "Comments", unattributed. **The fourteen comments are students' questions sent in; over what time span is not settled.** They are never called exam questions.

1.8 Not said on 15 October

Closed book — the syllabus says "Both exams are closed-book" (*syl* p. 4); the deck and the recording are silent. A clock time or a room beyond "It's on Tuesday, it's here" ⌚ 12:03 and the deck's "in class". Office hours before the exam. Points per part beyond five each, partial-credit rules beyond M2, or the grade's weights (the syllabus's Formula A/B, *syl* p. 3). What to bring beyond a calculator.

1.9 The methods Dávila demonstrated

Five calculations were worked with the room; each is worked in full below, starting from the method used in the room, and where an earlier note's route differs a *Two routes* passage says how and why both reach the same answer.

	Demonstration	The method, as shown	Earlier note's route	Worked in full
D1	21 pp. 7–8 example 1: C a year for ever, first payment in year 9	C/R values the stream one period before its first payment, at year 8; then divide by $(1 + R)^8$ ⌚ 24:39 – ⌚ 25:24	the same (Lecture 3, Section 2)	Worked problems, group 2
D2	21 pp. 7–8 example 2: C at years 3, 5, 7, 9	an annuity on a two-year clock: $1 + R_b = (1 + R)^2$; the annuity formula with four payments, dated at year 1; back one year at the annual R ⌚ 27:05 – ⌚ 29:25	the Lecture 4 note and WB 2.3 Q4 value a multi-year stream at year –1 and move it forward; same rule, other direction	Worked problems, group 2
D3	21 p. 12: an equally weighted portfolio as N grows	$1/N$ in every stock; uncorrelated case first, then the general split, "This goes to zero, this goes to one, so what we have next is the average covariance" ⌚ 37:35	the same (Lecture 7)	Worked problems, group 3
D4	21 p. 17 question 1: R_f and the premium from two stocks	one SML equation per stock, "you can solve it the way you want" ⌚ 46:56 ; 21 p. 18 subtracts the two	the Lecture 8 note's WB 4.2 Q1 (same givens, labels swapped) takes rise over run	Worked problems, group 4
D5	21 p. 17 question 2: three true/false claims	claim 1 from the SML's derivation and a better-deal argument; claims 2–3 by diagrams ⌚ 47:51 – ⌚ 51:59	the Lecture 8 note proves claims 2–3 with one identity, $SR_i = \rho_{iM} SR_M$	Self-check

2. Market efficiency, finished: do mutual funds beat the market? (05a pp. 22–27)

⌚ 00:30 · *the last five pages of the market-efficiency deck, taught before the review began; "that will be the last thing you need to know from the exam"*

This section establishes the practical test of market efficiency the course ends Block 5 on: whether actively managed funds beat a passive index. The answer has three parts, one page each — after costs the index wins on average (p. 24); before costs active managers' stock picks do beat the index, by less than the costs (p. 26); and a handful of winners proves little, because among a hundred managers with no skill five will look significant at the 5% level by luck (p. 25). The deck's p. 22 summary, recalled at the start, and its reading list (p. 27) frame it. Everything before p. 23 belongs to the Lecture 11 note (13 Oct, reconstructed); 21 p. 14's efficiency half restates the same outline (§11).

EXAM SIGNAL "the last thing you need to know from the exam" — 00:30

"I just want to finish these few slides that have been in, and then that will be the last thing you need to know from the exam, okay?" ⌚ 00:30 . These are the pages finished: 05a pp. 23–27. Block 5 is on the review deck's list of blocks (21 p. 5); 21 p. 14 restates 05a's outline without its mutual-fund bullet, which is not a contradiction — the deck lists the block as a whole.

2.1 The opening recap (05a p. 22, recalled)

⌚ 01:03 · *one recording; the lecturer's own summary of the previous lecture's market-efficiency material, which ended on p. 22*

The lecturer recalled the previous meeting's conclusions from what was called "kind of a good summary slide" ⌚ 01:21 , by content 05a p. 22, *Are Markets Efficient After All?* The page prints four findings and a conclusion. **Found:** short-run returns are not predictable; long-run returns are predictable; prices respond to new information in event studies; anomalies can be found "(given the joint hypothesis problem)".

Concluded: the efficient market hypothesis seems to hold partially; only superior information or skill earns money; market efficiency is the result of competition, and "Alphas – if they exist – will disappear quickly" (05a p. 22). The conclusion bullets were not read out **SLIDE ONLY**.

The lecturer opened with an apology, "because there's a lot of concepts here", and the hope that by now the class has "a sense of how these things work" and is getting "into the mindset of how finance economists think about these issues" ⌚ 01:09 . The recap, in the note's words with the lecturer's phrases: in the short run "returns are quite unpredictable" and "It's really hard to make money out of it", which is **weak-form efficiency**; "In the long run, that's not quite the case" ⌚ 01:36 ; the **semi-strong form**, "perhaps the most interesting test", is tested with event studies, looking at what happens before the event, at impact and after; then anomalies — "some things that were not quite compensated for risk", though "it's hard to make money out of this"; then alphas, the joint hypothesis, and how testing efficiency is "a challenging enterprise"; and finally the **strong form**. The lecturer said of it that it "is just not quite there. Because, you

know, if you have hard information, you will make money." ⌚ 02:10 . The deck's definition is the one to carry: strong-form efficiency asks "Do prices reflect all relevant information, including inside information?" — "Really strong criterion" — and the deck's verdict, on its "Empirical evidence: corporate insiders earn abnormal returns", is that "Markets are not strong form efficient" (05a p. 20). The spoken clause that introduced it — "this crazy idea that you could have profit from private information" — is reported here only as said; read against the deck, it describes the trading rather than the hypothesis (*the note's reading*). The recap names no day and no page; which pages the previous meeting reached is settled only at its end (§Continues in).

2.2 Two kinds of fund (05a p. 23)

⌚ 02:26 · one recording; four of its segments are the meeting's only low-confidence stretch, 02:44–03:13

The question "do mutual funds beat the market?" is, the lecturer said, "a version of these questions" — the efficiency question made "very tangible, very practical" ⌚ 02:26 . It needs two kinds of fund told apart, and 05a p. 23 prints both with figures.

DEFINITION Active and passive funds (05a p. 23)

An **active fund** is run by portfolio managers who bet on individual stocks, macro themes and style strategies. They trade often — turnover of about 100% a year — and pay for research and trading: expense ratios on the order of 1% a year, plus trading costs of about 0.5 to 1% a year on top. Example: the Fidelity Magellan Fund. A **passive fund** mechanically holds a basket of stocks: turnover about zero, expense ratios of maybe 0.2% a year. Example: the Vanguard S&P 500 Index Fund. *The four cost figures were not said aloud (slide only); read together they put an active fund's yearly cost at about 1.5 to 2% against about 0.2% for an index fund (explanation).*

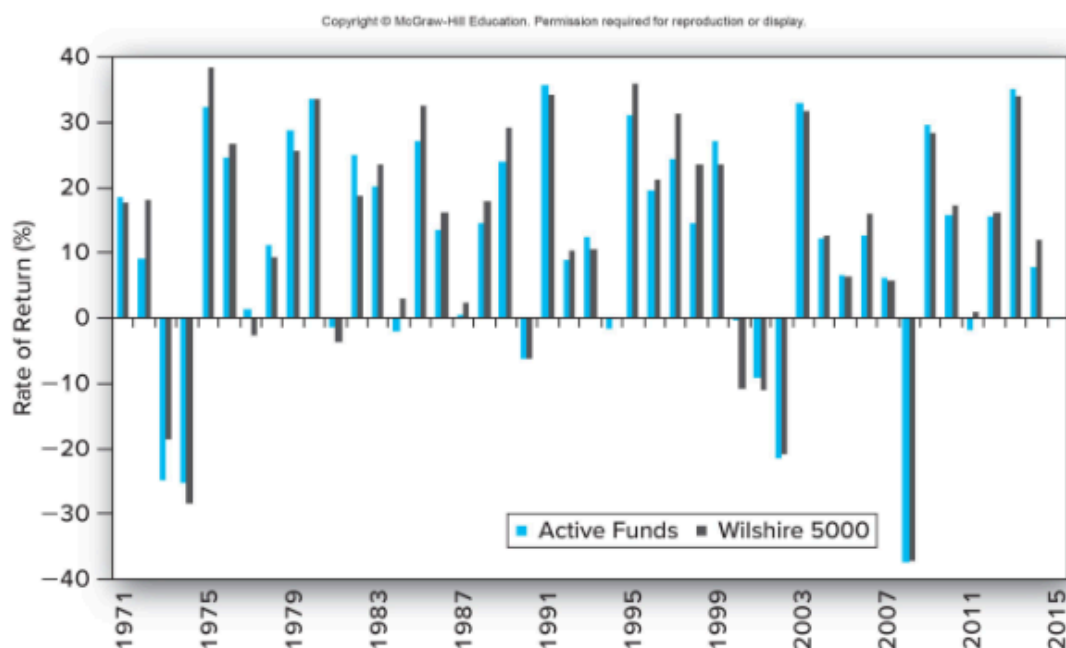
In the room: an active fund is "like AQR, this paper Toby was mentioning" ⌚ 02:40 — the guest lecturer's firm, recalled from 8 October. The next four segments, ⌚ 02:44 to ⌚ 03:13 , are the recording's low-confidence stretch and are not quoted: as far as they can be made out, they describe active managers as the page does — picking stocks and strategies, trading a lot, costing money — and the note attributes nothing more to them. Passive funds are the ones the course met early, with two-fund separation and the CAPM: "they just replicate an index, and they're essentially replicating the market portfolio" ⌚ 03:13 . "Vanguard, S&P 500 index fund, what this fund does is they just try to replicate the S&P 500"; "Fidelity Magellan Fund, it's an active fund, they try to have a style and a strategy" ⌚ 03:27 . Active funds "try to beat the market, they try to sort of earn alphas"; passive funds "are just sort of giving you exposure to the market" ⌚ 03:40 . The page's last line, "Which type of fund normally does better?", was put to the room at ⌚ 03:49 and answered on the next page; it is the first self-check.

Links back. Two-fund separation and why an index fund is the risky fund everyone should hold: Lecture 7, §8 and Lecture 8, §10.3, whose "later" lands on these pages. Active against passive strategies, and the Grossman–Stiglitz paradox they raise: Lecture 9, §3. Alpha as the measure of beating the market: Lecture 9, §2.2.

2.3 Active funds against the Wilshire 5000 (05a p. 24)

04:01 · one recording

Active vs Passive Funds



- ▶ Average index return is 1% above average fund return, and index beats funds in 27/45 years
- ▶ This comparison is of **returns to investors** so it subtracts trading costs and fund expenses from fund returns

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05a p. 24 — original slide: yearly returns to investors in active funds (cyan) and the Wilshire 5000 (grey), 1971–2014

How to read this: one pair of bars a year. The cyan bar is the average active fund's return to its investors, after fees; the grey bar is the Wilshire 5000, a broad index of about 5,000 US stocks. The two bars are close in most years; the grey one is the taller one more often than not. The page's bullet: "Average index return is 1% above average fund return, and index beats funds in 27/45 years".

"A nice picture", the lecturer said: the return of active funds against the Wilshire 5000, "a very broad index. Okay, S&P 500 is the top 500, this is the top 5,000, so it has a lot of stocks" ⌚ 04:01. The comparison is **after costs**: "these are comparisons once you've subtracted expenses". Why subtract them: the people who run the fund "always get a fee and they make money anyways", and the question is what the final investor makes. The picture shows no very strong difference — "In fact, some years, you'll be better off just putting your money in Wilshire 5000 funds, which is a passive fund" ⌚ 04:49 — and the bullet gives the average: the index return is 1% above the average fund return and beats the funds in 27 of 45 years ⌚ 05:05. "So in other words, the index funds, they are marginally better, or they even do better most of the year. So that tells you a lot about what happens in practice." ⌚ 05:14. (The remark "I'll put your money in the blue lines" is left as heard; on the page the active funds are the cyan bars and the index the grey ones.)

GAP Not on any written source — 04:36: the fee example

"if you put money in one of these active funds, So, if you run the funds, you may need to pay a fee, okay? There are different numbers, okay? But it'd be like 2% plus 15% on the return, although this can be negotiated, okay?" ⌚ 04:30. The figures 2% and 15% are the lecturer's illustration of a fund's fee structure; no course page prints them, and the note uses them nowhere else.

WARNING 05a p. 24's "27/45 years" against a chart of 44 years

The chart's bars run from 1971 to 2014, forty-four years; the slot under the "2015" label is empty. Read off the chart bar by bar, the index is clearly ahead in 23 years and the funds in 16, with 5 years too close to call (1980, 1990, 2004, 2005, 2008); the average gap is about one point a year in the index's favour, which agrees with the bullet's "1%". The bullet's "27/45" cannot be confirmed from the chart: 27 index wins would need four of the five close years, and there are 44 bars, not 45. **This is a discrepancy inside the slide, flagged and not corrected**; the lecturer read the bullet's numbers ⌚ 05:05. Item 14 of the closing list.

2.4 Before costs, and luck against skill (05a p. 25)

⌚ 05:27 · one recording; the page's two questions were put to the room and are the second self-check

05a p. 25 asks two questions. **(1) Can mutual fund managers pick stocks that beat the index before costs?** It cites Wermers (Journal of Finance, 2000), who looks at the actual stocks active funds hold, and says this gives a different picture — the next page's. **(2) Even if most active managers underperform, are there some that outperform?** Individual funds may, "e.g. Yale endowment"; but there is a **"Luck vs. skill problem"**: in a large population of unskilled managers a few will always perform particularly well by luck; "5 out of 100 will perform well enough to be significant at the 5% level"; so we should not be impressed by

evidence that the best few funds in a large population have statistically significant outperformance (05a p. 25). Wermers is named on the slide only; the lecturer called the paper "a very famous paper", "a bit of a classic reference", "a little bit old" ⌚ 07:41 .

What was said. "People have spent a lot of time thinking about this question" — can mutual fund managers beat the indexes before costs? ⌚ 05:27 . Perhaps active funds are not bad on average, or "some people are really skilled at choosing stocks, okay, maybe Warren Buffett" — a second name is garbled on both recordings — "I'm not saying these are the people who do them" — or "it's all like a little bit of luck" ⌚ 05:45 . The thought experiment: give everyone \$10,000 to invest today with different strategies; "5% of people will always be at the top, right, because by construction, they have to do well"; skill would show as the same people staying in the top 5% next week, "but it's hard to tell", because "there's going to be a lot of noise, that people did well in week one, may not do so well in week two" ⌚ 06:09 . "5% of people are in the top 5%, and they could just be lucky" ⌚ 06:41 . People invest in something that does really well and make a lot of money, "but it has nothing to do with them being skillful. They're just lucky, and testing these things is subtle and complicated. This is the kind of the key takeaway from this" ⌚ 06:50 . The lecturer compared sport, as the 8 October talk had: "there are players in the NBA who consistently score 30 points, right? That is not luck, okay?" — doing it every week is skill ⌚ 07:17 ; in finance it is much harder to tell who makes the money, "but if you're consistent over time, people say you're skilled" ⌚ 07:33 .

WARNING Said in class vs written — 06:41: two different statements about 5%

Said: "5% of people are in the top 5%, and they could just be lucky" — a definition: whoever the managers are, one in twenty is in the top twentieth. **Written:** "5 out of 100 will perform well enough to be significant at the 5% level" (05a p. 25) — a statement about hypothesis tests: a test at the 5% level wrongly declares an unskilled manager skilled with probability 0.05, so among 100 unskilled managers the expected number of false positives is $100 \times 0.05 = 5$. The second is the point the page makes and the one the self-check uses; the first is true but says nothing about significance.

The expected count needs no independence; the chance that *at least one* of the hundred looks significant does. If the hundred tests are independent, that chance is $1 - 0.95^{100} = 0.9941$ — worked as problem D of the Lecture 10 note (8 Oct), on the guest lecture's hundred-strategies version of the same point; it is not re-worked here.

Yale. "For Yale, for example, and I think I put it back here" — the page's "e.g. Yale endowment" — the lecturer recalled the prescription David Swensen gave when Swensen "used to come here": "we have a lot of expertise, so we can invest in all these active managers, and we're doing really well, but we spend a lot of time and effort to try to see what is each fund." ⌚ 09:45 . The lesson drawn: without particular information, trading a lot in an active style will not give a bigger return net of fees; "it will make the fund managers extremely wealthy, okay? At your expense." ⌚ 10:04 .

2.5 The key figure: gross and net fund returns (05a p. 26)

🕒 07:41 · one recording; "This is the key figure"

Performance before costs

1672

The Journal of Finance

Table II
Mutual Fund Returns

Year	S&P 500 Return	CRSP VW Return	No.	Merged Database			
				TNA-Avg Gross Returns (%/year)	EW-Avg Gross Return (%/year)	TNA-Avg Net Return (%/year)	EW-Avg Net Return (%/year)
1975–1979	15.8	18.8	241	20.2	22.8	16.8	18.2
1980–1984	15.5	15.1	459	16.3	17.3	14.6	15.0
1985–1989	20.8	19.0	676	20.2	19.5	17.4	16.4
1990–1994	9.3	9.5	1,567	10.8	11.3	9.7	10.1
1975–1994	15.4	15.6	1,788	16.9	17.7	14.6	14.9

- Stocks held by mutual funds beat the index, but the net fund returns are beaten by the index

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05a p. 26 — original slide: Wermers' Table II, "Mutual Fund Returns", with 15.4, 16.9 and 14.6 circled in the last row

How to read this: a scanned journal table, one row per five-year period and a last row for the whole 1975–1994 span. Read the last row: the S&P 500's return, the funds' gross return (before fees and costs) and the funds' net return (what investors received). The circles mark the three numbers the lecture compared.

The table's last row, 1975–1994, as printed (returns in percent a year):

S&P 500	CRSP value-weighted index	Number of funds	Funds' gross return (TNA-weighted)	Funds' gross return (equal-weighted)	Funds' net return (TNA-weighted)	Funds' net return (equal-weighted)
15.4	15.6	1,788	16.9	17.7	14.6	14.9

"What I want you to take a look is at the bottom row" ⌚ 07:48 . Over the long period the S&P 500 returned "around 15%" (the circled 15.4) ⌚ 07:52 . The authors merged a database of all these funds' returns; **gross returns** — "returns without subtracting all these fees and costs they have to pay" — came to "like a 17% return on average" (the circled 16.9) ⌚ 08:00 ; **net returns**, "this column over here", drop to 14.6% (circled) ⌚ 08:28 . The bottom line from the three numbers: "if you put your money on the market, you can earn 15%, but if you put your money, on average, on some of these funds, they are gonna make more money. So, in some sense, they are somewhat more skilled, let's put it that way, than putting your money on the market, but once you pay their fees, you're gonna be making less money, okay?" ⌚ 08:34 . In figures (*the note's subtractions of the printed numbers*): before costs the funds beat the index by $16.9 - 15.4 = 1.5$ points a year; after costs they trail it by $15.4 - 14.6 = 0.8$; the costs themselves are $16.9 - 14.6 = 2.3$ points — "That 2.3% that you're losing there, that's the money that the manager is getting that you're not getting" ⌚ 10:12 . (The spoken "around 15%" and "like a 17%" are roundings of 15.4 and 16.9, not conflicts.)

EXAM SIGNAL The key insight, "come back a few times in the course" — 08:57

"Okay, so the key insight here is, and I think this has come back a few times in the course, if you don't have any insight into what to do with the money, keep it to the best things, just to put in a well-diversified index fund, you're going to do well, okay?" ⌚ 08:57 . Active managers "are pocketing the difference, so they're going to be extremely wealthy", but "for the final investor, it's really not obvious that just putting your money in an actively managed fund is going to give you more money." The lecturer's own money is in well-diversified funds, "in the sense that I don't have any particular insight into that" ⌚ 09:28 .

INTUITION Why the picture before costs and after costs can both be true

Fund managers as a group do pick stocks a little better than the index — their holdings earned 1.5 points a year more over twenty years. But picking costs money: research, trading at the bid-ask spread, turnover of about 100% a year, and a fee of about 1%. Those costs (2.3 points a year in this sample) are larger than the picking advantage, so the investor ends up 0.8 points behind the index. The skill is real and the fund captures it; the investor pays for it and more. (*explanation, from 05a pp. 23 and 26*)

2.6 Reading recommendations, and the newspaper articles (05a p. 27)

⌚ 10:25 · one recording

05a p. 27 prints three book covers and no text: *A Random Walk Down Wall Street* by Burton G. Malkiel; *Irrational Exuberance* by Robert J. Shiller; *The Man Who Solved the Market: How Jim Simons Launched the Quant Revolution* by Gregory Zuckerman. The same three are on the lecturer's one-page list of extra

readings (*XR p. 1*). In the room: *Irrational Exuberance* had been mentioned before, and "it's a good book, so I think it's really going to be useful if you guys read it" ⌚ 10:25 . The Renaissance book is about "the most successful hedge fund in the last 25 years" and "how quant trading has changed the markets over the last 25 years and how successful some people have been, so this looks like skill to me in the way they are proceeding" ⌚ 10:47 (the title was not said; it is the cover's). *A Random Walk Down Wall Street* is "a bit of a more classic book from the 70s and 80s, that really goes over some of the issues at the beginning about testing the random walk hypothesis" ⌚ 11:07 . "So I think all three are really worth reading" — pick one over the break ⌚ 11:22 . The lecturer has also been posting Wall Street Journal articles "from the last few months on the website" as they come up, "a bunch of cool things there also to read" ⌚ 11:30 ; which articles was not said.

GAP The three books are not in the course materials

None of the three books is among the course files, so nothing from them is digested here; the note names them as the slide does. They are recommendations, not assigned reading: the course overview deck calls readings "Interesting and helpful but not required" (*00 p. 9*).

Self-check. Over 1975–1994 the average fund's stocks returned 16.9% a year against the S&P 500's 15.4%. A friend concludes that active funds beat the market. What is missing?

The 16.9% is the funds' **gross** return, before fees and trading costs. Investors received the **net** return, 14.6% — below the index. The 2.3-point gap between gross and net is what the fund kept. So the stocks the managers picked did beat the market, but the investors who paid them to pick did not (*05a p. 26*).

Self-check. A newspaper lists the ten mutual funds, out of 2,000, whose managers beat the market significantly at the 5% level over the past decade. Is that evidence of skill?

Weak evidence at best. If none of the 2,000 managers had any skill, a test at the 5% level would still flag about $2,000 \times 0.05 = 100$ of them as significant by luck alone; ten is far fewer than chance would produce. What would indicate skill is persistence — the same managers at the top period after period — not a one-off ranking (*05a p. 25; the thought experiment at* ⌚ 06:09).

Self-check. Why does the lecturer say index funds are the right choice for someone with no particular insight, when the evidence shows active managers do have some skill?

Because the investor does not receive the skill; the fund does. Gross of costs the managers' picks beat the index by about 1.5 points a year; net of costs the investor trails it by about 0.8 points. The index fund costs about 0.2% a year and delivers the market portfolio — which, by two-fund separation and the CAPM, is the risky portfolio every investor should hold anyway (05a pp. 23, 26; Lecture 8, §10.3).

3. The five blocks, in two minutes (21 p. 5)

🕒 20:02 · both recordings from 20:57; the review proper begins here, after the exam's rules (§1)

This section establishes the map of the course's first half as the lecturer drew it from the deck's overview page, with the one judgement that organises the rest of the review: the core is Blocks 3, 4 and 5.

What 21 p. 5 prints, under "Blocks of content so far": Block 1 Financial Instruments and Markets; Block 2 Present Value and Returns; Block 3 Portfolio Choice; Block 4 Capital Asset Pricing Model (CAPM); Block 5 Market Efficiency. The names are the course overview deck's (00 p. 12, items 1–5) without their lecture counts; Blocks 6–10 of that deck (equity valuation, no-arbitrage pricing, fixed income, options, financial intermediation) are not on this page or anywhere in the review deck. Block 5 was named aloud only inside "blocks three, four, and five" and the closing remark on testing (*the rest of the page: slide only*).

The walk through the five. "it's been only half a semester but I think, have you learned something guys?"

🕒 20:02 . At the start the course may have seemed "really straightforward, a bunch of concepts". **Block 1** was "kind of basic finance ideas to make sure everybody starts from a level field" — some students had a lot of finance experience, some none, "and we're trying to put everybody together here" 🕒 20:13 . **Block 2** moved into "the very basic sort of discounting cash flow type ideas, which is like how do we think about present values, future values, discounting" 🕒 20:41 . Then the core.

EXAM SIGNAL "the core of this first half of the semester" — 20:51

"And then really the core of this first half of the semester is sort of blocks three, four, and five."

🕒 20:51 , both recordings, word for word the same.

Block 3, portfolio choice: "If I have some money, how do I invest?" — all the combinations in turn, "First, two risky assets. One risky, one risk-free. One risk-free, many risky.", ending with "a bunch of interesting conclusions about the tangency portfolio", individual preferences and investment opportunity sets 🕒 20:57 .

Block 4: "what if we are in equilibrium, so everybody must hold the same thing, which is the market" —

which gives "very clear predictions for what the tangency should be in the market", and "very clear predictions about what is expected return we require in different stocks. And that's the SML, as well as beta."

🕒 21:24 . Why beta is the measure of risk, in the lecturer's words, is the Exam signal below.

EXAM SIGNAL "risk is always about covariance" — 21:49

"risk is always about covariance, because covariance is if I pick something, how does it move with what I have to, it's the covariance of what I'm thinking versus what I have to have in the market, so that's what beta tells us." 🕒 21:49 . On the page, beta is that covariance with the market divided by the market's variance (21 p. 4 item 10; 21 p. 13).

The CAPM is then a tool: "to benchmark investors", for capital budgeting — said as "capital evaluation" on both recordings; 21 p. 13 prints "Capital budgeting" — "how do I invest in a project", and "to start to think about whether the market is efficient or not" 🕒 22:04 . **Block 5** is that last use: "That's exactly what we saw in" the most recent lectures, "when we started to think about how to test the CAPM, whether markets are efficient or not. I think that's a pretty good summary in two minutes, or one minute, of what we've done so far." 🕒 22:17 .

WARNING Said in class vs written — 22:17: "the last two lectures" or "the last few lectures"

One recording renders the clause at 🕒 22:17 as "that's exactly what we saw in the last two lectures", the other as *the last few lectures*; one recording alone says "we just went over this in the last two lectures" at 🕒 41:23 . No written source counts the lectures that tested the CAPM, so **no count enters this note**. Which meetings taught the testing of the CAPM — 6 October, 8 October, 13 October — is item 13 of the closing list, and bears on the reconstructed Lecture 9 and Lecture 11 notes.

Links back. The ten blocks as announced on the first day, with their lecture counts: Lecture 1, §1.5.

4. Block 1: market making, spreads, short sales and orders (21 p. 6)

🕒 22:30 · both recordings

This section establishes what the review kept from Block 1: how a market maker works, what sets the bid-ask spread, short selling, and the types of orders — each a one-line pointer to the Lecture 2 note, plus the one workbook item the lecturer named.

What 21 p. 6 prints. "How market making works — The market maker (specialist) sets bid and ask prices to keep zero inventory (ask>bid)"; "What determines bid-ask spreads? — Volume, volatility of price,

competition"; "Understand short-selling"; "Understand types of orders". *Slide only*: the word "(specialist)", which appears on no other course page, and the zero-inventory wording; the idea was taught through the negative-inventory homework and is in the Lecture 2 note. The page lists the three determinants without their directions; the course's page gives them (*01b p. 9*): more trading **volume** narrows the spread, a more **volatile** equilibrium price widens it, more **competition** among market makers narrows it.

EXAM SIGNAL Block 1 on the exams — 22:30 to 23:28

Idea the lecturer "typically" puts on the exams (the verb is garbled on both recordings): "make sure you understand what market making is, how it works" ⌚ 22:30 ; "There's a, what used to be a homework question, on market making with negative inventory, which is I think the first question in the workbook, that people sometimes get confused, so I worked that one out" ⌚ 22:38 ; "ask and bid prices, what determines the bid-ask spread, volume, volatility of price, competition, these ideas of compensation for risk that came up later in the context of the CAPM, you can also see how to use them there" ⌚ 22:45 ; short selling — "after the break, it became really second nature because we're going to be doing short selling even more" ⌚ 23:06 ; "types of orders, how they work, and simple examples, and a lot of these questions in the practice workbook" ⌚ 23:14 . "These are the key main things from the first couple of days." ⌚ 23:28 .

In the note's words. A **market maker** quotes two prices, a **bid** at which it buys and a higher **ask** at which it sells, and sets both so as to hold its inventory near zero: when it has bought more than it has sold it lowers both quotes to attract buyers, and the reverse when it is short (*01b p. 8*). The spread is the market maker's compensation for the risk of carrying inventory while the price moves and for the chance of trading against someone better informed — the first appearance in the course of **compensation for risk**, which the lecturer connected forward to the CAPM. A **short sale** is selling a borrowed share now and buying it back later, so that it pays when the price falls (*01b pp. 13–15*). The order types are the **market order** (trade now at the best available price), the **limit order** (trade only at a stated price or better) and the **stop orders** (a sale, or a purchase, triggered once the price crosses a stated level) (*01b p. 10*); the names were garbled on the recording, and the page says only "types of orders".

Links back. Lecture 2, §6 (market making; "Inventory is pain", §6.3; what the spread pays for, §6.4); §7 (the three determinants); §8; §10; and the homework item the lecturer named, WB 1.2 Q1, the BioEngineering market maker, worked in Lecture 2's Worked problems.

Related practice elsewhere. Block 1's workbook items, all owned by the Lecture 2 note: WB 1.1 Q5, Q7, Q8, Q9, Q11, Q12, Q14, Q15 and WB 1.3 Q1, Q3, Q4; and four items adapted from past papers, *Adapted problems X02 and X03* and *Adapted self-checks X04 and X05*. The Block 1 key-concepts checklist is the Lecture 2 note's *WB 1.4 — the Block 1 key concepts this meeting covers*.

5. Block 2, discounting: two examples worked with the room (21 pp. 7–8)

🕒 23:34 · both recordings to 31:43; the printed answers, 21 p. 8, were never displayed or named — "I have the solution in the back", one recording, 🕒 25:24

This section establishes the one skill the review drilled in Block 2: **dating a present-value formula correctly** — knowing which date a perpetuity or annuity formula's value sits on, and moving it to today on the right clock. Two symbolic examples carry it: a perpetuity whose first payment is nine years away, and an annuity paid every two years. Both are worked in full, from the method used in the room, in Worked problems, group 2; this section gives the page, the method as said, and the traps named aloud.

EXAM SIGNAL Discounting is always asked; timing is the issue — 23:34

"Then, discounting. The midterm will have some discounting problems, because I always ask some discounting questions." 🕒 23:34 . One recording carries the next sentence: "Maybe a longer one, maybe a shorter one, that varies every year." 🕒 23:42 . "The main issue always is how to clear the timings, like when is the value computed." 🕒 23:47 .

Discounting

- ▶ Example: Your annual net discount rate/interest rate is R annually compounded. Compute the present value of the following cash-flows
 1. Yearly payments of C forever, with the first payment starting exactly nine years from now.
 2. Payments of C every two years for 6 years, starting 3 years from now (payments at years 3, 5, 7, 9).
- ▶ Review
 - ▶ Compounding at different frequencies
 - ▶ EAR vs APR $1 + EAR = \left(1 + \frac{APR}{m}\right)^m$
 - ▶ HPR, IRR

7 / 21

21 p. 7 — original slide: "Discounting", the two examples and the review list

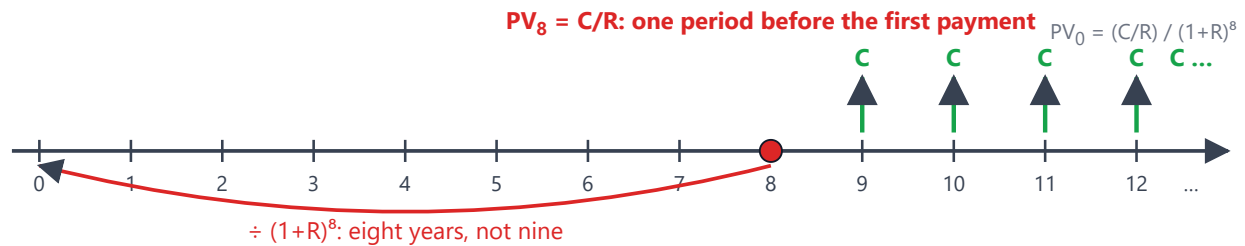
How to read this: one setting — an annual rate R , compounded annually — and two cash-flow patterns to value today. Below them, the review list: compounding at different frequencies, EAR against APR with its formula, HPR and IRR.

What 21 p. 7 prints. "Example: Your annual net discount rate/interest rate is R annually compounded. Compute the present value of the following cash-flows: 1. Yearly payments of C forever, with the first payment starting exactly nine years from now. 2. Payments of C every two years for 6 years, starting 3 years from now (payments at years 3, 5, 7, 9)." Then the review list — compounding at different frequencies; EAR against APR, with $1 + EAR = \left(1 + \frac{APR}{m}\right)^m$; HPR; IRR. The givens are letters, not numbers: the exercise is the dating, not the arithmetic. "I put a simple example here" 23:54.

5.1 Example 1: a perpetuity whose first payment is nine years away (21 pp. 7–8)

23:54 · demonstration D1; the class answered the two steps

Example 1 (21 p. 7): C a year for ever, first payment at year 9



Redrawn: the timeline of example 1 — payments of C at years 9, 10, 11, ... for ever; the perpetuity formula's value sits at year 8, one period before the first payment; the arrow carries it eight years back to year 0

How to read this: the board timeline, reconstructed from 21 p. 7 (the lecturer drew it at 24:21, one recording; no photograph). The formula C/R lands on the date one period before the first payment — year 8, not year 9. One further discount of eight years brings it to today.

The method, as said. The lecturer drew the years 1, 2, ..., 9 with C from year 9 onward (23:54), asked "what is the first step? ... Two steps. What do I do first?" (24:26), and took the room's answer: the perpetuity formula. "So perpetuity is C over R , right? And this value, again, is that 9 or 8?" — the room said 8 — "Very good. Right, it's at 8. Why is it at 8? Because remember, Now our formula is the first cash flow is at 1, so we compute the value at 0, so here the first cash flow is at 9, we have the value at 8" (24:50). Second step: "so if I want to compute the value at 0, I have this value at 8, which number do I put here?" — "Eight, right?" (one recording) — and the problem is done: "Perfect. Five points." (25:24). The exponent 8 is heard on one recording only; 21 p. 8 prints it.

The printed answer (21 p. 8, example 1): "1. First compute $PV_8 = \frac{C}{R}$. 2. Then discount for eight periods to find the present value at 0: $PV_0 = \frac{PV_8}{(1+R)^8}$."

$$PV_0 = \frac{C/R}{(1+R)^8}$$

The full working — each step with its reason, the check that the answer equals the sum $\sum_{t \geq 9} C/(1+R)^t$, the error of dating the perpetuity at year 9, the note's own numbers at $R = 5\%$ and $C = \$1$ (13.5368), and the workbook's twin, WB 2.3 Q3(b) — is Worked problems, group 2. **Same route as the earlier notes:** the Lecture 3 note's GM bond values a perpetuity one period before its first payment and discounts it like a zero-coupon bond (Lecture 3, §14.3); the Section 2 note's *small annuity plus a late perpetuity* does the same (Section 2, §8.3).

INTUITION Why the formula lands one period early

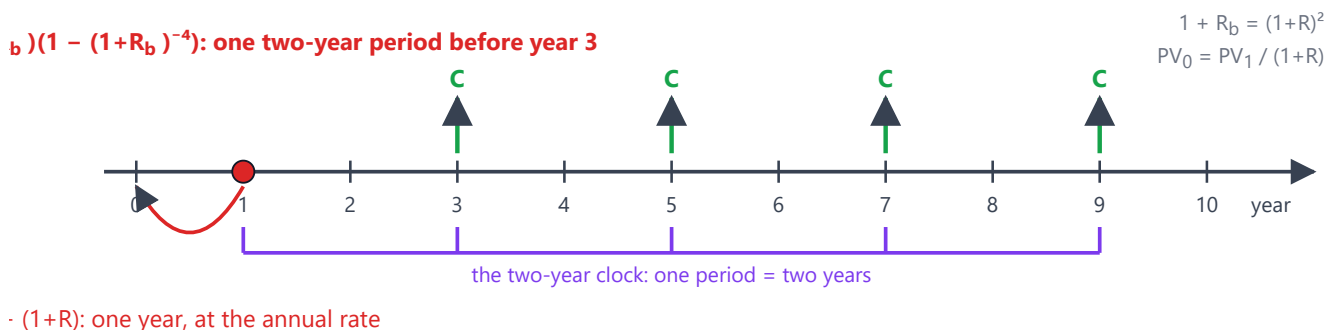
The perpetuity formula C/R is built for a stream whose first payment arrives one period after the valuation date: a dollar invested at R throws off $C = R \times (C/R)$ a period, starting next period. So wherever the first payment is, the formula's value sits one period before it. First payment at year 9: value at year 8. Then the question asks for year 0, which is eight years earlier, not nine. (*explanation; 02a p. 23; Lecture 3, §11.1*)

5.2 Example 2: payments every two years, at years 3, 5, 7 and 9 (21 pp. 7–8)

⌚ 25:44 · demonstration D2, "This one's going to be more fun. Let's try to do it together." (one recording, ⌚ 25:24); three student questions

Before the second example, a remark on the subject as a whole: discounting: "it's really very mechanical, but again, at the end with discounting, especially if you're understanding the fact that the money at different points in time has different valuations, so you cannot add up money together" ⌚ 25:44 .

Example 2 (21 p. 7): C every two years at years 3, 5, 7, 9



Redrawn: the timeline of example 2 — payments of C at years 3, 5, 7 and 9; on a two-year clock the annuity formula's value sits at year 1; one year's discount at the annual rate carries it to year 0

How to read this: the board timeline, reconstructed from 21 p. 7's parenthesis "(payments at years 3, 5, 7, 9)"; the lecturer's own drawing was misplaced and corrected aloud at ⌚ 26:05 – ⌚ 26:25 , and there is no photograph. The payments are two years apart, so the natural clock ticks every two years; the formula's value lands one tick before the first payment, at year 1; the last move, year 1 to year 0, is one year at the annual rate.

The method, as said, in five steps.

1. **Read the stream.** "So that one says payments of C every two years for six years, starting three years from now." ⌚ 25:59 . Drawing it, the lecturer first placed the payments wrongly, then corrected: "Oh, actually, I have a ninth year." The page pins the dates: "I'm not going to try to trick you with a timeline. If there's any ambiguity, you can always ask and I'll address it, okay? But I put it there, in the 3, 5, 7, 9. Okay." ⌚ 26:31 .
2. **Reject the long way, name the security.** One could "do it really in a really sort of dumb way and tell me, oh, C divided by one plus R cubed," and so on payment by payment — "which I'm sure some of you are already thinking to do it, but what would be the natural way to do this? What type of security is this one?" ⌚ 26:49 . On a clock whose period is two years, four equal payments one period apart: an annuity. "So it's an annuity." ⌚ 27:05 .
3. **Build the rate for the two-year clock.** "And what interest rate should I be using if I want to use the annuity formula? I should be computing a rate over two years, right?" ⌚ 27:24 . Call it R_b (said "r-bar"): "let me call it r-bar, should be equal to 1 plus r-squared, right? Right, because r is my annual rate, this, when I solve for this one, is r bar is one plus r squared minus one, okay, this is my biannual rate" ⌚ 27:46 . That is $1 + R_b = (1 + R)^2$, so $R_b = (1 + R)^2 - 1$; "bi-annual" here means once every two years (21 p. 8; 02b p. 10).
4. **Apply the annuity formula, and date it.** " C over R " with the bar, "and then it's 1 minus, I think I know this, let me look at it, I know the formula" ⌚ 28:16 — the bracket's exponent, -4 for four payments, was said on neither recording; it is the page's. Then the question that matters: "Okay, now, important, this value is computed when? Which year, except for here? Some people are saying two. Anybody has another answer? ... We're in one, right? Because we're looking at a two-year period here, okay? So this value The value is at 1." ⌚ 28:46 . One two-year period before the first payment at year 3 is year 1.
5. **Back to today, one year at the annual rate.** "But I asked you the value at 0, so the value at 0 is the value at 1 divided by 1 plus r . This r is not this, okay? This r is the original one" ⌚ 29:18 — the annual R , not R_b .

The printed answer (21 p. 8, example 2): "1. First compute the bi-annual rate: $1 + R_b = (1 + R)^2$. 2. Then find the value of the annuity: $PV_1 = \frac{C}{R_b} (1 - (1 + R_b)^{-4})$ — We use the bi-annual rate because the annuity is paid every two years — The present value of the annuity is at date 1, because this is one period before the first payment, and a period in this case accounts for two years. 3. Then discount for one period to find the present value at 0: $PV_0 = \frac{PV_1}{1+R}$."

$$PV_0 = \frac{1}{1+R} \cdot \frac{C}{R_b} (1 - (1 + R_b)^{-4}), \quad 1 + R_b = (1 + R)^2$$

WARNING 21 p. 8, example 2, step 3: "discount for one period" means one year

The page's second sub-bullet defines a period as two years ("a period in this case accounts for two years"); its step 3 then says "discount for one period" while its formula divides by $1 + R$, which is **one year** at the annual rate — from date 1 to date 0. The formula is right, and it is what the lecturer said ("divided by 1 plus r. This r is not this ... This r is the original one", ⌚ 29:18); the word "period" in step 3 is loose. Dividing by $1 + R_b$ instead would carry the value to year -1 , one year too far. Read step 3 as *discount for one year*.

Three questions from the room. *Why not a perpetuity?* (the question itself is garbled, ⌚ 29:32): because the payments stop — "if this forever, that would be a perpetuity. Only payments 3, 5, 7, 9" ⌚ 29:36; with a perpetuity "it would be kind of the same issue" — the bi-annual rate and the one-period-before dating would be the same, "if you're looking at two-year periods, the value is at 1" ⌚ 29:54. *Why go back to zero at all?* — "In your case, why do you do zero and zero?" ⌚ 30:17 (the other recording renders it as *why you do v_0 and v_n*): "because I asked you about present value. Okay? So, if I mean present value, I mean kind of at zero"; the formula gives the value at 1, "so I need to do this extra step to do it, because remember, value of money at 0 and money at 1 are different things, sir, apples and bananas, different objects, I cannot put them together." ⌚ 30:20. *Would summing the four payments by hand have been marked wrong?* — the question below.

WARNING Said in class vs written — 30:50: the student's question

A student asked a question twice (⌚ 30:50, ⌚ 30:54); one recording renders it "Would it have been marked wrong in an exam to manually calculate it?" and the other "What is the most common reason for making a transcript?" Only the first fits the answer. The note reports the question in the first rendering and states neither as what was said.

The lecturer's answer, which both recordings carry: "Probably not, because it's right, but I would probably make it more complicated. So you wouldn't be able to do it." ⌚ 30:57; "Or if I do it with numbers or something, I can do 15 cash flows, and then you're going to waste an hour, so it will not be worth your time, I think, to do it." ⌚ 31:04.

EXAM SIGNAL What the review's second example trains — 26:31 to 31:32

No tricks with timelines; ask if a timeline is ambiguous (🕒 26:31). Summing each payment is right but will be made impractical ("15 cash flows ... waste an hour", 🕒 31:04); the natural way is the annuity formula on the right clock, with its value on the right date. "a lot of them in the workbook problems just works to them, and there's honestly a finite number of things I can do" 🕒 31:32.

The full working — every step with its reason, the sum check, the two slips that both land on the year -1 value, the note's own numbers ($R = 5\%$: $R_b = 10.25\%$, $PV_1 = 3.1528$, $PV_0 = 3.0027$ per dollar of C), and the workbook's cruise, WB 2.3 Q4 — is Worked problems, group 2.

INTUITION Two routes: back from year 1, or forward from year -1

The earlier notes met two-year clocks on streams whose **first** payment was one year away — the bi-annual perpetuity of 02b p. 10 and the workbook's cruise (Lecture 4, §5.3), and the Section 2 note's step-up perpetuity (Section 2, §14). One two-year period before a payment at year 1 is year -1 , before today, so the last step there carries the value **forward** one year, multiplying by $1 + R$. Here the first payment is three years away, one period before it is year $+1$, after today, and the last step goes **back**, dividing by $1 + R$. The back route was already met once, on a longer clock: the Section 2 note's perpetuity on a multi-year clock lands its formula value at year 1 and discounts it back one year at the annual rate (Section 2, §9). It is one rule — the annuity or perpetuity formula on the two-year clock puts its value one two-year period before the first payment, and a single one-year move at the annual rate takes it to today — read in the direction the dates demand. The review's route is taught first; the forward route is the check in the worked problem, and gives the same number.

5.3 The review list: compounding, EAR against APR, HPR, IRR (21 p. 7)

🕒 31:44 · both recordings; named, not worked

The bottom of 21 p. 7 lists the rest of Block 2, and the lecturer named each item without working one: "Compound at different frequencies." 🕒 31:44; the effective annual rate against the APR (one recording, garbled); an example — "The interest rate is 6% compounded monthly, so you have to do a lot of that, and there's a very nice handout that it took me a lot of time to write years ago that goes through all of this in great detail, I hope you guys have read it," 🕒 31:50; and "HPR, IRR, all the things we've thrown over, so I think that should be straightforward, you have the solution." 🕒 32:10. Then the sentence whose middle the recordings render differently (§1.6, S9): "because again, these are the key ideas." 🕒 32:15.

The content, in one paragraph each, with the note that teaches it. **Compounding at different frequencies and EAR against APR:** a quoted annual percentage rate APR compounded m times a year

pays APR/m per period, and a dollar grows over a year to $(1 + APR/m)^m$; the **effective annual rate** is that growth less one, $1 + EAR = (1 + APR/m)^m$ (21 p. 7; formula sheet item 5). The handout's example, the one the lecturer quoted: 6% APR compounded monthly pays 0.5% a month, and $1.005^{12} - 1 = 6.1678\%$ (exact 6.16778%; the handout prints 6.1677%, a truncation already noted in the Lecture 4 note) (handout 02 p. 4). The handout is digested in full at Lecture 4, §2.7; the three names for a rate at Section 2, §10. **HPR**: the holding-period return, what an investment actually earned over the time it was held, $\$1 + HPR = \$ \text{end value over start value}$, and how it is annualised: Lecture 4, §6. **IRR**: the internal rate of return, the discount rate at which a project's present value is zero, and when it is unique: Lecture 5, §1. No APR/EAR, HPR or IRR example was worked on 15 October (the EAR formula on the page: slide only).

Related practice elsewhere. The two examples' workbook twins: WB 2.3 Q3 (Lecture 3) and WB 2.3 Q4, the cruise (Lecture 4); the closest workbook analogue of the page's frame, WB 2.4 Q1 (Lecture 4). The review list's items, all in the Lecture 4 note: APR and EAR — WB 2.1 Q6, Q7, Q11, Q13, Q15, Q21; 2.3 Q5, Q6; 2.4 Q3, Q4, Q6, Q8; holding-period returns — WB 2.2 Q2, Q3(a); 2.3 Q7; IRR — WB 2.2 Q3(b), Q6(d), Q6(e) in the Lecture 5 note. The Block 2 items adapted from past papers are in the Lecture 3 and Lecture 4 notes under their display ids, *Adapted problems X06 to X18, X83 and X92*. The Block 2 key-concepts checklist is split by meeting: Lecture 3, WB 2.5 — the Block 2 key concepts this meeting covers; Lecture 4, WB 2.5 — the Block 2 key concepts this meeting covers; Lecture 5, WB 2.5 — the Block 2 key concept this meeting covers (the IRR).

Self-check. A perpetuity pays C a year, first payment four years from today, at an annual rate R . Where does C/R sit, and what is the value today?

C/R sits at year 3, one period before the first payment. The value today is $\frac{C/R}{(1+R)^3}$ — three years of discounting, not four. Dating it at year 4 and dividing by $(1+R)^4$ would undervalue it by a factor $1+R$ (21 p. 8, example 1's rule).

Self-check. Payments of C arrive every three years at years 2, 5, 8 and 11; the annual rate is R . Which rate goes into the annuity formula, where does its value sit, and how do you reach today?

The three-year rate, $R_3 = (1+R)^3 - 1$, with $T = 4$ payments. The formula's value sits one three-year period before the first payment: year $2 - 3 = -1$. Today is one year later, so multiply by $(1+R)$ once — the forward direction, because the first payment is less than one period away. Check: this is the forward route of the Lecture 4 note's bi-annual perpetuity (§5.3) and the Section 2 note's step-up perpetuity (§14), on a three-year clock, and the reverse of 21 p. 8's step 3.

Self-check. In 21 p. 8's example 2, a student divides PV_1 by $1 + R_b$ to discount what the page calls one period. What date does the result sit on, and by how much is it wrong?

On year -1 : a two-year period back from year 1. It is too small by one year's discount, a factor $1 + R$ — in the worked problem's illustration at $R = 5\%$, 2.8597 instead of 3.0027. The period in step 3 is one year, because the move is from year 1 to year 0 (21 p. 8; the Warning above).

6. Portfolio choice: the two moments (21 p. 9)

🕒 32:21 · both recordings

This section establishes the two formulas that open Block 3 on the review deck, both on the formula sheet: a portfolio's expected return and its variance. The lecturer's point was not the algebra but the reading — what each Greek letter stands for.

What 21 p. 9 prints (and nothing else):

$$\mathbb{E}[R_p] = \sum_{i=1}^N w_i \mathbb{E}[R_i]$$

$$\text{Var}[R_p] = \sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 \text{Cov}[w_1 R_1, w_2 R_2] = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \rho_{12} \sigma_1 \sigma_2$$

The first is formula-sheet item 7; the second is item 8's two-asset line, written with the covariance step shown (03a pp. 5–6). A portfolio's expected return is the weighted average of its assets' expected returns; its variance is not — the cross term $2w_1 w_2 \rho_{12} \sigma_1 \sigma_2$ is where diversification lives, and it is smaller the lower the correlation.

EXAM SIGNAL "you have it in your cheat sheet" — 32:21

"Portfolio choice. Next, we got these type of equations. Okay, the first one, how you compute expected returns. The second one, how you compute the variance. Again, it's nasty, but you have it in your cheat sheet, so you know, you don't have to worry, because you have it there. All you have to do is know how to interpret all those Greek letters we got over there, okay? Rows and sigmas and whatnot, okay?"

🕒 32:21 . Then, just before the two-asset page: "Okay, conceptually, there are always questions on this, okay?" 🕒 32:43 (the other recording renders the last words as *on the list*); the note attaches the remark to the two-asset material that follows and adds no referent.

Reading the letters. w_i is the fraction of wealth in asset i (negative for a short position; the weights sum to one); σ_i is asset i 's volatility, the standard deviation of its return; ρ_{12} is the correlation of the two assets' returns, between -1 and 1 ; $\rho_{12}\sigma_1\sigma_2$ is their covariance. The N -asset variance on the formula sheet, $\sum_i w_i^2 \sigma_i^2 + 2 \sum_i \sum_{j>i} w_i w_j \rho_{ij} \sigma_i \sigma_j$, is the same thing with one cross term for every pair.

Links back. Lecture 5, §7.1 and §7.2 (the two moments, with the IBM/Dell example); the covariance rule they rest on, Lecture 5, §3.3; the many-asset form, Lecture 7, §11.1.

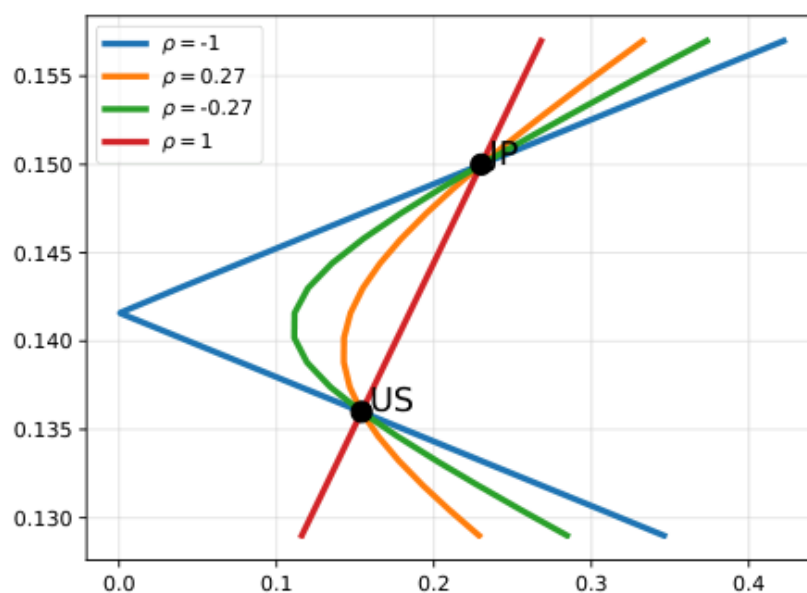
Related practice elsewhere. In the Lecture 5 note: WB 3.1 Q3, Q23, Q33, Q40, Q44, Q50; 3.2 Q5; 3.3 Q1; 3.4 Q1, Q8; and two items adapted from past papers, *Adapted problems X24 and X25*. In the Lecture 7 note: WB 3.2 Q3.

7. The two-asset case: the opportunity set at four correlations (21 p. 10)

 32:49 · both recordings; "there are always questions on this"

This section establishes the picture every two-asset question is read off: the set of mean–standard-deviation pairs two risky assets can reach, how its shape depends on their correlation, and the four terms that name its parts.

Two asset case



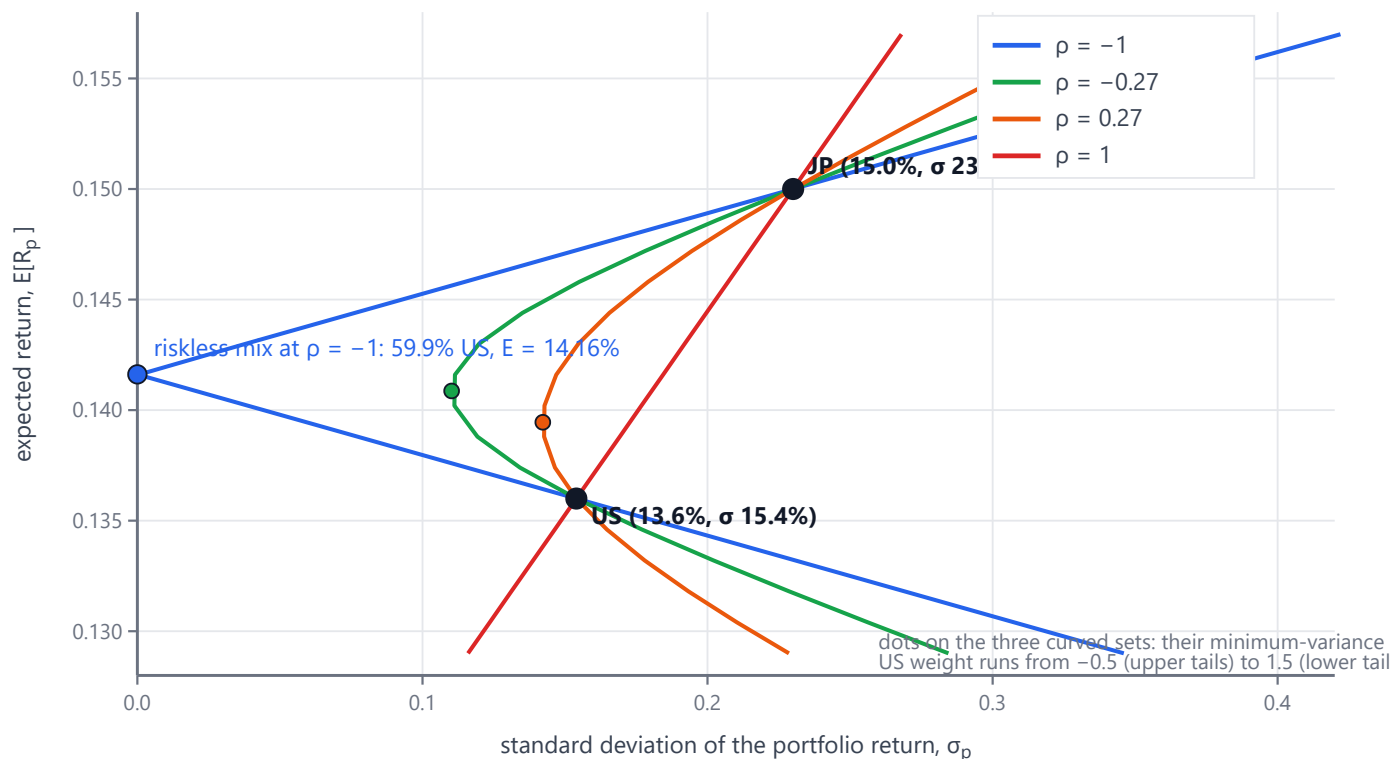
- Investment opportunity set, Efficient portfolio, Efficient frontier, Minimum variance portfolio, indifference curves (risk aversion)

10 / 21

21 p. 10 — original slide: the US/JP opportunity set at $\rho = -1$ (blue), 0.27 (orange), -0.27 (green) and 1 (red); the two assets as black dots

How to read this: standard deviation across (0 to 0.4), expected return up (0.130 to 0.155) — the page prints no axis titles. Every curve passes through both assets, US at (0.154, 0.136) and JP at (0.230, 0.150), because a 100% position in either is on every curve. The lower the correlation, the further left the curve bends: at $\rho = 1$ it is a straight segment, at $\rho = -1$ two straight segments meeting on the vertical axis at a riskless portfolio. The curves run past both assets because short sales are allowed.

Two risky assets, four correlations: the US/JP opportunity set (21 p. 10, redrawn)



Redrawn: the same four opportunity sets from 03a p. 8's inputs, axes titled, the minimum-variance portfolios and the riskless mix at $\rho = -1$ marked

How to read this: the same drawing with the names on it. US: expected return 13.6%, volatility 15.4%; JP: 15.0%, 23.0% (03a p. 8). The weight on US runs from -0.5 (shorting US to buy more JP, the upper tail) to 1.5 (shorting JP, the lower tail). At $\rho = -1$ the mix with 59.9% in US has zero volatility and an expected return of 14.16%. The dot on each of the three curved sets is its minimum-variance portfolio, and the branch above it is the efficient frontier; at $\rho = 1$ the set is a straight segment whose minimum (a heavy short position in JP) lies outside the plotted weights.

Interactive spec — Two risky assets: the opportunity set as the correlation changes

The sliders arrive with the site; the previewer shows the spec.

```
id: two-asset-frontier
title: Two risky assets: the opportunity set as the correlation changes
inputs:
  - {name: rho, label: "Correlation of US and JP returns,  $\rho$ ", min: -1, max: 1, step: 0.01, default: 0.27}
series:
  - {name: "opportunity set", x: w, x_values: [-0.5, -0.4, -0.3, -0.2, -0.1, 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5], xplot: "((w*0.154)**2 + ((1-w)*0.230)**2 + 2*w*(1-w)*rho*0.154*0.230)**0.5", y: "w*0.136 + (1-w)*0.150"}
  - {name: "US", xplot: 0.154, y: 0.136, style: point}
  - {name: "JP", xplot: 0.230, y: 0.150, style: point}
x_axis: {label: "standard deviation of the portfolio return,  $\sigma_p$ ", min: 0, max: 0.4}
y_axis: {label: "expected return,  $E[R_p]$ ", min: 0.128, max: 0.158}
readout: ["(0.230**2 - rho*0.154*0.230)/(0.154**2 + 0.230**2 - 2*rho*0.154*0.230)", "0.136*((0.230**2 - rho*0.154*0.230)/(0.154**2 + 0.230**2 - 2*rho*0.154*0.230)) + 0.150*(1 - (0.230**2 - rho*0.154*0.230)/(0.154**2 + 0.230**2 - 2*rho*0.154*0.230))"]
```

What 21 p. 10 prints. The chart, which is 03a p. 18's drawing scaled up with its three closing questions dropped, and one bullet: "Investment opportunity set, Efficient portfolio, Efficient frontier, Minimum variance portfolio, indifference curves (risk aversion)". *Slide only:* the $\rho = 0.27$ and $\rho = -0.27$ curves, which were not discussed (only $\rho = 1$ and $\rho = -1$ were), and the untitled axes.

DEFINITION The four terms (03a p. 13), and the fifth the page adds

The **investment opportunity set** is every mean–standard-deviation pair a portfolio of the assets can reach — with two assets, the curve itself; with more, a region. An **efficient portfolio** gives the highest expected return for its standard deviation. The **efficient frontier** is "the upper portion of the investment opportunity set starting at the minimum variance portfolio". The **minimum variance portfolio** (MVP) is the one with the lowest standard deviation of all. **Indifference curves** describe what the investor wants — the trade-off of return against risk at a given risk aversion — and pick one point on the frontier (03b pp. 8–11; §15 gives the lecturer's own account of these words).

What was said. "Here we have the two asset case, number two risky assets, all the notions." — what is an investment opportunity set, an efficient portfolio, the efficient frontier, the minimum variance portfolio? 32:49 . Then the questions the picture is for: where can the chosen portfolio end up once preferences are added? Not on the bottom part of the set — "It has to be on the efficient part." 33:08 . Could it be to the left of Japan, to the right of Japan? "Yeah, it can be." 33:17 . Are short sales allowed here? Yes: "If I didn't allow for short sales, I'll have to stop at U.S. and Japan, because I'm extending the set." 33:21 — the curves run past both assets because one can be sold short to buy more of the other. Then the correlations: "Correlation one, there's no backward bending piece. There's no gain from diversification. Correlation negative one, the gains are maximal. You can even create a risk-free investment combining two risky securities if they have only when they have negative one, like correlation. I guess that's the blue thing." 33:32 — on the page the $\rho = -1$ curve is the blue one. "All of these are key ideas you have to keep in mind." 33:54 .

The two extreme cases, in formulas. With weights w on asset 1 and $1 - w$ on asset 2, the variance is $w^2\sigma_1^2 + (1 - w)^2\sigma_2^2 + 2w(1 - w)\rho_{12}\sigma_1\sigma_2$. At $\rho_{12} = 1$ it is $(w\sigma_1 + (1 - w)\sigma_2)^2$, so the standard deviation is the weighted average of the two and the set is a straight line — no diversification. At $\rho_{12} = -1$ it is $(w\sigma_1 - (1 - w)\sigma_2)^2$, which is zero at $w = \sigma_2/(\sigma_1 + \sigma_2)$ (03a p. 16): for US and JP, $0.230/(0.154 + 0.230) = 0.599$ in US, with expected return $0.599 \times 0.136 + 0.401 \times 0.150 = 0.1416$ — the vertex of the blue curve on the vertical axis. The riskless mix is why $\rho = -1$ is the case where the arbitrage question arises (that riskless portfolio must earn the risk-free rate); the Lecture 5 note works it.

EXAM SIGNAL "there are always questions on this" — 32:43; the correlation cases — 52:54

"Okay, conceptually, there are always questions on this, okay?" 32:43 . Later, answering a written comment: "you should understand how things work when you have different correlations. So, yeah, it would be good to know what happens if positive one, negative one, etc. So, maybe spend a bit of time on that slide again." 52:54 — this is the slide (§15).

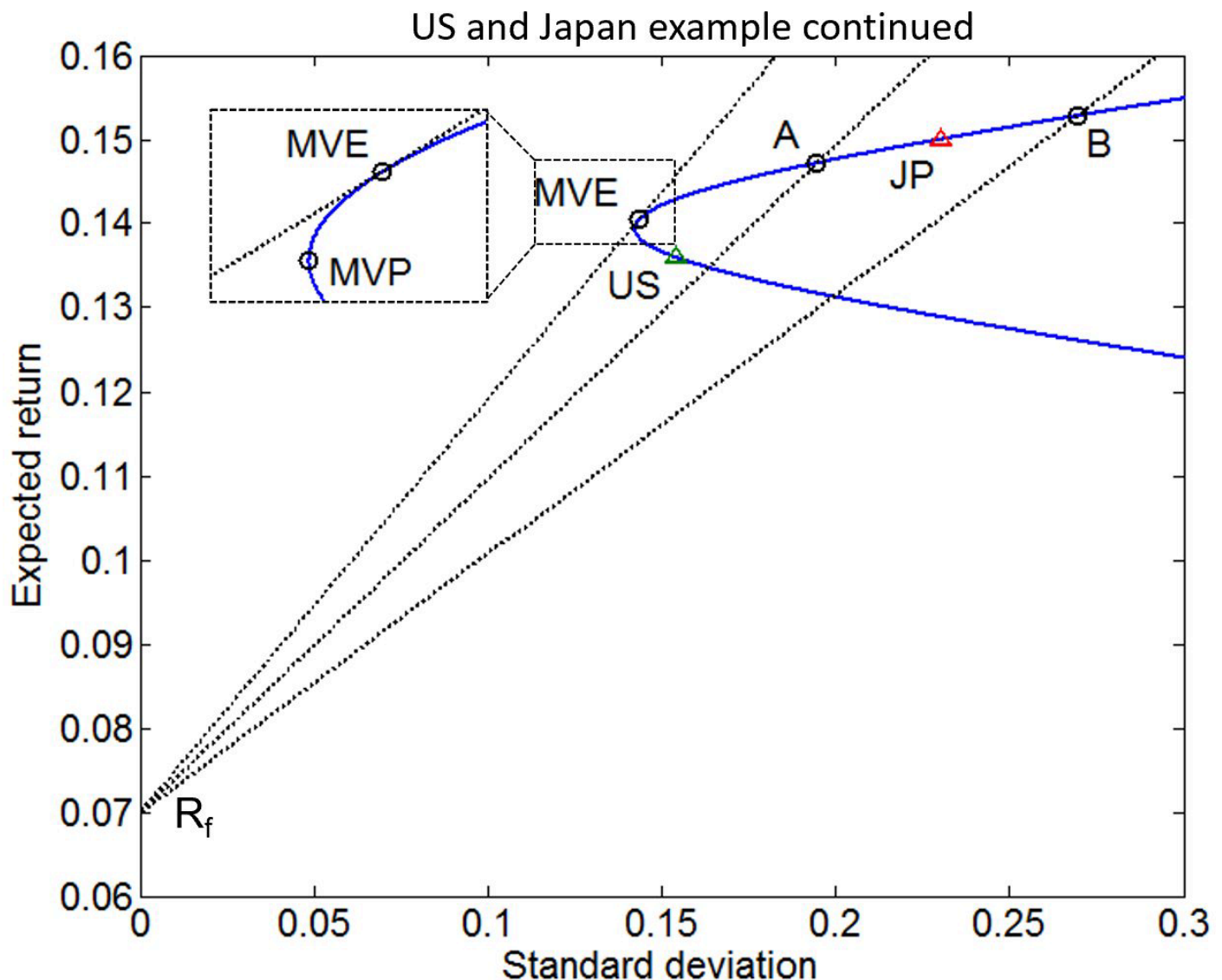
Links back. Lecture 5, §9.3 (short sales extend the set), §9.4 (the four terms on the figure), §10 ($\rho = \pm 1$), §11 (the MVP in full), §12.1 (this very chart and its questions); Lecture 6, §3 (indifference curves) and §7.1 (the optimum of two risky assets).

Related practice elsewhere. In the Lecture 5 note: WB 3.1 Q5, Q6, Q7, Q8, Q21, Q28, Q30, Q36, Q45, Q48, Q51; 3.2 Q1, Q5; 3.3 Q1, Q3; 3.4 Q1, Q2, Q9; its Block 3 key concepts; and *Adapted self-check X29*. In the Lecture 6 note: WB 3.1 Q9, Q11, Q29, Q39, Q41; 3.4 Q12; its key concepts; and part of *Adapted problem X20*. In the Lecture 7 note: WB 3.4 Q3.

8. Two risky assets and the risk-free asset: the capital allocation lines, the tangency portfolio, two-fund separation (21 p. 11)

34:02 · both recordings; two-fund separation, "Super important result" 35:59

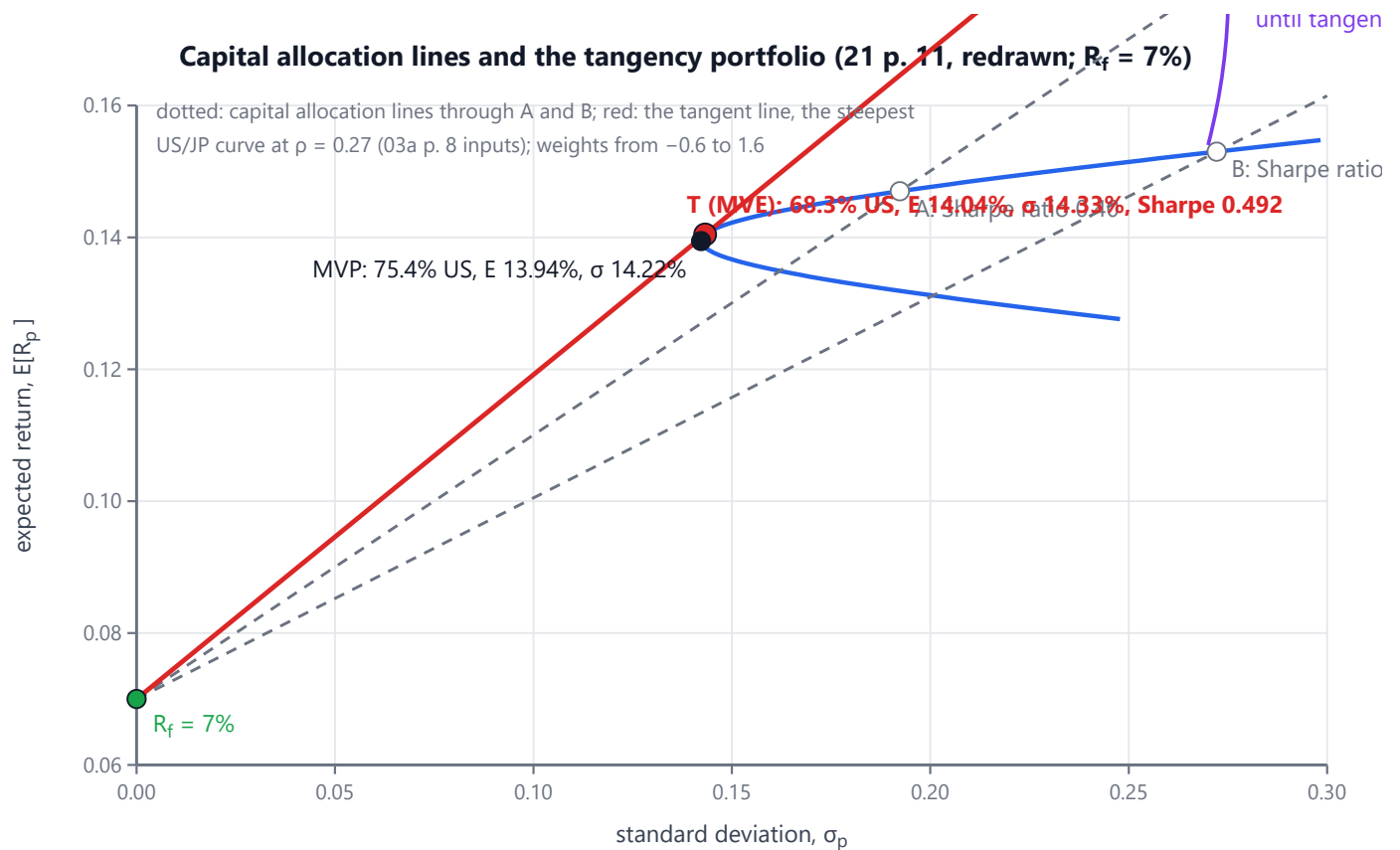
This section establishes what adding a risk-free asset does to the picture: every mix of the risk-free asset with one risky portfolio is a straight line from R_f , the best such line is tangent to the risky frontier, and the portfolio it touches — the tangency portfolio — is the one risky portfolio every investor holds, in a mix with the risk-free asset that depends only on risk aversion.



21 p. 11 — original slide: the US/Japan curve with the risk-free rate at 7% and three dotted capital allocation lines, the steepest tangent at MVE; MVP, A and B labelled

How to read this: standard deviation across, expected return up; $R_f = 0.07$ on the vertical axis. Each dotted line from R_f through a point of the curve is a capital allocation line; its slope is that risky portfolio's Sharpe

ratio. The steepest one touches the curve at MVE, the mean-variance efficient (tangency) portfolio; the inset magnifies the curve's nose, where MVP, the minimum-variance portfolio, sits just below MVE. The labels MVE, MVP, A and B were not named aloud (slide only).



Redrawn: the same curve from 03a p. 8's inputs at $\rho = 0.27$, with the risk-free rate at 7%; the capital allocation lines through two risky portfolios and the tangency line, with the rotation that finds it

How to read this: start a line at R_f and pass it through any risky portfolio; rotate it upward until it just touches the curve. The touching point is the tangency portfolio: 68.3% in US, expected return 14.04%, volatility 14.33%, Sharpe ratio 0.492. The minimum-variance portfolio (75.4% in US, 13.94%, 14.22%) sits just below it on the curve. Every point on the tangent line is a mix of the risk-free asset and the tangency portfolio; points beyond it borrow. This is also the diagram for claim 2 of 21 p. 17 (§14.2).

Interactive spec — The capital allocation line through the tangency portfolio, as the risk-free rate moves

The sliders arrive with the site; the previewer shows the spec.

```
id: cal-tangency
title: The capital allocation line through the tangency portfolio, as the risk-free rate moves
inputs:
  - {name: Rf, label: "Risk-free rate, R_f", min: 0.00, max: 0.12, step: 0.005, default: 0.07, format: percent}
series:
  - {name: "opportunity set (p = 0.27)", x: w, x_values: [-0.5, -0.4, -0.3, -0.2, -0.1, 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5], xplot: "((w*0.154)**2 + ((1-w)*0.230)**2 + 2*w*(1-w)*0.27*0.154*0.230)**0.5", y: "w*0.136 + (1-w)*0.150"}
  - {name: "tangency line (CAL through T)", x: s, x_values: [0, 0.3], xplot: "s", y: "Rf + s*((wT*0.136 + (1-wT)*0.150 - Rf)/(((wT*0.154)**2 + ((1-wT)*0.230)**2 + 2*wT*(1-wT)*0.27*0.154*0.230)**0.5))", where: {wT: "((0.136-Rf)*0.230**2 - (0.150-Rf)*0.27*0.154*0.230)/((0.136-Rf)*0.230**2 + (0.150-Rf)*0.154**2 - (0.136-Rf+0.150-Rf)*0.27*0.154*0.230)"}
x_axis: {label: "standard deviation, σ_p", min: 0, max: 0.3}
y_axis: {label: "expected return, E[R_p]", min: 0.06, max: 0.16}
readout: ["((0.136-Rf)*0.230**2 - (0.150-Rf)*0.27*0.154*0.230)/((0.136-Rf)*0.230**2 + (0.150-Rf)*0.154**2 - (0.136-Rf+0.150-Rf)*0.27*0.154*0.230)"]
```

What 21 p. 11 prints. The chart (03c p. 17's image, byte for byte) and two topic names: "Two-fund separation theorem"; "Optimal portfolios". The page's own 7% risk-free rate is the chart's; the course's worked example used 5% (03c p. 8).

What was said. "Now, what if we bring now the risk-free asset? Well, remember when we have a risk-free asset in a portfolio, we got those lines. How do we call those dash lines? Capital what?" — the room: allocation lines — "very good. They can combine any portfolio, so it's very risky security" ⌚ 34:02. Then the argument, in the lecturer's rhythm: "if there is a free rate, let me combine it with this one, let me combine it with this one, let me combine it with this one, and then you get to this one, there's one here that is tangent, okay, so that is sort of what I'm going to end up doing, because then, that gives me higher Sharpe ratio" ⌚ 34:11. Preferences come after: "if I want to add some, For preferences, I'm going to add that somewhere up there. Yeah, that is what we call the tangency portfolio." ⌚ 34:57.

DEFINITION The capital allocation line, the Sharpe ratio, the tangency portfolio (03c p. 7; 03d p. 4)

Mixing the risk-free asset with one risky portfolio i gives every point on the **capital allocation line** $\mathbb{E}[R_p] = R_f + SR_i \sigma_p$, whose slope $SR_i = (\mathbb{E}[R_i] - R_f)/\sigma_i$ is portfolio i 's **Sharpe ratio**, "compensation per unit of risk" (03c p. 7). Among all risky portfolios, the **tangency portfolio** is the one with the highest Sharpe ratio — the one whose line from R_f just touches the frontier:
 $\max_{w_1, \dots, w_N} (\mathbb{E}[R_p] - R_f)/\sigma_p$ subject to $w_1 + \dots + w_N = 1$ (03d p. 4). 21 p. 11's chart labels it MVE.

Two-fund separation, from the room. "Who can explain to me two-fund separation in simple terms?"

🕒 35:04 . A student: "Well, you invest in a risk-free asset and the same mutual fund." 🕒 35:15 .

The lecturer: "Perfect, right. And if you're going to have different risk aversion? What's the only difference between you and me?" — only the mix (a class answer the recording does not decode): "All that changes is this mix", while the risky portfolio is "the exact same for the two of us" (the end of the sentence is garbled)

🕒 35:20 . "It's a really remarkable result. I mean, this really shook people's heads for a long time because people always thought, well, if I'm" more tolerant of risk (the word is garbled) "I may invest more in some stocks that are super risky, but that's not what we're doing." The deck's statement: all investors hold combinations of the same two "mutual funds", the risk-free asset and the tangency portfolio; risk aversion sets only the fraction in the risk-free asset; the rest goes in the tangency portfolio "Independently of their degree of risk aversion (!!!)" — Tobin's result, which "debunks the idea of individually tailored portfolios" and predates the mutual-fund industry (03c p. 21).

EXAM SIGNAL "one of the main results you learn in this course" — 35:59

"Super important result, really interesting," 🕒 35:59 — a garbled clause — "I think it's one of the main results you learn in this course, okay?" Two-fund separation is also the premise of the CAPM's first insight on 21 p. 13 (tangency = market) and of the mutual-fund discussion of §2.2.

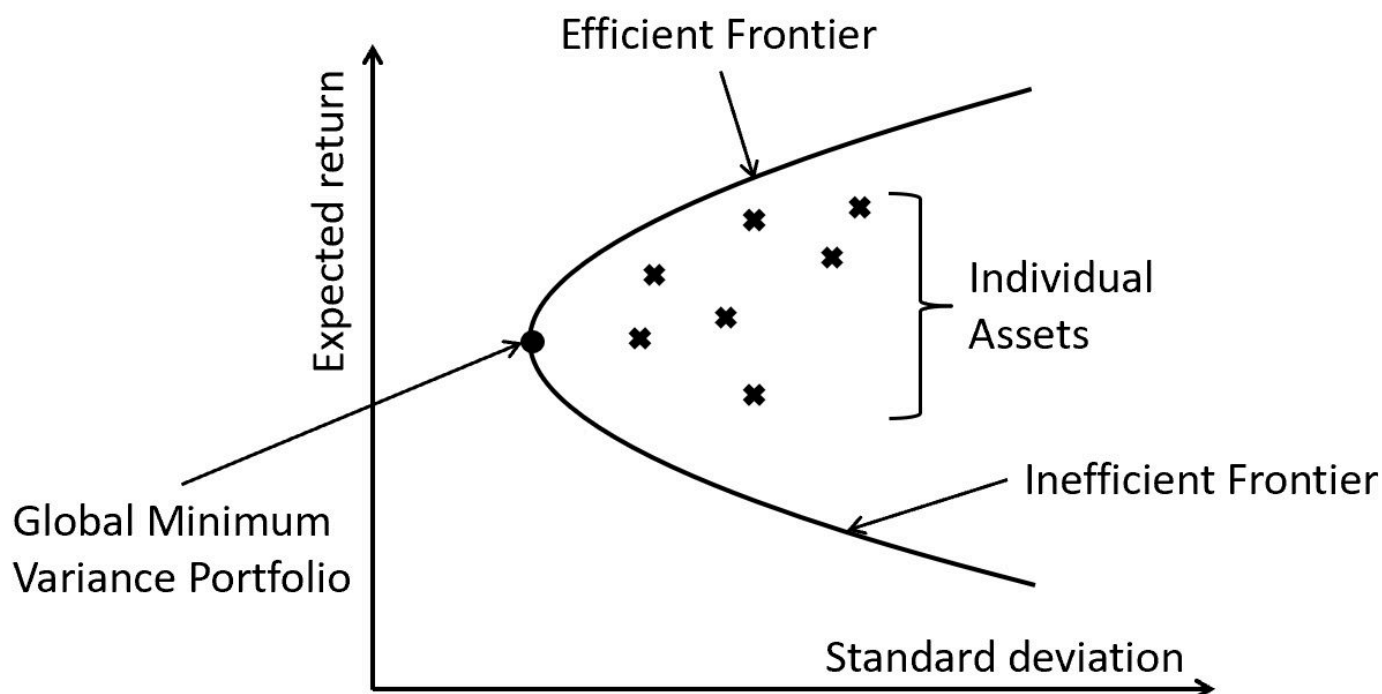
Links back. Lecture 6, §10.3 (the CAL and the Sharpe ratio); Lecture 7, §6 (the tangency portfolio, with the closed form), §8 (the theorem, and the resistance to it), §9.5 (it survives many assets).

Related practice elsewhere. In the Lecture 7 note: WB 3.1 Q15, Q16, Q17, Q22, Q32, Q34, Q46; 3.2 Q3; 3.3 Q2; 3.4 Q3, Q4, Q6, Q10, Q11, Q13, Q15; its key concepts; and *Adapted self-checks X21, X22, X27 and X28* and *Adapted problem X31*. In the Lecture 8 note: *Adapted self-check X23*. In the Lecture 6 note: its key concepts.

9. Many risky assets: diversification and the average covariance (21 p. 12)

🕒 36:16 · both recordings to 37:05, then one; demonstration D3

This section establishes what changes with many assets — the individual assets fall inside the frontier — and the one calculation the review recalled: the variance of an equally weighted portfolio, and its limit as the number of assets grows. The limit is the average covariance, which is why risk is co-movement.



21 p. 12 — original slide: the many-asset frontier, the global minimum variance portfolio at its nose, the individual assets inside; the equal-weight variance split beneath

How to read this: a schematic with no numbers (03d p. 3's drawing). The curve is the boundary of everything portfolios of the assets can reach; its upper branch is the efficient frontier, its lower branch the inefficient one; the dot is the global minimum variance portfolio. The seven crosses are the individual assets, all strictly inside: a mix of them reaches the same expected return at lower risk.

What 21 p. 12 prints. "Diversification", with sub-bullets "Variance of a portfolio" and "Idiosyncratic vs systematic risk", and the formula

$$\sigma_p^2 = \underbrace{\frac{1}{N} \left[\frac{\sum_{i=1}^N \sigma_i^2}{N} \right]}_{\text{Avg. Variance}} + \left(1 - \frac{1}{N} \right) \underbrace{\left[\frac{1}{\frac{N(N-1)}{2}} \sum_{i=1}^N \sum_{j>i}^N \text{Cov} [R_i, R_j] \right]}_{\text{Avg. Covariance}}$$

WARNING 21 p. 12 prints the split without its condition: equal weights

The formula holds only for an **equally weighted** portfolio, $w_i = 1/N$ for every i , which the course's page states first (03d p. 10) and this page omits. Under the heading *Variance of a portfolio* it reads as general, and it is not: with unequal weights the portfolio variance is not this combination of the average variance and the average covariance (the worked problem checks a 75/25 counter-example). The lecturer said the condition — "the case where we had number one over n in all the stocks" 🕒 37:06. Read the page as "variance of an equally weighted portfolio". This is a dropped condition, not a wrong value.

What was said. With two risky assets the set is one curve; with many, "the only difference, remember, when you have many risky is the fact that you have sort of, yeah, all of these things inside" 🕒 36:16 — every pair of assets traces its own curve, so "nothing is at the boundary, everything is inside, that's why I put the portfolios inside, that's something that would be hard to see unless you do the math, but it kind of makes sense" 🕒 36:31. (The lecturer called the curves "parabolas"; the course's own name for the many-asset boundary is the *super parabola* of 03d p. 3, and in mean–standard-deviation space the curve is not a parabola, so this note says *curve* or *frontier*.) Then the demonstration: "we went through these equations, which were a bit of an analytical exploration into idiosyncratic risk and systematic risk" 🕒 36:54 — ways to formalise how diversification reduces a portfolio's variance. **First case**, $1/N$ in every stock and zero correlation: the variance is "one over n times the constant, when n goes to infinity, this whole thing goes to zero" 🕒 37:06. **General case:** "This goes to zero, this goes to one, so what we have next is the average covariance." 🕒 37:35 — the $1/N$ on the average variance goes to zero, the $1 - 1/N$ on the average covariance goes to one. So the risk is measured by how different securities co-move with each other (one recording's rendering of a sentence the other garbles, 🕒 37:40).

$$\lim_{N \rightarrow \infty} \sigma_p^2 = \overline{\text{Cov}}$$

The limit is 03d p. 11's line, said aloud but not printed on 21 p. 12. The derivation from formula-sheet item 8 with $w_i = 1/N$, both limits, the equal-weights check, and the Lecture 7 note's illustration (volatility 40% each, correlation 0.25: one stock 40%, ten stocks 22.80%, a hundred 20.30%, floor $\sqrt{0.25} \times 40\% = 20\%$; uncorrelated stocks of 20% volatility: 10% at four, 2% at a hundred) are Worked problems, group 3. **Same route as the Lecture 7 note.**

EXAM SIGNAL Not on the formula sheet: derive it from item 8 — 37:48 to 38:27

Asked whether these equations are on the paper (the speaker is not identifiable), the lecturer's answer, in the body's words: "They're not? Well, it doesn't make me want to keep them in mind. Again, it's not that hard. You know these things." — the equations follow "from the equation over there. If you do one over n , you get exactly" the same formula (37:49). **Settled by the two pages: 21 p. 4 does not carry 21 p. 12's split; set $w_i = 1/N$ in item 8 and it falls out.** The wording of the lecturer's rule for the list is not settled (§1.1's second Warning).

GAP The Excel spreadsheet — 38:30

"Remember we had a spreadsheet in Excel, and that gave a lot of insight" (38:30) — the live portfolio optimizer of 29 September (03d p. 6), worth a look again. The spreadsheet itself is not among the course files; what it showed is recorded in Lecture 7, §10.

Links back. Lecture 7, §9.1 (the many-asset frontier), §11.2 (independent returns), §12.1 and §12.2 (the split and the limit); Lecture 8, §1 and §2 (systematic against idiosyncratic risk, the model).

Related practice elsewhere. In the Lecture 7 note: WB 3.1 Q18 and Q19 (the two limits as multiple-choice items), its key concepts, and *Adapted self-check X30*; in the Section 3 note: *Adapted self-check X26*. In the Lecture 8 note: WB 3.1 Q24; 3.4 Q7, Q14; 4.4 Q6; its key concepts.

Self-check. Two portfolios have the same expected return; one is on the curve of 21 p. 10 and one strictly inside the region of 21 p. 12. What do you know about their standard deviations, and which is efficient?

With two assets every feasible portfolio is *on* the curve, so "inside" has no meaning there; with many assets the inside point has a higher standard deviation than the frontier point at the same expected return, so the frontier point dominates it. The frontier point is efficient only if it is on the upper branch (above the minimum-variance portfolio); a point on the lower branch is dominated by the point straight above it (03a p. 13; 03d p. 3).

Self-check. The risk-free rate rises from 7% to 9% on 21 p. 11's chart. Does the tangency portfolio move, and in which direction along the curve?

It moves. The tangency portfolio is where a line from R_f touches the curve; raising R_f pivots every capital allocation line's starting point upward, and the touching point slides *up and to the right* along the frontier, into higher-return, higher-risk mixes (here, less in US, more in JP). The minimum-variance portfolio does not move — it depends only on the two assets' variances and covariance (03c p. 17; the slider above).

Self-check. A hundred stocks each have volatility 30% and pairwise correlation 0.4. What is the smallest volatility an equally weighted portfolio of them can approach as more such stocks are added, and why can it not reach zero?

The limit of the variance is the average covariance, $0.4 \times 0.30 \times 0.30 = 0.036$, so the volatility approaches $\sqrt{0.036} = 18.97\%$. It cannot reach zero because the covariance term's weight $1 - 1/N$ goes to one, not zero: what the stocks have in common — their co-movement — cannot be diversified away. Only the $1/N$ term, the stocks' own variances, disappears (21 p. 12; 03d pp. 10–11).

10. The CAPM: two insights, and why high beta pays (21 p. 13)

🕒 38:36 · one recording (recording 2 stops at 40:28, inside this page)

This section establishes the review deck's one-page CAPM: an equilibrium model with assumptions, whose two insights are that everyone holds the market portfolio with the risk-free asset, so the capital market line runs through the market, and that a stock's equilibrium expected return depends on its beta, through the security market line. The lecturer spent the time on the second insight's logic — "If you understand this, you understand the SML."

What 21 p. 13 prints. "Equilibrium model + Assumptions". "Two main insights": **1. Combination of market portfolio + risk-free asset** — "Tangency = market"; the market weights

$$w_M^i = \frac{p_i \times n_i}{\sum_j p_j \times n_j} = \frac{\text{Market Capitalization of Security } i}{\text{Total Market Capitalization}}$$

and the CML, $\mathbb{E}[R_p] = R_f + \underbrace{\left(\frac{\mathbb{E}[R_M] - R_f}{\sigma_M} \right)}_{\text{Sharpe Ratio of the Market}} \sigma_p$. **2. Equilibrium expected returns depend on beta**

— the security market line $\mathbb{E}[R_i] = R_f + \beta_i (\mathbb{E}[R_M] - R_f)$; $\beta_i = \text{Cov}[R_i, R_M] / \text{Var}[R_M]$; and the

risk decomposition $\underbrace{\sigma_i^2}_{\text{Total Risk}} = \underbrace{\beta_i^2 \sigma_M^2}_{\text{Systematic}} + \underbrace{\sigma_\varepsilon^2}_{\text{Idiosyncratic}}$. "Broader implications": "Capital budgeting";

"Performance evaluation". The skeleton is the CAPM deck's outline (04a p. 3) with its formulas inserted (04a pp. 6, 8, 12, 19). *Slide only*: the market-weight formula, the risk decomposition, and the assumptions, which were named ("has some assumptions") and not listed. (The page writes the sum's index in w_M^i as i , reusing the numerator's; read it as $\sum_j p_j n_j$. σ_ε^2 carries no i ; it is stock i 's. Both are 04a's notation, carried.)

DEFINITION The six assumptions (04a p. 4), named on 21 p. 13 and not listed

1. The market is in a competitive equilibrium: investors take prices as given.
 2. A common single-period investment horizon.
 3. All assets are tradable, so there is a market portfolio.
 4. No transaction costs, no taxes.
 5. Investors are rational mean-variance optimizers.
 6. All investors perceive the same means, variances and covariances of returns.
- The deck's own gloss: "All assumptions are inaccurate/wrong, but useful". What "the CAPM holds" means in a question is §17's third comment: all of these, and all the predictions, true.

Insight 1, as said. "we talked about equilibrium model, has some assumptions. Capital Market Line, which says tangency equals to market, and this line now goes to the market. We call it the capital market line, the particular capital market line, and we know what the market is" ⌚ 38:36 ; its slope "is the Sharpe ratio of the market, I'll talk about slopes in a second" ⌚ 38:52 . The market portfolio holds every risky asset in proportion to its market capitalisation — price times shares outstanding, over the total: with prices 2 and 4 and 25 and 50 shares, capitalisations of 50 and 200, weights 20% and 80% (04a p. 6's example). Why the tangency portfolio must be the market: every investor holds the same tangency portfolio (two-fund separation, §8), and in equilibrium what everyone holds must add up to what exists (04a p. 5).

EXAM SIGNAL The SML's logic, "once again" — 38:52 and 39:29

"I think I said this five times, I'm going to say it once again." ⌚ 38:52 — why high-beta stocks require high expected returns. "If you understand this, you understand the SML." ⌚ 39:29 .

Insight 2, as said. "Why is it the case that high beta stocks require high expected returns in the market?" ⌚ 39:23 . "Remember the logic, okay? You're holding everything. You give me any security that, you know, whatever is in the market, I have to hold, okay?" ⌚ 39:29 . What co-moves with the market is risky and so unpleasant to hold, "because I don't like risky stuff" ⌚ 39:40 ; "So, I have to hold, what's going to happen to the price in equilibrium? It's going to be low. And the expectation is going to be high." ⌚ 39:50 . "So, the way to read the equation is always the same. Because high beta stocks are unpleasant to hold, they make me unhappy because they increase my variance, I'm only willing to hold them in equilibrium if the price is low, which means that the expectation has to be high." ⌚ 39:59 . "because it's an equilibrium, you have to

be happy holding everything, okay, and to make you happy, you need to get" a good deal — "so it is compensation for risk" — a principle that runs through the course from the bid-ask spread onward, "we've got many different versions of this idea of compensation for risk, I think it's the nicest one" ⌚ 40:17 . "if you go out there and ask people what the SML says, they won't be able to explain to you so clearly ... but you thankfully won't be confused." ⌚ 40:45 .

INTUITION The SML read as price × quantity of risk

$\mathbb{E}[R_i] - R_f = \beta_i (\mathbb{E}[R_M] - R_f)$: the excess return a stock must offer is a **quantity** of risk, β_i — how much the stock adds to the risk of the market portfolio you already hold, which is its covariance with the market scaled by the market's variance — times a **price** of risk, the market risk premium, which is the same for every stock (*handout 04 p. 4, eq. (13); 04a p. 14*). A high-beta stock adds a lot of variance to everyone's portfolio; to be held at all it must be cheap, and cheap means a high expected return. A low-beta stock is "good at diversifying risk", in high demand, expensive, low expected return (*04a p. 14*). The deck's takeaway: in equilibrium a security's risk is not its variance but its covariance, beta.

The risk decomposition (*slide only here; 04a p. 19*). Write stock i 's return as its characteristic line, $R_i = \alpha_i + \beta_i R_M + \varepsilon_i$ with $\text{Cov}[\varepsilon_i, R_M] = 0$; then its variance splits into a systematic part the market moves, $\beta_i^2 \sigma_M^2$, and an idiosyncratic part, $\sigma_{\varepsilon_i}^2$, which diversification removes. Only the first is priced — which is why total risk is on the CML's axis but only beta is on the SML's (§12).

The broader implications, as said. Beta is also used for "performance evaluation" (the example that followed is garbled) and "a little bit of capital budgeting for projects" ⌚ 41:08 ; what was said of their difficulty is the Exam signal below. *Performance evaluation*: a fund's alpha relative to the SML; *capital budgeting*: discounting a project's cash flows at the CAPM's required return for its beta (*04a pp. 26–33*); both are the Lecture 9 note's.

EXAM SIGNAL Capital budgeting and performance evaluation — 41:08

"I don't think this is super, super challenging, we'll have to understand the basic idea." ⌚ 41:08 — of the two applications 21 p. 13 lists under "Broader implications".

Links back. Lecture 8, §7 (what equilibrium adds), §8, §9 (tangency = market, the weights, how the equilibrium is reached), §10, §11.2, §11.3, §12.2, §14; Lecture 9, §2 and §4.

Related practice elsewhere. In the Lecture 8 note: WB 3.1 Q20; 4.1 Q1, Q2, Q3, Q4, Q5, Q7, Q8, Q10, Q13, Q14, Q25, Q27, Q29, Q30, Q32, Q34, Q35; 4.2 Q3, Q4 (a)–(g), (i); 4.3 Q1; 4.4 Q2, Q5, Q6, Q11; its Block 4 key

concepts; and the items adapted from past papers *Adapted self-checks X23, X34 and X48* and *Adapted problems X32, X37 (in part), X40, X41, X45, X84, X85, X86, X90 and X93*.

11. Beyond the CAPM, and the efficient market hypothesis (21 p. 14)

🕒 41:23 · one recording; the page's second half was not spoken to with the deck up

This section establishes the two halves of the review deck's last content page: how the CAPM was tested and what replaced it, which the lecturer recalled in a minute, and the market-efficiency outline, which the page lists and the opening recap had already covered.

What 21 p. 14 prints. Three bullets with sub-bullets: the Fama-MacBeth test — conclusions, "Flat SML"; the Fama-French 3-factor model (printed as a sub-bullet of the test); the Efficient Market Hypothesis (EMH) — weak form, semi-strong form, strong form; and "Some Key Concepts" — random walk hypothesis, long-run predictability, event studies, anomalies, joint-test issue/data-mining, Grossman-Stiglitz paradox. The efficiency half is 05a p. 2's outline word for word, less its last bullet on mutual funds (which §2 covers; the block as a whole is on 21 p. 5's list). *Slide only:* the whole efficiency half — it was taught through the opening recap (§2.1) and the data-snooping answer (§15); the Grossman–Stiglitz paradox was never said on 15 October.

The testing half, as said. "we just went over this in the last two lectures" (one recording; the count is unsettled, §3's Warning) — how to test the CAPM, "which is a cross-sectional test. SML is flat, and Toby also remarked that, you know, when you look at the SML, it's like this, but it's kind of flat, okay, that's the betting against beta that Toby told you about, okay, the high beta, high beta stocks seem to not be paying what it is due" 🕒 41:23 . Then Fama–French: with the market alone, size and value portfolios show alpha — "They have an alpha against the CAPM, but then you move to a new level, such that value and size don't have an alpha against" the new model (the end is garbled) 🕒 41:30 . "That was the idea. All the stuff we saw in the last few lectures." 🕒 42:06 ; the sentence that follows is garbled and is not used.

The content behind the three lines, in the note's words. The **Fama–MacBeth test** has two steps (04b pp. 6–7): first, for every stock, estimate its beta from its characteristic line over a short window (five years of monthly returns); second, regress the stocks' average excess returns on those betas, $\bar{R}_i - \bar{R}_f = \gamma_0 + \gamma_1 \beta_i + \varepsilon_i$. If the CAPM is true, $\gamma_0 \rightarrow 0$ and $\gamma_1 \rightarrow \bar{R}_M - \bar{R}_f$: the fitted line is the SML. In practice the second step is run on ten portfolios sorted by beta, because a portfolio's beta is estimated far more precisely than a stock's (04b p. 8). **The conclusion, "Flat SML"** (04b pp. 10–11): for value-weighted beta deciles over 1927–2016 the lowest-beta decile was predicted 9.2% and earned 9.5% (an alpha of +0.3 points), the highest was predicted 19.5% and earned 12.5% (an alpha of –7 points); a "qualitative success" — higher beta does earn more — and a "quantitative failure" — not nearly as much more as the model says. The guest lecturer's **betting against beta** is the trade this suggests: buy low-beta stocks, short high-beta ones. **Fama–French's three-factor model** (04b pp. 16–18) adds a size factor and a value factor to the

market: the small-stock and value portfolios that had alpha against the CAPM have none against the three factors. The lecturer drew the test's picture again at (🕒 60:30), answering a written comment; the redrawn chart and the time-series alternative are in §16.

The efficiency half, one line each, and where it is taught. The **EMH** says prices reflect available information; its three forms differ in which information — past prices (**weak**), all public information (**semi-strong**), all information including inside information (**strong**) (05a pp. 3–4). The **random walk hypothesis** is the weak form's model: tomorrow's price is today's plus an unpredictable step. **Long-run predictability:** over years, valuation ratios such as the price–earnings ratio do forecast returns. **Event studies** test the semi-strong form by watching abnormal returns before, at and after an announcement. **Anomalies** are patterns that earn more than their risk seems to warrant; the **joint-test issue** is that any such finding tests the pricing model and efficiency at once, and **data mining** is that a pattern searched for among enough candidates will be found by chance. The **Grossman–Stiglitz paradox:** if prices reflected all information perfectly, no one would pay to gather it, so prices could not reflect it — efficiency needs some inefficiency to pay for itself. The weak and semi-strong forms, the random walk, long-run predictability, event studies, anomalies and data mining are the Lecture 11 note (13 Oct), whose content the opening recap summarised (§2.1); the Grossman–Stiglitz paradox is Lecture 9, §3; data mining in the guest's version is Lecture 10, §13.3.

EXAM SIGNAL What was said about this page's topics, elsewhere in the meeting

The cross-sectional test is the one the course reviewed; the time-series test will not be asked (🕒 61:28, §16). Four- and five-factor models: "I could ask you this question" (🕒 53:16, §15). The mutual-fund pages of Block 5 are "the last thing you need to know from the exam" (🕒 00:30, §2). The deck prints everything on this page; "everything that is in the slides is fair game for the exam" (🕒 17:32).

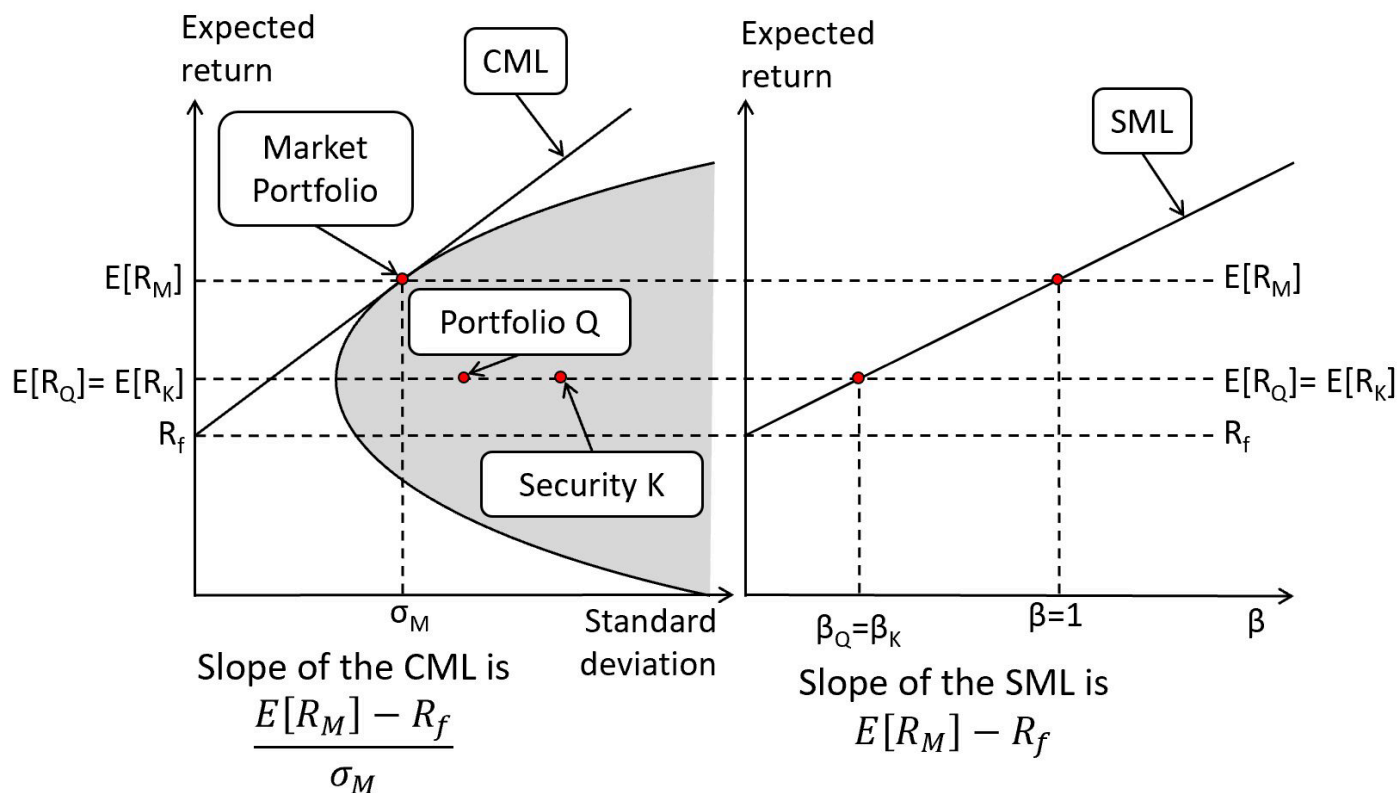
Links back. Lecture 9, §5, §6, §7; Lecture 10, §5 and §8; the Lecture 11 note for the efficiency half.

Related practice elsewhere. In the Lecture 9 note: WB 4.4 Q4; 4.1 Q17, Q18; its key concepts; and *Adapted self-checks X36 and X44* and *Adapted problems X33, X46 and X89*. The Block 5 items — WB 5.1 Q1, Q3–Q6; 5.2 Q1–Q3; the Block 5 key concepts, bullets 1–8; and *Adapted self-checks X50, X51, X52, X53 and X88* — are the Lecture 11 note's; bullet 9 of the Block 5 key concepts is this note's (Key takeaways).

12. CML against SML: the same idea twice (21 p. 15)

(🕒 42:17) · one recording; "a good summary ... This puts together most of the first part"

This section establishes the two-panel picture that holds the CAPM's two insights side by side — the capital market line in mean–standard-deviation space and the security market line in mean–beta space — and the lecturer's reading of their two slopes as two prices of risk.



21 p. 15 — original slide: left, the risky set, the CML through the market portfolio, a portfolio Q and a security K at the same expected return; right, the SML, with Q and K at the same beta and the market at $\beta = 1$

How to read this: dashed horizontals at $E[R_M]$, at $E[R_Q] = E[R_K]$ and at R_f run across both panels. Q and K have the same expected return but K has the larger standard deviation; on the SML they have the same beta. Only beta is priced. The CML's slope is $(E[R_M] - R_f)/\sigma_M$, the SML's is $E[R_M] - R_f$ (the captions on the page).



Redrawn: the two panels with 21 p. 18's numbers — the risk-free rate 4%, the market's expected return 8% — the SML's negative-beta region below the risk-free line shaded, and Q and K placed as on the slide

Redrawn: the two panels with 21 p. 18's numbers — the risk-free rate 4%, the market's expected return 8% — the SML's negative-beta region below the risk-free line shaded, and Q and K placed as on the slide

How to read this: the left panel's horizontal axis is total risk; only the market portfolio and its mixes with the risk-free asset lie on the CML, every other security or portfolio below it. The right panel's horizontal axis is beta; under the CAPM every security lies on the SML. The shaded strip below R_f on the right is where a negative-beta security sits — "I don't have it in the picture, you could have beta negative that have the return below the risk-free rate" 43:09 ; it is also the diagram for claim 3 of 21 p. 17 (§14.2).

Interactive spec — The two lines as the risk-free rate and the market risk premium move

The sliders arrive with the site; the previewer shows the spec.

```
id: cml-sml
title: The two lines as the risk-free rate and the market risk premium move
inputs:
  - {name: Rf, label: "Risk-free rate,  $R_f$ ", min: 0.00, max: 0.08, step: 0.005, default: 0.04, format: percent}
  - {name: MRP, label: "Market risk premium,  $E[R_M] - R_f$ ", min: 0.01, max: 0.10, step: 0.005, default: 0.04, format: percent}
  - {name: sigmaM, label: "Market volatility,  $\sigma_M$ ", min: 0.10, max: 0.30, step: 0.01, default: 0.20}
series:
  - {name: "CML (left panel)", x: sigma, x_values: [0, 0.40], y: "Rf + (MRP/sigmaM)*sigma"}
  - {name: "SML (right panel)", x: beta, x_values: [-0.5, 2.0], y: "Rf + beta*MRP"}
  - {name: "market", x: 1, y: "Rf + MRP", style: point}
x_axis: {label: "left: standard deviation  $\sigma_p$ ; right: beta  $\beta_i$ "}
y_axis: {label: "expected return"}
readout: ["MRP/sigmaM", "MRP"]
```

What 21 p. 15 prints. The two-panel figure (04a p. 17's image, byte for byte) under the title "CML vs. SML". Left: "Expected return" against "Standard deviation"; the grey risky set; the CML from R_f through the red "Market Portfolio" dot, a dashed vertical to σ_M ; two red dots at the level $E[R_Q] = E[R_K]$, "Portfolio Q" inside the set and "Security K" further right; the caption "Slope of the CML is $\frac{E[R_M] - R_f}{\sigma_M}$ ". Right: "Expected return" against " β "; the SML from R_f ; dots at $\beta_Q = \beta_K$ and at $\beta = 1$ (the level $E[R_M]$); the caption "Slope of the SML is $E[R_M] - R_f$ ". No values. (Q is drawn neither on the CML nor on the frontier; the figure does not say what Q is beyond a portfolio with K's expected return.)

What was said. "On the left, you have the portfolio choice view of the CAPM, and this is more like the CML view, this is the SML view." ⌚ 42:27 . "CML is what we said before, that's the market portfolio, you have different securities or portfolios inside, they have" different standard deviations, "but again, it has to be that their expected returns are connected to their beta." ⌚ 42:38 . "So, for example, that portfolio and that security, they both have some expected return and have different volatility, that must be because they both have the same beta over here. Market, beta one, risk-free rate, beta zero." ⌚ 42:58 . The slopes: "on the left, Sharpe ratio, on the right, market risk premium or expected return on the market, both tapping into

interpretations of compensation for risk." ⌚ 43:16 . "Here it says that if I have a portfolio with higher standard deviation, I'm compensated according to the Sharpe ratio of the market. Here it says if I take stock with a higher beta, I'm compensated per unit of beta in terms of the market risk premium." ⌚ 43:27 . "But here we're looking at the risk of the overall portfolio, you are looking at the risk of an individual security." ⌚ 43:43 . Together they are "a snapshot of what should be happening in the universe, like this sort of beautiful image of what's going on in the economy" ⌚ 43:52 .

WARNING Said in class vs written — 43:16: the SML's slope

Said: the right-hand slope is "market risk premium or expected return on the market". **Written:** "Slope of the SML is $E[R_M] - R_f$ " (21 p. 15) — the premium, the market's expected return **less the risk-free rate**. The two coincide only if $R_f = 0$. The slope is the premium; "expected return on the market" was a loose second name for it.

INTUITION Two prices of risk, two measures of quantity

On the CML the quantity of risk is a portfolio's standard deviation and the price per unit is the market's Sharpe ratio — the right picture for a whole portfolio, which is what an investor holds. On the SML the quantity is beta and the price per unit is the market risk premium — the right picture for one security, because the only risk a security adds to a diversified portfolio is its co-movement with the market. A security with a lot of idiosyncratic risk (K) has the same beta as a portfolio without it (Q), the same expected return, and sits far to the right on the left panel: the extra standard deviation earns nothing because it diversifies away (*handout 04 p. 4; 04a p. 17; Lecture 8, §13*).

Links back. Lecture 8, §13 (this figure, taught), §12.1 (reading the SML chart), §12.3 (the bands of beta on the mean–standard-deviation picture, including $\beta < 0$ below R_f).

Related practice elsewhere. In the Lecture 8 note: WB 3.1 Q35; 4.1 Q2, Q4, Q6, Q8, Q14, Q20, Q21, Q22, Q26, Q27, Q32; 4.2 Q1, Q4 (a)–(g), (i); 4.3 Q1, Q2; 4.4 Q3, Q5, Q11; its key concepts; and *Adapted self-checks* X35, X38, X42, X48, X49 and X87 and *Adapted problems* X32, X40, X45, X84, X85, X86, X90 and X93.

13. The four lines: CAL, CML, SML, SCL (21 p. 16)

⌚ 44:11 · one recording; "Everybody always asks me"

This section establishes the page the lecturer keeps for the question everyone asks — which line is which — and the distinction that sorts them: three are equations of the model, the fourth is a regression on data.

What 21 p. 16 prints, four bullets with a display each:

Line	As printed	Axes	What it is
Capital Allocation Line	$\mathbb{E}[R_p] = R_f + \underbrace{\left(\frac{\mathbb{E}[R_i] - R_f}{\sigma_i} \right)}_{\text{Sharpe Ratio of } i} \sigma_p$	$\sigma_p, \mathbb{E}[R_p]$	the mixes of the risk-free asset with one risky asset or portfolio i (03c p. 7)
Capital Market Line " (goes through market portfolio)"	$\mathbb{E}[R_p] = R_f + \underbrace{\left(\frac{\mathbb{E}[R_M] - R_f}{\sigma_M} \right)}_{\text{Sharpe Ratio of the Market}} \sigma_p$	$\sigma_p, \mathbb{E}[R_p]$	the CAL when the risky portfolio is the market — the best CAL under the CAPM (04a p. 8)
Security Market Line	$\mathbb{E}[R_i] = R_f + \beta_i (\mathbb{E}[R_M] - R_f)$	$\beta_i, \mathbb{E}[R_i]$	the required return of any security, given its beta (04a p. 12)
Security Characteristic Line	$R_i(t) - R_f = \alpha_i + \beta_i (R_M(t) - R_f) + \varepsilon_i(t)$	$R_M(t) - R_f, R_i(t) - R_f$	a regression on data, one security at a time; its slope estimates β_i , its intercept is α_i (04a p. 21)

What was said. "Everybody always asks me, oh, I saw these lines and I get a little confused. Can you help me sort them out? Remember, we have these four lines." ⌚ 44:11 . The **CAL**, "we talked about it before" ⌚ 44:16 . The **CML** "is the same, the only difference is that instead of being for a particular asset type, it's for the market, so the equation is the same, the only difference is that the first one is for a security, the second one is when that security is the market" ⌚ 44:24 (the end of the sentence is garbled; the page's own words are the rule: the CML is the capital allocation line that "goes through market portfolio"). The **SML**: "it's not about the combinations of expected return on volatility we can get when we invest, those are the first two. This is about what is the required return on a particular security" per unit of beta, "which is a measure of risk once we're in equilibrium" ⌚ 44:40 . The **SCL** "is really a regression line": "return the market minus the risk-free rate, return the security minus the risk-free rate, and you have like a bunch of data points ... it's the line we use to estimate beta, it's really a regression line, it's a linear regression" ⌚ 44:58 . "The other three are not statistical regression lines, they're really behavioral equations that come out of the model, it's like what happens in the model, this is really what happens in the data, how we estimate the" beta "in practice, and the intercept, we call alpha, and the slope, we call beta, and we calculate security per security" ⌚ 45:12 . The CAL and CML describe an overall investment; the SML and SCL are security by security (the closing sentence is garbled) ⌚ 45:37 .

DEFINITION Sorting the four lines

Ask two questions. **Is it a line in the model or a line through data?** The CAL, CML and SML are predictions of the theory; the SCL is a least-squares fit to observed returns, and its slope is the estimate of beta the SML then uses. **Is the horizontal axis total risk, or beta, or the market's return?**

Standard deviation: CAL and CML — the risk–return combinations an investor can *hold*, by mixing the risk-free asset with one risky portfolio (any portfolio for a CAL; the market for the CML). Beta: the SML — the return a security must *offer*, given how it co-moves with the market. The market's excess return, period by period: the SCL. (*explanation, from 21 p. 16 and the lecturer's sorting at* ⌚ 44:11 – ⌚ 46:03)

Two forms of the SCL on one deck. 21 p. 16 writes the SCL with a constant R_f ; 21 p. 4's item 11 (the formula sheet's) writes $R_f(t)$, a risk-free rate that moves period by period, as the testing deck does (04b p. 6). Both are course forms, not a slip. With a constant R_f the regression slope is exactly $\text{Cov}[R_i, R_M] / \text{Var}[R_M]$, the CAPM's beta.

Links back. Lecture 6, §10.3 (the CAL); Lecture 8, §10.1 (the CML as the second of four lines), §11.3 (the SML), §15.1 (the SCL as the fourth).

Related practice elsewhere. In the Lecture 6 note: WB 3.1 Q9, Q13, Q14, Q43; 3.2 Q2, Q6; its key concepts; and *Adapted problem X19*. In the Lecture 7 note: WB 3.2 Q3. In the Lecture 8 note: WB 3.1 Q20; 4.1 Q1, Q4, Q5, Q7, Q8, Q9, Q14, Q25, Q30, Q32, Q34, Q35; 4.2 Q3, Q4 (a)–(g), (i); 4.4 Q5, Q6, Q11; its key concepts; and *Adapted self-checks X23 and X48* and *Adapted problems X32, X37 (in part), X41, X45, X84, X90 and X93*. The Lecture 9 note's Block 4 key concepts.

Self-check. A stock's expected return equals the market's but its standard deviation is twice the market's. Where is it on each panel of 21 p. 15, and what is its beta?

Right panel: on the SML at $\beta = 1$ — the same expected return as the market means the same beta, under the CAPM. Left panel: at the market's height but twice as far right, far below the CML. The extra standard deviation is idiosyncratic risk, which a diversified holder does not bear and the market does not pay for (21 p. 15's Q and K).

Self-check. You are given sixty months of a stock's and the market's excess returns and asked for the stock's required return. Which two of the four lines do you use, in which order?

First the **SCL**: regress the stock's excess returns on the market's; the slope is the estimate of β_i (and the intercept is α_i). Then the **SML**: $\mathbb{E}[R_i] = R_f + \hat{\beta}_i (\mathbb{E}[R_M] - R_f)$ gives the required return. The CAL and CML are about portfolios an investor can hold, not about one stock's required return (21 p. 16; 44:58).

Self-check. Why does the review deck call the CML a special case of the CAL rather than a different line?

Because the equation is the same, $\mathbb{E}[R_p] = R_f + SR_i \sigma_p$, with the risky portfolio i set to the market: the CML is the CAL "(goes through market portfolio)". What the CAPM adds is that the market is the tangency portfolio, so the CML is the steepest of all the CALs — the one every investor is on (21 p. 16; 21 p. 13 "Tangency = market").

14. Two CAPM questions worked with the room (21 pp. 17–18)

46:13 · one recording; "I'm done with the review." — then the deck's two practice questions; their printed answers, 21 p. 18, were never displayed ("I have a solution", 51:59)

This section establishes the two kinds of CAPM question the review modelled: a numerical one — two stocks on the security market line, find the risk-free rate and the premium — and a short-question one — three true/false claims, each to be explained in a line or three, with a diagram if you like. The first is Worked problems, group 4; the second is the Self-check; this section gives the pages, the method as said, and what was said about each as an exam question.

14.1 Question 1: the risk-free rate and the market risk premium from two stocks (21 pp. 17–18)

46:28 · demonstration D4; no answer was said aloud

What 21 p. 17 prints. "1. Stock A has an expected return according to the CAPM of 8% and stock B of 10%. The beta of stock A is 1 and the beta of stock B is 1.5. The CAPM holds. — Find the risk-free rate and the market risk premium."

What was said. The question was read with its numbers 46:28 (parts garbled), then: "Yeah, I'm just saying, well, how do you tackle this question? In this question, you say, well, the first one has to be true,

right? So 8% with beta of 1, that, you know, you can solve it the way you want, 10% with beta of 1.5, you can kind of solve for both of them, at least just a little bit, you know, be creative on how you're going to use the equations" ⌚ 46:52. That is the whole method: **the SML must hold for each stock, which gives two equations in the two unknowns**, R_f and $\mathbb{E}[R_M] - R_f$. The lecturer did not solve them aloud; "These have to be more interesting, okay? So let's see if we can do this most quickly." ⌚ 47:14 moved to question 2.

The printed answer (21 p. 18, never displayed): "The SML gives one equation per stock — $8\% = R_f + 1 \times (\mathbb{E}[R_M] - R_f)$ and $10\% = R_f + 1.5 \times (\mathbb{E}[R_M] - R_f)$ — Subtracting the first equation from the second gives $2\% = 0.5 \times (\mathbb{E}[R_M] - R_f)$, so the market risk premium is $\mathbb{E}[R_M] - R_f = 4\%$ — Then $R_f = 8\% - 4\% = 4\%$ and $\mathbb{E}[R_M] = 8\%$ (stock A has $\beta_A = 1$, so it earns the expected return of the market)."

$$\mathbb{E}[R_M] - R_f = 4\%, \quad R_f = 4\%, \quad \mathbb{E}[R_M] = 8\%$$

The full working, with every step's reason, the sanity checks and what varies, is Worked problems, group 4.

Two routes, in one line each: the review's answer subtracts the two SML equations; the Lecture 8 note's WB 4.2 Q1 — the same two points with the labels A and B swapped, asking only for R_f — reads the SML as a straight line in beta and takes its slope as rise over run, $(10\% - 8\%)/(1.5 - 1) = 4\%$, then the intercept $8\% - 1 \times 4\% = 4\%$. Same line, same arithmetic; "you can solve it the way you want". (WB 4.2 Q1's printed " $\beta_A = 1$ " for its stock B is a typo the Lecture 8 note flags; it is not re-ruled here.)

EXAM SIGNAL "how do you tackle this question?" — 46:48

"Yeah, I'm just saying, well, how do you tackle this question?" ⌚ 46:48. The form of a numerical CAPM question: an equilibrium relation that must hold for every security, stated for each one given, then solved. 21 p. 3: numerical questions are "similar to the exercises made in class and to the practice problems".

14.2 Question 2: three claims about the CAPM, true or false (21 pp. 17–18)

⌚ 47:17 · demonstration D5; "You can do pictures, by the way." ⌚ 49:25

What 21 p. 17 prints. "2. Suppose the assumptions behind the Capital Asset Pricing Model (CAPM) hold. Which of the following statements are true or false? — If you hold the market portfolio, adding or subtracting a little of any stock should give the same increase in expected return per unit of additional standard deviation as the Capital Market Line. — All stocks must have Sharpe ratios that are less than or equal to that of the market portfolio. — A stock with a negative Sharpe ratio must have a negative market beta." The three are the workbook's WB 4.1 Q14, re-ordered and re-posed as true/false one at a time — the

midterm's short-question form, where "just true or false comes zero, it's all about the explanation" ⌚ 13:03 .

21 p. 18: "All three statements are true."

Claim 1, as said — the one the lecturer called hard. "So it's kind of hard, I would say, because I didn't spend a lot of time on this, so somewhere buried in the notes." ⌚ 47:34 . "What do you guys think? That's kind of true, right?" "Again, this would be, I think, unduly hard, because I haven't spent a lot of time on this, so I'm not going to ask this one over," ⌚ 47:51 — but the answer is straight: it was worked out when the SML was derived, "we essentially argued that these two things had to be the same for all ... stocks" (a word as heard is left out; the claim is about any stock), "so yeah, I mean, this has to be true. The way to see it is like, if these were not true, what would you do, you would like, you have something that gives you more than the market, you would buy this thing" — and the market could not be the portfolio everyone holds. "not the most exciting question, okay, but it will be something, this is kind of buried in some of the slides and some of the content of the workbook, so keep an eye out for it, the other two are more natural" ⌚ 48:15 . "I will consider this a kind of very hard question ... if you have worked out all the workbook, you will know the answer" ⌚ 48:43 ; "Okay, so maybe like one or two questions of this, of this." ⌚ 48:57 . **The printed reason** (21 p. 18): adding a little of stock i to the market, financed at R_f , raises expected return by $\mathbb{E}[R_i] - R_f$ and standard deviation by $\beta_i \sigma_M$ per unit invested; by the SML their ratio is $(\mathbb{E}[R_M] - R_f) / \sigma_M$, the CML's slope; "This is the condition that makes the market the tangency portfolio". The rate is a first-order one — exact for a small change, approximate for a large one; the algebra, step by step, is in the self-check.

Claim 2, as said — the one called "very standard". "All stocks will have Sharpe ratios that are less than or equal to that of the market portfolio." "This is just very standard, like very easy in a sense for somebody who is here." ⌚ 49:09 . "I hear a lot of truths. Anybody wants to walk me through why?" "You can do pictures, by the way. So like if you answer with a diagram." ⌚ 49:25 "I love diagrams, as you may have noticed." ⌚ 49:29 . A student answered with the picture: rotate the lines from the risk-free rate, "increasing the slope until we get through the tangency portfolio" ⌚ 49:32 — the market, under the CAPM, which therefore has the highest Sharpe ratio of all. That is the figure of §8: every stock lies on or below the CML, so the line from R_f through any stock is no steeper than the CML. 21 p. 18: "True. Draw diagram. The market is the tangency portfolio, so it has the highest Sharpe ratio".

GAP Not recoverable — 49:38

After the student's answer, 23.8 seconds of the recording decode to nothing — in the overlapping stretches of the meeting every silence this long had words on the other recording, so this is probably quiet speech, not a pause; recording 2 had stopped by then. Whatever was said there is not on file. Nothing in the answer depends on it: the claim's reason is the diagram described before the gap and printed on 21 p. 18.

Claim 3, as said. "Some with a negative Sharpe ratio must have a negative market beta. What do you guys think?" (as heard; the page's "A stock with ...") ⌚ 50:02. The lecturer asked for voices from students who had not spoken; from the room, as heard, "Negative Sharpe ratio is a negative market value." and then "It's less than the risk-free rate" — "Wonderful. Thank you. That's the right answer." ⌚ 50:44 (class answers, not the lecturer's). Then the diagram: "Let me not keep drawing, and you can see here right away, right? So you have anything down here, right below the lower horizontal dash line, in this part here, that is, that's a negative Sharpe ratio, right? It pays something less than the risk-free rate, but by, by construction here it should be on this part of the, of the diagram with the negative beta, okay?" ⌚ 50:54. "if the expected return is less than the risk-free rate, which is the consequence of the negative Sharpe ratio, it must be at the" negative-beta part (the last word is garbled) ⌚ 51:22. The diagram is the right panel of §12: the dashed line at R_f ; below it the expected return is below R_f , the Sharpe ratio's numerator is negative, and on the SML that happens only left of $\beta = 0$. 21 p. 18: "True. Draw diagram."

WARNING 21 p. 18, claim 3: it needs a positive market risk premium

The SML gives $\mathbb{E}[R_i] - R_f = \beta_i (\mathbb{E}[R_M] - R_f)$. A negative Sharpe ratio means $\mathbb{E}[R_i] < R_f$, so the left side is negative; that forces $\beta_i < 0$ **only if** $\mathbb{E}[R_M] - R_f > 0$. The page does not state the condition. It is the CAPM's normal case — risk-averse investors hold the risky market only if it pays more than R_f , and the course puts the premium at "4% to 6% in practice" (04a p. 14) — but with a negative premium a positive-beta stock would have the negative Sharpe ratio. An answer that states the condition is complete; item 7 of the closing list asks whether the exam expects it.

EXAM SIGNAL What was said about these three as exam questions — 47:51 to 51:59

Claim 1: "unduly hard ... I'm not going to ask this one over" ⌚ 47:51, then "maybe like one or two questions of this, of this" ⌚ 48:57 — what "this" refers to is unsettled (§1.6). Claim 2: "very standard, like very easy" ⌚ 49:09. After claim 3: "I'm not trying to trick anyone, this is just your understanding of the framework." ⌚ 51:36; "I'll try to ask questions that either I spend time on, or it can be easily derived with a little bit of logic from things that we have worked on." ⌚ 51:44; "I have a solution. Again, draw a diagram, draw a diagram. Diagrams are super helpful to make this work." ⌚ 51:59.

Two routes. The review answered claims 2 and 3 with diagrams — the capital allocation lines rotating up to the tangency portfolio (§8's redrawn figure), and the SML's region below the risk-free line (§12's). The Lecture 8 note's self-check on WB 4.1 Q14 proves both with one identity, $SR_i = \rho_{iM} SR_M$: claim 2 is $\rho_{iM} \leq 1$, claim 3 is that $\rho_{iM} < 0$ means $\beta_i < 0$ (with $SR_M > 0$). The diagrams are that identity drawn, and the review's route — the picture, then one or two lines — is the one the lecturer recommended: "You can do pictures, by the way." ⌚ 49:25; "draw a diagram, draw a diagram" ⌚ 51:59. The full self-check, with

each claim's reason, why "false" is tempting and wrong, and the algebra, is in the Self-check; the two diagrams were described in the room, not printed, and the redrawn figures are reconstructed from the recording on the base of 21 pp. 11 and 15 (no photograph of the board exists).

Links back. Lecture 8: WB 4.2 Q1 and the SML's sign in Worked problems E; WB 4.1 Q14 in its Self-check; §11.2 (claim 1's condition, the beta representation theorem), §12.1 and §12.3 (negative beta on both diagrams); Lecture 9, §8.1 (the theorem proved), and its worked problem *Handout 04 p. 1 — the ratio is 4 for Apple and 2 for Microsoft: what would you do?* (the equal ratio behind claim 1).

Self-check. In question 1, stock B's expected return is raised to 11% with everything else unchanged. What are the premium and the risk-free rate now, and why does $\mathbb{E}[R_M]$ not move?

Subtracting the SML equations: $11\% - 8\% = 0.5 (\mathbb{E}[R_M] - R_f)$, so the premium is 6%; then $R_f = 8\% - 1 \times 6\% = 2\%$. $\mathbb{E}[R_M] = R_f + 6\% = 8\%$ still, because stock A has $\beta_A = 1$ and its 8% did not change: a beta-one stock is the market's expected return (21 p. 18's route).

Self-check. Someone argues: *Stock B expects 10% and the market 8%, so stock B has the better Sharpe ratio.* Which claim of question 2 does this contradict, and where is the error?

Claim 2. The Sharpe ratio divides the excess return by the stock's *total* volatility, which includes idiosyncratic risk the market has diversified away. Stock B's higher expected return comes with proportionally more systematic risk ($\beta = 1.5$) and, in addition, its own idiosyncratic risk; per unit of total risk it cannot beat the tangency portfolio. On the diagram, the line from R_f through B is flatter than the CML (21 p. 18; §8).

15. Comments (1): the MVP, the correlation cases, more factors, efficient, optimal, feasible, desirable (21 p. 19)

🕒 52:06 · one recording; "these are questions that, you know, some people have asked me" — the first of three pages of students' written questions, read out and answered

This section establishes the lecturer's answers to the first four questions students sent in, each with the course's written answer beside it where the spoken one was short. The questions are printed unattributed on 21 p. 19; no answers are printed.

15.1 "Do we need to know how to calculate MVP in the general case?"

As said. "Okay, I'll say it is in the slides, we work it out, so in principle yes." ⌚ 52:20 . "Okay, I don't know if I'm going to ask for now, but I mean, I don't think, it would not be unfair if I ask you something about" — the end is garbled.

The written answer (03a p. 17, "MVP: General Case"). With two risky assets and weights w_1 and $1 - w_1$, the portfolio variance is $\sigma_p^2 = w_1^2 \sigma_1^2 + (1 - w_1)^2 \sigma_2^2 + 2w_1(1 - w_1)\sigma_{12}$, where $\sigma_{12} = \rho_{12}\sigma_1\sigma_2$ is the covariance. Setting its derivative to zero, $2w_1 [\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}] - 2 [\sigma_2^2 - \sigma_{12}] = 0$, gives the **minimum-variance portfolio**

$$w_1^G = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}, \quad w_2^G = \frac{\sigma_1^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}} = 1 - w_1^G$$

Two special cases the page prints: uncorrelated assets, $\sigma_{12} = 0$, give $w_1^G = \sigma_2^2 / (\sigma_1^2 + \sigma_2^2)$; equal variances give $w_1^G = w_2^G = 1/2$. Check on 21 p. 11's chart (the note's arithmetic): US and JP with $\sigma_1 = 0.154$, $\sigma_2 = 0.230$, $\rho_{12} = 0.27$ give $\sigma_{12} = 0.27 \times 0.154 \times 0.230 = 0.009563$ and $w_1^G = (0.0529 - 0.009563) / (0.023716 + 0.0529 - 0.019127) = 0.754$: 75.4% in US, expected return 13.94%, volatility 14.22% — the chart's MVP. The full model — assumptions, objective, first- and second-order conditions, comparative statics in the correlation — is Lecture 5, §11; the many-asset MVP is the handout's eq. (3), Lecture 7, §9.3.

15.2 "Should we memorize the formulas or learn how to optimize in various correlation cases?"

As said. The question was read in a garbled sentence ⌚ 52:36 ; the answer: "If you don't memorize anything, remember, it's all about thinking here. But you should understand how things work when you have different correlations." ⌚ 52:43 . "So, yeah, it would be good to know what happens if positive one, negative one, etc. So, maybe spend a bit of time on that slide again." ⌚ 52:54 — the slide with the four correlations, 21 p. 10 (§7). "I have asked questions about these in the past. I cannot tell you what's happening exactly this year, but it's happening." ⌚ 53:01 .

The written answer. The MVP formula above is on no formula sheet; what the deck gives is the variance formula (item 8) from which it is one derivative away. The correlation cases (03a pp. 15–16, 18): at $\rho_{12} = 1$ the set is a straight line and the MVP is a corner (or a short position), no diversification; at $\rho_{12} = -1$ the MVP is riskless, $w_1^G = \sigma_2 / (\sigma_1 + \sigma_2)$; in between, the lower the correlation the further left the curve bends and the lower the minimum variance. Being able to *draw* the three cases and *derive* the riskless weight is what "optimize" means here.

EXAM SIGNAL The MVP and the correlation cases — 52:20 to 53:07

The MVP in general: "it is in the slides, we work it out, so in principle yes"; "it would not be unfair if I ask you something" ⌚ 52:20 . The correlation cases: "it would be good to know what happens if positive one, negative one, etc. So, maybe spend a bit of time on that slide again." ⌚ 52:54 .

15.3 "I don't understand why we need further 4-factor and 5-factor models when the Fama-French model demonstrates that alpha is equal to zero."

As said. "Very good question. I could ask you this question. Anybody wants to help me answer the question? That would be a great-sounding question. I should ask it somewhere." ⌚ 53:08 . Put to the class: what is the point of a four- or five-factor model if the three-factor model gets a zero alpha? ⌚ 53:26 . From ⌚ 53:37 , 10.8 seconds decode to nothing (the Gap below). The lecturer's answer: "We showed that Fama-French three-factor gets zero alpha relative to the Fama-French portfolios, or the value and size portfolio, but if I come up with a new factor, then I can find alpha against the portfolios that Fama-French came up with based on value and size." ⌚ 53:47 . A new sort of portfolios shows alpha against the three factors; a fourth factor kills it; sort again, find alpha again, add a fifth — "So at the end you keep playing this game, okay? You can always find alpha essentially against any model, and then you add a new factor, and then you kill the alpha against the new one." ⌚ 54:35 . Once more: "CAPM, doesn't work great, okay, Fama-French, come and say, hey, you know how my better model, then you get no alphas, then somebody says, oh, let me sort by momentum or something, and then alphas pop up, and then I put the fourth factor to kill the momentum" ⌚ 54:44 . "The limit to that is the logic that we call data snooping or, yeah, this data snooping idea is just like overfitting the data, okay, and as we saw, the Chinese zodiac seems to have very good predictive power" ⌚ 55:33 — the tail is garbled — "it's a very good question".

GAP Not recoverable — 53:37

After the question was put to the class, 10.8 seconds of the only recording decode to nothing; recording 2 had stopped. Whatever was said there is not on file. The lecturer's answer that follows is complete without it.

The written answer. A **factor model** says a security's expected excess return is a sum of exposures to a few factors times those factors' premia; its **alpha** is what the model leaves unexplained. The three-factor model (04b p. 16) — market, size, value — is the deck's "Solution" to the alphas the CAPM leaves on value and small stocks (04b pp. 12–16); it prices those portfolios with no alpha (04b p. 18). Then the Carhart four-factor model added **momentum** — buy the past year's winners, short its losers; the deck reports that this sort "delivers a 10% alpha relative to the Fama-French 3-factor model" (04b p. 19); short-term reversal and

others followed; and the deck's own verdict is "Big problem: theoretical motivation absent": what risk a momentum factor is compensation for is unclear, where the CAPM's single factor has a clear meaning (04b p. 19). The limit the lecturer named, **data snooping**, is 05a pp. 16–17's point: search enough sorts and some will show alpha by chance, as the Chinese zodiac appears to predict returns on 05a p. 19 — the example recalled here, taught in the Lecture 11 note. Each new factor is at once a repair of the pricing model and a candidate for over-fitting; the joint-test problem is that the data cannot tell the two apart.

EXAM SIGNAL "I could ask you this question" — 53:16

"Very good question. I could ask you this question." ⌚ 53:16 . The answer in one line: a model's zero alpha is zero against the portfolios it was built to price; new sorts find new alphas, new factors kill them, and the only brake is the data-snooping objection.

15.4 "I'm a bit confused about the distinctions between 'efficient' 'optimal' 'feasible' and 'desirable' ... What does 'compensation for risk' refer to?"

As said. "This is great. I mean, this should be very clear. You should have no doubt." ⌚ 56:02 . **Efficient** portfolios "are the portfolios on the top part, right on the investment opportunity set": of two portfolios with the same standard deviation, "The efficient one is up here. If you have many, you'll be the one at the top" ⌚ 56:13 . **Optimal** "is a bit of a different thing because optimal combines preferences, okay? So optimal is given preferences, it's the one out of the efficient ones that I'm going to be picking. So all the efficiencies are good candidates to be optimal, but only one for a preference specification would be the optimal" ⌚ 56:35 . **Feasible** is everything attainable: with two assets "feasible is just the parabola. This is not feasible. If we have more than two, all of this is feasible. And once I put the risk-free asset, anything below is feasible." ⌚ 56:50 — "these things kind of look similar, the actual feasible set looks pretty different" ⌚ 57:12 . **Desirable** is what preferences say: "desirable are the indifference curves, okay, and the higher up you go, the more desirable these things are" ⌚ 57:25 . **Compensation for risk** "means different things in different ways, okay, it's always this idea that the more risk you take one from one another, you're going to get some compensation" — "these slopes are compensations for risk, they're prices of risk, and then beta is" a quantity of risk (garbled) — "and the product of the two is your overall compensation for risk, how much you get per unit of risk" — "that's what all finance is all about, how much risk you take, how much compensation you get per unit of risk" ⌚ 57:53 .

DEFINITION The four words, and the one picture they sit on

Feasible: every mean–standard-deviation pair you can actually reach — the curve with two risky assets, the region inside the frontier with more, everything below the capital allocation line once a risk-free asset is added (03a p. 13). **Efficient:** the feasible points no other feasible point beats — the upper branch of the frontier, from the minimum-variance portfolio up (03a p. 13). **Desirable:** what the investor prefers, drawn as indifference curves, higher ones better (03b pp. 8–11). **Optimal:** the single efficient point the highest reachable indifference curve touches — "Optimal portfolio reconciles what is desirable: described by the indifference or utility curves; feasible: described by the upper portion of the investment opportunity set, the efficient frontier" (03b p. 20). The deck's p. 20 uses "feasible" for the efficient frontier, the part of the set worth choosing from; the lecturer's answer used it for the whole attainable set and kept "efficient" for its top — the same picture, the word drawn tighter or looser.

Compensation for risk, written. The CAL's and CML's slopes, Sharpe ratios, are the **price of risk** per unit of standard deviation; the SML's slope, the market risk premium, is the price per unit of beta; beta is the **quantity** of systematic risk a security carries; and a security's excess return is price times quantity, $\beta_i (\mathbb{E}[R_M] - R_f)$ (handout 04 p. 4; 03c p. 8; 04a p. 14). The same idea appeared first as the bid-ask spread, the market maker's compensation for inventory risk (§4).

Links back. Lecture 6, §1, §7.1, §11.3; Lecture 5, §9.4 and §12; Lecture 8, §12.2; Lecture 9, §7.4; Lecture 10, §13.3.

16. Comments (2): the lines, APR against EAR, arbitrage, negative beta, the two approaches (21 p. 20)

🕒 58:25 · one recording

This section establishes the answers to the second page of written questions. Two were answered "We already did this" and are pointers; three were answered at length — arbitrage, negative beta, and the two ways of testing the CAPM.

"Review of SCL vs SML vs CML and when to apply each" — "We already did this." 🕒 58:25 : §13 and §12.

"Review of APR vs EAR would also be nice and maybe some more complex discounting problems" — "We already did this." 🕒 58:28 : the review list of §5.3 with the handout's 6%-compounded-monthly example, and the two discounting examples of §5.

16.1 "Can we clarify the meaning of arbitrage and examples of it that we've discussed in class?"

As said. "As I've said many times, arbitrage won't show up explicitly in the midterms." ⌚ 58:34 . "I will not ask a question like, oh, is this an arbitrage or not?" ⌚ 58:38 . A sentence about the final is garbled and is not quoted. "But remember, arbitrage is the idea that you turn a profit, you make a profit, without putting too much effort into it." ⌚ 58:45 . Then 22.7 seconds the recording does not decode. "We're going to spend a lot of time talking about it after the break, but again, that is the notion. It is not alpha. Alpha is not an arbitrage. Alpha is an abnormal compensation for holding risk and excess money that you pocket, but you can lose money doing alpha investing. Very important." ⌚ 59:13 .

WARNING Said in class vs written — 58:45: what an arbitrage is

Said: "you make a profit, without putting too much effort into it". **Written:** the course's fourth principle, no arbitrage — an **arbitrage** is a set of trades that costs nothing today and pays a sure profit, or pays something today and never requires a payment later: no net investment and **no risk** (*01a pp. 3–7; the mayonnaise and Nutella shelf prices of 01a pp. 6–7 are its everyday examples*). The written definition governs. The spoken phrase reads as a gloss on "free" (*the note's reading*). The next sentence holds on the written definition, and the lecturer's own reason is the risk: alpha is not an arbitrage — "you can lose money doing alpha investing".

GAP Not recoverable — 58:51

Between "without putting too much effort into it" and "We're going to spend a lot of time talking about it after the break", 22.7 seconds of audio decode to nothing. The examples the comment asked for, if any were given, are in that stretch; the two the course taught are the supermarket shelves of 01a pp. 6–7 and the no-arbitrage pricing argument of the Section 2 note (§3).

EXAM SIGNAL Arbitrage on the midterm — 58:34; alpha against arbitrage — 59:20

"arbitrage won't show up explicitly in the midterms. I will not ask a question like, oh, is this an arbitrage or not?" ⌚ 58:34 . And "Alpha is not an arbitrage ... you can lose money doing alpha investing. Very important." ⌚ 59:20 — the question the CAPM deck asks of a stock with a positive alpha (*04a p. 28; 04b p. 4*), answered in Lecture 9, §2.4 and in its Self-check WB 4.1 Q16 and *Adapted self-check X47*; the principle itself is Lecture 1, §3.4.

16.2 "What does a negative beta mean with respect to risk? (negative risk?)"

As said. "negative beta means that, I mean, you're not just" uncorrelated — the asset moves opposite to the market (garbled, ⌚ 59:33). "And I will not say it's negative risk, but yes, I mean, if you have a negative beta

stock, it's going to help you reduce the risk of your portfolio. I wouldn't use the word negative risk, but I will say it's very good for diversification. It's going to allow you to reduce the risk of your portfolio, because that's what negative beta really means." ⌚ 59:47 . "It's a wonderful hedge. That's the word. I will not use negative risk. It's a bit of a weird thing. Risk reducing, I think that would be a good terminology." ⌚ 60:09 .

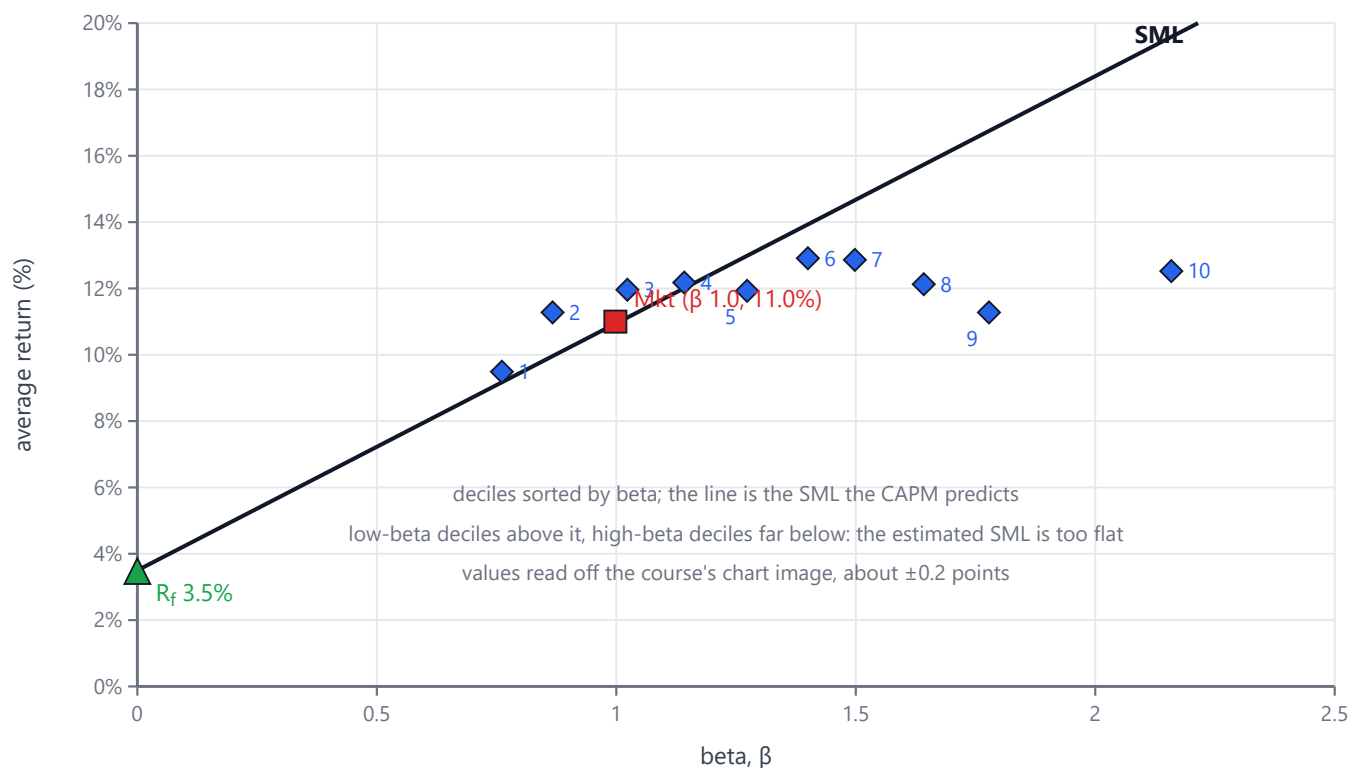
The written answer. $\beta_i = \text{Cov}[R_i, R_M] / \sigma_M^2 < 0$ means the stock tends to rise when the market falls. Added to the market portfolio it *lowers* the portfolio's variance, so investors want it as insurance and bid its price up; on the SML its expected return is **below the risk-free rate**, $\mathbb{E}[R_i] = R_f + \beta_i(\mathbb{E}[R_M] - R_f) < R_f$ — the band below R_f on 04a p. 15's figure and the shaded strip of §12's redrawn SML (04a pp. 13, 15). The stock's own return is still uncertain; what is "negative" is its contribution to the portfolio's risk, not its risk. It is also the case behind claim 3 of §14.2: a negative Sharpe ratio is a negative beta.

Links back. Lecture 8, §12.1 ("Can β be 0? And negative?"), §12.3, and its Self-checks on WB 4.1 Q6 (a stock with a negative beta) and WB 4.1 Q22 (its expected return).

16.3 "Can you review the two approaches to the CAPM (cross-sectional approach and time-series approach)?"

As said. "Let's be honest, we only reviewed the cross-sectional approach, which is the Fama-MacBeth approach. Remember how we do that? You go through all the stocks, you compute the beta stock by stock, and then we kind of plot them in the SML." ⌚ 60:26 . The lecturer drew the picture — "I'm not too good at this" — betas against expected returns: "The estimated SML is too flat and that's how we do it." ⌚ 61:02 . Each plotted point is a portfolio, "lowest deciles to highest deciles", and "the key step is you estimate the betas" (parts garbled, ⌚ 61:13); portfolios are used because estimates carry noise — "in finance there's all this risk floating around, so you put them in an average" and the estimation error "disappears" (garbled, ⌚ 61:28). "So that's cross-sectional. We never do time series. I'm not going to ask about time series." ⌚ 61:28 . Time series "is a bit different, because instead of ... trying to do this picture, taking one kind of section, what you do is you do the security characteristic lines" and test all the alphas at once: "if you do time-series econometrics and you look at something like a security characteristic line, you see how, like, you have 2,000 stocks, you have 2,000 systems of equations with 2,000 of them, you can do a joint test for all the alpha sequences." ⌚ 61:58 . "That would be a time-series test, which we don't do here because it requires some of the econometrics, and honestly, it's going to be hard to see," ⌚ 62:12 . "The book I recommended," a book whose name the recording garbles, "has a lot of detail on how you do it." ⌚ 62:23 .

Value-weighted beta decile portfolios, 1927–2016 (redrawn from 04b p. 10)



Redrawn from 04b p. 10: average return against beta for ten beta-sorted portfolios, 1927–2016, with the SML through the risk-free rate and the market

How to read this: the picture the lecturer drew on the board at ⌚ 60:30 (no photograph exists), redrawn from the course's own chart, 04b p. 10, whose points were read off its image (about ± 0.2 points). The line is the SML the CAPM predicts, from R_f (3.5%) through the market (beta 1, 11%). The diamonds are the ten decile portfolios: beta rises from about 0.76 to 2.16, average return only from about 9.5% to 12–13%. The low-beta deciles sit above the line (positive alpha, +0.3 points for decile 1), the high-beta ones far below (decile 10 about 7 points below). Flatter than the SML — a "Qualitative success" and a "Quantitative failure" (04b p. 11).

The written answer (04b pp. 5–9). **Cross-sectional** (Fama–MacBeth, 1973): step 1, estimate each stock's beta from its characteristic line over a short window, say five years of monthly data (04b p. 6); step 2, regress average excess returns on those betas, $\bar{R}_i - \bar{R}_f = \gamma_0 + \gamma_1 \beta_i + \varepsilon_i$, and test $\gamma_0 = 0$, $\gamma_1 = \bar{R}_M - \bar{R}_f$ (04b p. 7) — on beta-sorted decile portfolios, because a portfolio's beta is estimated far more precisely than a stock's when the measurement errors are uncorrelated (04b p. 8). **Time-series**: a single step — in the regression of each asset's excess return on the market's excess return, is alpha zero? Easy for one asset, hard for many at once; done optimally, the joint test of all the alphas is equivalent to the cross-sectional approach, so "the two approaches are more similar than they first appear" (04b p. 9). The deck itself then focuses on the cross-sectional approach, which is what the lecturer recalled.

EXAM SIGNAL Only the cross-sectional test — 60:26 and 61:28

"we only reviewed the cross-sectional approach, which is the Fama-MacBeth approach" ⌚ 60:26 ; "We never do time series. I'm not going to ask about time series." ⌚ 61:28 .

GAP "The book I recommended" — 62:23

The book's name is garbled on the only recording. The nearest reference in the course materials is the testing deck's pointer to *Empirical Asset Pricing: The Cross Section of Stock Returns* by Bali, Engle and Murray (2016), "Advanced but accessible" (04b p. 5) — an inference from the subject, not a fact on file; the book is not in the course materials. Item 10 of the closing list.

Links back. Lecture 9, §5.3, §5.4–§5.6, §5.7, §5.8 (this chart, in full); Lecture 10, §5.2 (why portfolios); the Lecture 9 note's WB 4.4 Q4 (the flat SML).

17. Comments (3): the CAL against the frontier, low price and high return, "the CAPM holds", where the equations come from, the workbook (21 p. 21)

⌚ 62:31 · one recording; "Okay, my last slide, and then you go."

This section establishes the answers to the last five written questions, including two the note completes from the written sources: the capital allocation line's relation to the efficient frontier, and why a low price means a high return.

17.1 "Is the CAL just the efficient frontier for combinations of risk-free/risky assets, or is the CAL the same as the efficient frontier in general (for any combination of assets)?"

As said. "Well, that's not a very good question. It's not very well formulated." ⌚ 62:35 . "The Capital Allocation Line is an efficient frontier only if you have one risk and one risk-free asset, but if you have many, it's not part of the efficient frontier, so it will not be an efficient frontier in general for any combination of" assets (garbled) ⌚ 62:47 . "So the Capital Allocation Line is just a combination so you can achieve using one risk and one risk-free, but in general, no, that's not a good question. The answer is no." ⌚ 63:03 .

WARNING Said in class vs written — 62:47: when a CAL is the efficient frontier

Said: with many assets a CAL — "it's not part of the efficient frontier" 62:47 — "The answer is no."

63:03 . **Written:** with a risk-free asset and any number of risky assets, the efficient frontier **is** a capital allocation line — the one through the tangency portfolio: every efficient combination of all the assets lies on it, and every other CAL (through a non-tangency risky portfolio) is dominated by it (03c pp. 7, 17–18; 03d p. 4). Without a risk-free asset the efficient frontier is the upper branch of the risky curve. **So the answer is not in general: a CAL is the efficient frontier only when it is the best one, the CML under the CAPM; a CAL through an arbitrary risky asset is not.** That is the sense of the spoken answer — the comment's second alternative, "the same as the efficient frontier in general (for any combination of assets)", is false — and the note does not state that a CAL is never the efficient frontier. Item 8 of the closing list.

Links back. Lecture 6, §10.3; Lecture 7, §6 and §9.4, and its Self-check on WB 3.1 Q15 (the efficient frontier with two risky assets and a risk-free asset).

17.2 "For risky assets, why does low price lead to high returns? (such as when we discuss how a decrease in demand decreases prices, which increases returns)"

As said. "this person listened to me, because I said how many of them said, high price, low returns, like 25 or something" 63:13 . The reason: "implicitly in all these models we're sort of holding fixed the payoff, right? We're kind of always saying, like, given whatever I'm going to get in the future, if the price is high, then the return has to be low" — "I should probably always say, given payoffs, high prices, low returns, and low prices, high returns, of course." 63:49 . "If you have a world in which like prices today go up and payoffs in the future go up, then how do we know what can happen to the return? We don't know, okay?"

63:56 . For the CAPM the return is "sort of proportional": "given payoffs, low price means high return, vice versa" 64:04 . "for fixed income, it's very clear, right, because the payoffs are always fixed ... If the price goes up, the returns have to go down." 64:26 . For a stock, if price and payoffs move together, "God knows, but typically that's not what we're assuming. We're assuming that we're fixing payoffs and then we're looking back at price and returns." 64:42 .

DERIVATION Full derivation (not done in class) — with the payoff fixed, the return falls as the price rises: $dR/dP = -X/P^2 < 0$

Let an asset cost P today and pay X for sure next period.

$$R = \frac{X}{P} - 1$$

Why this step follows: the net return is what you get back per dollar paid, less the dollar; for a risky asset read X as the expected payoff and R as the expected return — the same algebra.

$$\frac{dR}{dP} = -\frac{X}{P^2}$$

Why this step follows: differentiate $X P^{-1}$ with respect to P , holding X fixed; the result is negative for any positive payoff and price.

$$R(95) - R(100) = \frac{110}{95} - \frac{110}{100} = 1.1579 - 1.1000 = 0.0579$$

Why this step follows: a numerical check with $X = 110$: at a price of 100 the return is 10%; at 95 it is 15.79% — a lower price, a higher return.

If the payoff moves too, $dR = dX/P - X dP/P^2$, and the sign depends on which moves more — "We don't know." The CAPM's equilibrium works through this channel: excess demand for stock i raises $p_i n_i$, "which lower the expected return on i ", until supply and demand balance (04a p. 7); a bond's fixed coupons make the same relation exact (Lecture 3, §13).

Links back. Lecture 3, §13 (prices and rates move opposite ways); Lecture 8, §9.3, §12.2, and its Self-check WB 4.1 Q27 (a perpetuity whose beta doubles: the price falls, the required return rises).

17.3 "What is the big picture with the CAPM? What does it mean to say 'the CAPM holds'?"

As said. "Well, when you get the CAPM holds, it means that all the equations are true. You may have noticed that I use it always in my question, because, again, we know that CAPM doesn't really hold in practice, but we, you know, when I ask you questions, I say, assume the CAPM holds, it means the CML is true, the SML is true, that the framework we are considering is true. The assumptions are true, the predictions are true, that's the way I typically ask the questions, okay?"  64:58 .

EXAM SIGNAL "assume the CAPM holds" — 64:58

In a question, "the CAPM holds" means: the six assumptions of 04a p. 4 (§10) are in force; the tangency portfolio is the market; the CML holds for every efficient portfolio; the SML holds for every security — all of them at once. The empirical failure of the model (§11) is a separate matter: "we know that CAPM doesn't really hold in practice". 21 p. 17's two questions both open with the phrase.

17.4 "I'm still a bit confused on CAPM, like where the equations come from"

As said. "Well, the CML, I think, is very straightforward. The SML, as I said, I didn't give you the full proof," — a clause about "the question yesterday" is garbled — "so don't worry, I'm not going to ask you to derive the SML, okay?" ⌚ 65:35. "But the idea is, as before, you've got to be indifferent between holding everything, and when you have that, go through some of the handouts where I have the more formal proofs, then you end up getting the SML. That's where the equations are coming from." ⌚ 65:45. "but again, I care more to understand the economics than the exact mathematics. To do it well, I need to spend a little bit more time doing some linear algebra, things like this. I do this in some other sort of higher-level courses." ⌚ 65:55.

The written answer, in a paragraph. The **CML** is the capital allocation line through the tangency portfolio, which equilibrium makes the market: $\mathbb{E}[R_p] = R_f + SR_M \sigma_p$, straight from the two-fund mix of the risk-free asset and the market (04a p. 8). The **SML** comes from the condition that the market is the tangency portfolio — that no investor can raise the portfolio's Sharpe ratio by adding or subtracting a little of any stock, the "indifferent between holding everything" of the spoken answer. The CAPM handout proves it two ways: the beta representation theorem in §1, and in §2 the first risk–reward relationship, $\frac{\mathbb{E}[R_i] - R_f}{\partial \sigma_m / \partial \omega_i} = \frac{\mathbb{E}[R_m] - R_f}{\sigma_m}$ for every security, where ω_i is security i 's weight in the market portfolio and the handout's lower-case m is the market, this note's M — the extra return per unit of extra risk is the same for all, and equals the market's price of risk — which, with $\partial \sigma_m / \partial \omega_i = \beta_i \sigma_m$, becomes the SML, its eq. (13) (handout 04 pp. 2–4). The handout itself says of the derivative's derivation, "You are not responsible for this derivation" (handout 04 p. 2). This is also the content of claim 1 in §14.2.

GAP "the question yesterday" — 65:35

The clause "and even the question yesterday was a little bit tricky" ends in a garbled passage, and nothing on file says which question, asked where, is meant. Item 11 of the closing list.

EXAM SIGNAL Not asked: deriving the SML — 65:35

"don't worry, I'm not going to ask you to derive the SML, okay?" ⌚ 65:35 ; "I care more to understand the economics than the exact mathematics" ⌚ 65:55 .

Links back. Lecture 8, §11.2 and §12.2; Lecture 9, §8 (the handout's proofs in full); Section 4, §12 (the theorem on the board).

17.5 "I found some of the practice problems more difficult compared to the questions we cover in class"

As said. "I thought this is a very good one to end. ... Absolutely, right? The workbook is a little bit harder because if I spent, I don't know, 20 minutes here doing a complicated problem in class, they wouldn't have time to do anything. But because you've been working on them slowly for the last six weeks, you're confident now and you're going to do pretty well." ⌚ 66:14 . And the close: "Alright, so let me close it there." — "get a great sleep, and see you next time." ⌚ 66:37 .

EXAM SIGNAL The workbook is harder than class, on purpose — 66:21

"The workbook is a little bit harder because if I spent, I don't know, 20 minutes here doing a complicated problem in class, they wouldn't have time to do anything." ⌚ 66:21 . 21 p. 3: numerical questions are "similar to the exercises made in class and to the practice problems".

Self-check. A stock has beta -0.3 ; the risk-free rate is 4% and the market risk premium 5%. What does the CAPM say it should earn, and why would anyone hold it?

$\mathbb{E}[R_i] = 4\% + (-0.3)(5\%) = 2.5\%$, below the risk-free rate. It is held because it rises when the market falls: adding it lowers the portfolio's variance — "a wonderful hedge" ⌚ 60:09 — and investors accept a return below R_f for that insurance (04a pp. 13–15; §16.2).

Self-check. With one risky asset and a risk-free asset, is the capital allocation line the efficient frontier? With ten risky assets and a risk-free asset?

Yes in the first case: every feasible portfolio is on that one line, and the part above R_f is efficient. In the second case the efficient frontier is again a capital allocation line — but only the one through the tangency portfolio; the CAL through any other risky portfolio lies below it and is dominated (03c pp. 7, 17; 03d p. 4; the Warning in §17.1).

Continues in

The next meeting is the **midterm, Tuesday 20 October, "in class"** (21 p. 2); it is not a note. After it: the equity-valuation block will be delivered as a video (\$1.5, Gap 00:45; the deck is 06a); arbitrage returns after the break — "We're going to spend a lot of time talking about it after the break" (59:13) (decks 07a and 07b); and the fixed-income remark at (64:26) — a bond's payoffs are fixed, so its price and its return move in opposite directions — points to the fixed-income decks, 08a–08d, with Lecture 3, §7 the course's page on it so far. Nothing after the midterm is named in clear words on the recording. Sideways: the Lecture 11 note (13 Oct) ends where this meeting's opening recap begins, on 05a p. 22; its continuation is §2 here.

Worked problems

(23:54) · the review's demonstrations, worked in full from the method used in the room; the two printed questions of 05a and the three true/false claims are in the Self-check

This meeting assigned no workbook items of its own: what it worked are the review deck's practice items and two printed questions put to the room, and those are this note's exercises. The basis for working them is the course's own: "There are no graded homeworks, but you are expected to work through the concept questions and the practice problems every week" (syl p. 3). The midterm of 20 October and the final are never worked here. Every worked answer below that a workbook item also covers was checked against the workbook's official solution — WB 2.3 Q3(b) and Q4 for the two discounting examples, WB 3.1 Q18 and Q19 for the diversification limit, WB 4.2 Q1 and WB 4.1 Q14 for the two CAPM questions — and every one agrees; no erratum was found in the review deck. Where a worked problem uses numbers, it says whose they are: the deck's examples are symbolic, so the discounting problems carry the note's own illustration ($R = 5\%$, $C = \$1$), labelled as such.

Lecture examples, and where the note works them

Demonstration	Where it was worked aloud	Worked in full
21 p. 7 example 1 — a perpetuity whose first payment is nine years away	\$5.1, ⌚ 23:54 – ⌚ 25:42	Group 2 below
21 p. 7 example 2 — payments every two years, at years 3, 5, 7 and 9	\$5.2, ⌚ 25:44 – ⌚ 31:43	Group 2 below
21 p. 12 — the equally weighted portfolio as N grows	\$9, ⌚ 36:54 – ⌚ 37:45	Group 3 below
21 p. 17 question 1 — the risk-free rate and the premium from two stocks	\$14.1, ⌚ 46:28 – ⌚ 47:14	Group 4 below
21 p. 17 question 2 — three claims about the CAPM, true or false	\$14.2, ⌚ 47:17 – ⌚ 52:04	Self-check
05a p. 23 — "Which type of fund normally does better?"	\$2.2, ⌚ 03:49	Self-check
05a p. 25 — before costs; luck against skill	\$2.4, ⌚ 05:27	Self-check
WB 5.3, key concept 9 — do mutual funds beat the market?	\$2	Key takeaways

Group 1. Market efficiency: the fund evidence (05a pp. 23–26)

Two concept questions the deck prints and the lecturer put to the room. They train reading the fund evidence the way the course does: the comparison that matters to an investor is **after costs** (05a p. 24's chart, where the index wins by about a point a year), the comparison that measures **skill** is before costs (Wermers' table, where the funds' stocks win by 1.5 points and the costs take 2.3), and a few significant winners in a large population are what luck alone produces (five of a hundred at the 5% level). Both are in the Self-check, with the tempting wrong answers named.

Group 2. Discounting: the date a formula's value lands on (21 pp. 7–8)

The two review examples, worked from the lecturer's own steps. The skill is one: a perpetuity or annuity formula values its stream **one period before the first payment, on the clock the rate is quoted in** — so a perpetuity starting at year 9 is valued at year 8 and discounted eight years, and an annuity paid every two years is valued with the two-year rate $R_b = (1 + R)^2 - 1$, one two-year period before its first payment, then moved to today by **one year** at the annual rate. Each problem carries the sum of the discounted payments as its check, the dating error named aloud, the note's own illustration at $R = 5\%$, and its

workbook twin. The second carries a *Two routes* passage: the review's route (value at year 1, back one year) against the Lecture 4 note's (value at year -1 , forward one year), which reach the same number.

21 p. 7 example 1 — a perpetuity whose first payment is nine years away

Source: 21 p. 7, example 1 (the question), and 21 p. 8 (the printed answer); worked aloud with the class, ⌚ 23:54 – ⌚ 25:42. The deck's example is symbolic; the numbers in Step 7 are the note's own illustration, not the lecture's.

SETUP

At an annual rate R , compounded annually, what is the value today of C a year forever, when the first payment arrives exactly nine years from now?

GIVEN

Input	Value	Units	Source
Payment C	the same amount every year, forever	dollars per payment	21 p. 7
First payment	year 9, then 10, 11, ...	years from today	21 p. 7
Rate R	annual net discount rate, compounded annually	per year	21 p. 7
Perpetuity formula	$PV_{\text{perpetuity}} = C/R$ for payments starting at $t = 1$, so the value sits at $t = 0$, one period before the first payment	dollars	21 p. 4 item 4; 02a p. 23
One sum moved through time	$FV = PV(1 + R)^T$	dollars	21 p. 4 item 1

STRATEGY

This is Dávila's method, in two steps ("Two steps. What do I do first?" ⌚ 24:39). First value the whole stream with the perpetuity formula, at the date that formula lands on: one year before the first payment, year 8 ("here the first cash flow is at 9, we have the value at 8" ⌚ 25:04). Then treat that value as a single sum of money at year 8 and discount it eight years to today. The date is the point of the exercise: the review calls the timing, when each value is computed, the main issue in every discounting question.

SOLUTION

Step 1. Draw the timeline.

Payments of C sit at years 9, 10, 11, ... and nothing is paid at years 1 to 8.

Why this step follows: 21 p. 7's "first payment starting exactly nine years from now", then one payment a year, forever.

Step 2. Value the stream with the perpetuity formula.

$$PV_8 = \frac{C}{R}$$

Why this step follows: standing at year 8, the payments arrive 1, 2, 3, ... years later, which is exactly the stream the formula prices (first payment one period after the valuation date, 02a p. 23).

Step 3. Date that value.

$$\text{valuation date} = 9 - 1 = 8$$

Why this step follows: the formula's value sits one period before the first payment; a period is one year and the first payment is at year 9. Dating it at 9 instead would treat every payment as arriving a year later than it does.

Step 4. Discount the year-8 value to today.

$$PV_0 = \frac{PV_8}{(1 + R)^8}$$

Why this step follows: PV_8 is now one sum of money at year 8, so item 1 solved for PV , with $T = 8$, moves it back eight years — 21 p. 8's "discount for eight periods".

Step 5. Substitute Step 2 into Step 4.

$$PV_0 = \frac{C}{R(1 + R)^8}$$

Why this step follows: replace PV_8 by C/R and write the two denominators as one product.

Step 6. Check by adding up every payment (the note's own check).

$$\sum_{t=9}^{\infty} \frac{C}{(1 + R)^t} = \frac{C}{(1 + R)^9} \cdot \frac{1}{1 - \frac{1}{1+R}}$$

Why this step follows: 02a p. 18's present value is the sum of the discounted payments; factoring out the first term leaves a geometric series with ratio $\frac{1}{1+R}$.

$$\frac{1}{1 - \frac{1}{1+R}} = \frac{1+R}{R}$$

Why this step follows: over the common denominator $1+R$, the bottom is $\frac{R}{1+R}$; inverting it gives $\frac{1+R}{R}$.

$$\frac{C}{(1+R)^9} \cdot \frac{1+R}{R} = \frac{C}{R(1+R)^8}$$

Why this step follows: one factor of $1+R$ cancels against the ninth power, leaving the eighth. This is Step 5.
✓

Step 7. A numerical check (the note's own illustration, not the lecture's: $R = 5\%$, $C = \$1$).

$$PV_8 = \frac{1}{0.05} = 20$$

Why this step follows: Step 2 with these numbers.

$$(1.05)^8 = 1.477455$$

Why this step follows: eight years of growth at 5% a year, compounded annually.

$$PV_0 = \frac{20}{1.477455} = 13.5368$$

Why this step follows: Step 4 with these numbers.

ANSWER

$$PV_0 = \frac{PV_8}{(1+R)^8} = \frac{C/R}{(1+R)^8}$$

in dollars today, with $PV_8 = C/R$ dated at year 8 — 21 p. 8's answer. In the illustration ($R = 5\%$, $C = \$1$), **\$13.5368** today.

SANITY CHECK

- **Positive, and below the undelayed perpetuity.** Today's value is C/R scaled by $1/(1+R)^8$, a number below one (0.6768 at 5%), so the illustration's 13.5368 sits below 20. A stream that starts later must be worth less. ✓

- **Perpetuity minus annuity** (*the note's own check*). A perpetuity starting at year 1 is worth C/R ; the eight payments at years 1 to 8 are an annuity worth $\frac{C}{R} (1 - (1 + R)^{-8})$ (21 p. 4 item 3). Taking them away leaves $\frac{C}{R}(1 + R)^{-8}$, the same answer: 02a p. 23's identity, an annuity is the difference of two perpetuities, read the other way. ✓
- **The workbook's own deferred perpetuity agrees.** WB 2.3 Q3(b) (p. 21) values \$10,000 a year with the first payment in year 11 by the same two steps: \$200,000 at year 10 at 5% (\$100,000 at 10%), then a division by $(1 + r)^{10}$, giving the official \$122,782 (exact \$122,782.65) and \$38,554. This problem's formula, with 10 in place of 8, gives exactly those. So does the course's first example of the method, 02a pp. 28–29's GM bond: \$8,000 a year from year 6 is worth \$80,000 at year 5, and \$49,673 today (exact \$49,673.71) after dividing by 1.1^5 . ✓
- **The tempting slip.** Dating C/R at year 9 and dividing by $(1 + R)^9$ gives 12.8922 in the illustration: too low by exactly the factor $1 + R$, because it shifts every payment one year later.

WHAT VARIES

Input	Direction	Size of the effect (illustration: $R = 5\%$, $C = \$1$, value 13.5368)
The rate R (decisive)	higher R lowers the value twice over: a smaller C/R and a heavier eight-year discount	$R = 4\%$: 18.2673; $R = 6\%$: 10.4569
The delay	each extra year before the first payment divides the value by $1 + R$	first payment in year 10: 12.8922
The payment C	proportional	doubling C doubles the value

LINKS

- The same dating rule, first met: the GM bond in the Lecture 3 note (10 Sep), and *a small annuity, plus a late perpetuity* in the Section 2 note (18 Sep). WB 2.3 Q3 is worked in full in the Lecture 3 note.
- The same rule on a two-year clock: example 2 of 21 p. 7, the next problem in this note.

21 p. 7 example 2 — payments every two years, at years 3, 5, 7 and 9

Source: 21 p. 7, example 2 (the question), and 21 p. 8 (the printed answer, three steps); worked aloud with the class, ⌚ 25:44 – ⌚ 31:43. The deck's example is symbolic; the numbers in Step 9 are the note's

own illustration, not the lecture's.

SETUP

At an annual rate R , compounded annually, what is the value today of payments of C every two years for six years, starting three years from now (payments at years 3, 5, 7 and 9)?

GIVEN

Input	Value	Units	Source
Payment C	the same amount on each payment date	dollars per payment	21 p. 7
Payment dates	years 3, 5, 7, 9: four payments, two years apart	years from today	21 p. 7 (the parenthesis)
Rate R	annual net discount rate, compounded annually	per year	21 p. 7
Annuity formula	$PV_{\text{annuity}} = \frac{C}{R} (1 - (1 + R)^{-T})$ for T payments, one per period, starting one period after the valuation date	dollars	21 p. 4 item 3; 02a p. 21
One sum moved through time	$FV = PV(1 + R)^T$	dollars	21 p. 4 item 1

STRATEGY

This is Dávila's route, step for step. Recognise the stream as an annuity on a two-year clock ("What type of security is this one?" ⌚ 27:05); build the rate for that clock from the annual one ("I should be computing a rate over two years, right?" ⌚ 27:32); apply the annuity formula with four payments; date its value one two-year period before the first payment, year 1 ("The value is at 1." ⌚ 29:15); then bring it back to today across one year, at the annual rate. Adding up the four discounted payments one by one, the route Dávila called "a really sort of dumb way" ⌚ 26:55 , reaches the same number and is kept as the check (Step 8).

SOLUTION

Step 1. Draw the timeline.

Payments of C at years 3, 5, 7 and 9; nothing at years 1, 2, 4, 6 and 8.

Why this step follows: 21 p. 7's parenthesis, "payments at years 3, 5, 7, 9". The "six years" run from the first payment to the last.

Step 2. Name the security.

On a clock whose period is two years, the stream pays C once a period for four periods: an annuity.

Why this step follows: equal payments, equally spaced, a finite number of them, is 02a p. 21's annuity, with the period stretched to two years.

Step 3. The rate for one two-year period.

$$1 + R_b = (1 + R)^2$$

Why this step follows: a dollar left for two years at R a year, compounded annually, grows to $(1 + R)^2$ (item 1 with $T = 2$); R_b is the one rate that gives the same growth over a single two-year period. Dávila calls it "r-bar" ("this is my biannual rate" [027:46](#)); 21 p. 8 writes R_b ; bi-annual means once every two years.

$$R_b = (1 + R)^2 - 1$$

Why this step follows: subtract 1 from both sides.

Step 4. The annuity formula on the two-year clock.

$$PV_1 = \frac{C}{R_b} (1 - (1 + R_b)^{-4})$$

Why this step follows: item 3 with the per-period rate R_b and $T = 4$ periods. The rate and the payments must count time in the same unit, here two-year periods (21 p. 8: "We use the bi-annual rate because the annuity is paid every two years").

Step 5. Date that value.

$$\text{valuation date} = 3 - 2 = 1$$

Why this step follows: the formula's value sits one period before the first payment; the first payment is at year 3 and a period is two years. 21 p. 8: "one period before the first payment, and a period in this case accounts for two years". Some in the room answered two; the answer is one ([028:56](#) – [029:15](#)).

Step 6. Back from year 1 to today.

$$PV_0 = \frac{PV_1}{1 + R}$$

Why this step follows: from year 1 to year 0 is one year, so the one-year factor at the annual rate applies: "the value at 0 is the value at 1 divided by 1 plus r . This r is not this, okay? This r is the original one" ⌚ 29:18. 21 p. 8 words this step "discount for one period"; the period meant is one year, not Step 3's two-year period. Dividing by $1 + R_b$ would carry the value to year -1 .

Step 7. One formula in R alone.

$$(1 + R_b)^{-4} = (1 + R)^{-8}$$

Why this step follows: raise Step 3's identity to the power -4 ; four two-year periods are eight years.

$$PV_0 = \frac{C(1 - (1 + R)^{-8})}{((1 + R)^2 - 1)(1 + R)}$$

Why this step follows: substitute Step 3's R_b and the line above into Step 4, then divide by $1 + R$ as in Step 6.

Step 8. Check by adding up the four payments (the "dumb way", the note's own working). Write $q = (1 + R)^{-2}$ for two years of discounting.

$$PV_0 = \frac{C}{(1 + R)^3} + \frac{C}{(1 + R)^5} + \frac{C}{(1 + R)^7} + \frac{C}{(1 + R)^9}$$

Why this step follows: 02a p. 18, discount each payment to year 0 and then add; amounts can be added only once they are dated at the same point in time.

$$PV_0 = \frac{C}{(1 + R)^3} (1 + q + q^2 + q^3)$$

Why this step follows: factor out the first term; each later payment carries two more years of discounting, one more factor q .

$$1 + q + q^2 + q^3 = \frac{1 - q^4}{1 - q}$$

Why this step follows: the finite geometric sum.

$$1 - q = \frac{(1 + R)^2 - 1}{(1 + R)^2}$$

Why this step follows: put $1 - (1 + R)^{-2}$ over the common denominator $(1 + R)^2$.

$$PV_0 = \frac{C(1 - (1 + R)^{-8})}{((1 + R)^2 - 1)(1 + R)}$$

Why this step follows: substitute the last two lines, use $q^4 = (1 + R)^{-8}$, and cancel $(1 + R)^2$ against $(1 + R)^3$, which leaves one factor $1 + R$. This is Step 7. ✓

Step 9. A numerical check (the note's own illustration, not the lecture's: $R = 5\%$, $C = \$1$).

$$R_b = 1.05^2 - 1 = 0.1025$$

Why this step follows: Step 3; 10.25% per two years.

$$(1.1025)^{-4} = 0.676839$$

Why this step follows: four two-year periods of discounting; the same number as 1.05^{-8} .

$$PV_1 = \frac{1 - 0.676839}{0.1025} = 3.1528$$

Why this step follows: Step 4 with $C = 1$.

$$PV_0 = \frac{3.1528}{1.05} = 3.0027$$

Why this step follows: Step 6.

The four payments one by one: $0.863838 + 0.783526 + 0.710681 + 0.644609 = 3.0027$. ✓

ANSWER

$$PV_0 = \frac{1}{1 + R} \cdot \frac{C}{R_b} (1 - (1 + R_b)^{-4})$$

in dollars today, with $1 + R_b = (1 + R)^2$ and the annuity's value PV_1 dated at year 1 — 21 p. 8's answer. In the illustration ($R = 5\%$, $C = \$1$), **\$3.0027** today.

SANITY CHECK

- **Between the bounds.** Every payment is discounted by at least three and at most nine years, so the value lies between $4C/(1 + R)^9$ and $4C/(1 + R)^3$: in the illustration, $2.5784 < 3.0027 < 3.4554$. ✓
- **The two slips land on the same wrong number.** Dividing PV_1 by $1 + R_b$ (discounting "one period" of two years), or dating PV_1 at year 2 and dividing by $(1 + R)^2$, both give 2.8597, which is the value at year -1 : too low by exactly one year's discount, $3.0027/1.05$. ✓

- **The workbook's two-yearly annuity agrees.** WB 2.3 Q4 (p. 22) prices a cruise every two years with the same rule; the TWO ROUTES block below reproduces its official \$15,427.34 to the cent. ✓

WHAT VARIES

Input	Direction	Size of the effect (illustration: $R = 5\%$, $C = \$1$, value 3.0027)
The rate R (decisive)	higher R lowers the value: a higher R_b in the annuity and a heavier final year of discount	$R = 4\%$: 3.1734; $R = 6\%$: 2.8438
The start date	each year of delay divides the whole value by $1 + R$	payments at 4, 6, 8, 10: 2.8597
The number of payments	each extra payment adds its own discounted value	a fifth payment at year 11 adds 0.5847: 3.5873
The payment C	proportional	doubling C doubles the value

TWO ROUTES

Back from year 1 (the review) or forward from year –1 (the Lecture 4 note). The Lecture 4 note (17 Sep) values two-yearly streams whose first payment is **one** year away: the bi-annual perpetuity of 02b p. 10 and the workbook's cruise. There, one two-year period before the first payment is year –1, before today, so the last step carries the value **forward** one year. Here the first payment is **three** years away, one period before it is year +1, after today, and the last step goes **back**.

- **Route A, the review's** (Steps 3 to 6): the annuity at R_b with four payments, dated at year 1, divided by $1 + R$.
- **Route B, forward from year –1** (*the note's own construction, to show the Lecture 4 direction on this stream*): add a payment of C at year 1 that the asset does not make. The five payments at 1, 3, 5, 7, 9 are an annuity on the two-year clock, valued at year –1:

$$V_{-1} = \frac{C}{R_b} (1 - (1 + R_b)^{-5})$$

Why this step follows: item 3 with $T = 5$; the first of the five payments is at year 1, one two-year period after year –1.

$$PV_0 = V_{-1}(1 + R) - \frac{C}{1 + R}$$

Why this step follows: carry the year -1 value forward one year at the annual rate, then take away the added payment, worth $C/(1 + R)$ today.

In the illustration, $V_{-1} = 3.7667$, forward one year 3.9550, less 0.952381, gives 3.0027, as in Route A. ✓

Why they agree. Both are the present value of the same four payments; Route B adds a payment and later removes exactly its value. And the rule behind both is one rule: the annuity formula at the two-year rate puts its value one two-year period before the first payment, and a single one-year move at the annual rate takes that value to today, forward if it lies before today, back if it lies after. The workbook's cruise, WB 2.3 Q4 (p. 22), is route B's direction with real numbers: \$3,000 at years 1, 3, ..., 41 at 10% a year, a two-year rate of $1.1^2 - 1 = 21\%$, 21 payments valued at year -1 , then multiplied by 1.1: the official **\$15,427.34**.

Which the review used: route A, "The value is at 1." then "divided by 1 plus r" (⌚ 29:15 — ⌚ 29:18).

LINKS

- The one-year move **forward** on a two-year clock, first met: 5. *The bi-annual perpetuity* in the Lecture 4 note (17 Sep), with WB 2.3 Q4 (the cruise) worked there; *The hardest problem today: a step-up perpetuity every two years* in the Section 2 note (18 Sep).
- The one-year move **back**, this problem's direction, already met on a longer clock: 9. *A perpetuity on a multi-year clock* in the Section 2 note (18 Sep), whose formula value lands at year 1 and is discounted back one year at the annual rate.
- The same dating rule on a one-year clock: example 1 of 21 p. 7, the previous problem in this note.

Group 3. Diversification with many assets (21 p. 12)

The derivation the lecturer recalled and the formula sheet does not print. From item 8 with every weight equal to $1/N$, the portfolio variance splits into $1/N$ times the average variance plus $(1 - 1/N)$ times the average covariance; as N grows the first term vanishes and the second tends to the average covariance — zero if the assets are uncorrelated. The problem shows why the split needs equal weights (the condition 21 p. 12 omits), checks the two limits against WB 3.1 Q18 and Q19, and illustrates with the Lecture 7 note's numbers.

The review's diversification limit — an equally weighted portfolio as N grows

Source: 21 p. 12 (the equal-weight variance split) and 21 p. 4 item 8; recalled aloud, ⌚ 36:54 —

⌚ 37:45 . No question is printed: this works the derivation Dávila recalled, whose result 03d pp. 8–11 teach (the Lecture 7 note). The numbers in Step 8 are the Lecture 7 note's illustration, not the lecture's.

SETUP

A portfolio holds $1/N$ in each of N risky assets: what is its variance, and what does that variance approach as N grows, first when the assets are uncorrelated and then in general?

GIVEN

Input	Value	Units	Source
Portfolio variance	$\text{Var}[R_p] = \sum_i w_i^2 \sigma_i^2 + 2 \sum_i \sum_{j>i} w_i w_j \rho_{ij} \sigma_i \sigma_j$	return squared	21 p. 4 item 8
Weights	$w_i = 1/N$ for every asset	—	03d p. 10; ⌚ 37:06
Covariance	$\rho_{ij} \sigma_i \sigma_j = \text{Cov}[R_i, R_j]$	return squared	the definition of correlation
Average variance	$\overline{\sigma^2} = \frac{1}{N} \sum_i \sigma_i^2$	return squared	21 p. 12 ("Avg. Variance")
Average covariance	$\overline{\text{Cov}}$: the sum of the $\text{Cov}[R_i, R_j]$ over pairs $j > i$, divided by the number of pairs, $\frac{N(N-1)}{2}$	return squared	21 p. 12 ("Avg. Covariance")

STRATEGY

Follow Dávila's order (⌚ 37:06 – ⌚ 37:35): put "one over n in all the stocks" into item 8 and regroup the two sums as averages. With zero correlations only the first part survives, "one over n times the constant", and it vanishes as N grows. In general the weight on the average variance goes to zero and the weight on the average covariance goes to one: "This goes to zero, this goes to one, so what we have next is the average covariance." The split is not on 21 p. 4's list; it takes four lines from item 8, which is the skill this exercise trains.

SOLUTION

Step 1. Put the equal weights into item 8.

$$\sigma_p^2 = \sum_{i=1}^N \frac{\sigma_i^2}{N^2} + 2 \sum_{i=1}^N \sum_{j>i} \frac{\text{Cov}[R_i, R_j]}{N^2}$$

Why this step follows: with $w_i = 1/N$, both w_i^2 and $w_i w_j$ equal $1/N^2$; and $\rho_{ij} \sigma_i \sigma_j$ is $\text{Cov}[R_i, R_j]$.

Step 2. The first sum as an average.

$$\sum_{i=1}^N \frac{\sigma_i^2}{N^2} = \frac{1}{N} \overline{\sigma^2}$$

Why this step follows: split $1/N^2$ into $\frac{1}{N} \cdot \frac{1}{N}$; one $\frac{1}{N}$ times the sum of the variances is their average.

Step 3. Count the pairs.

$$\sum_{i=1}^N \sum_{j>i} \text{Cov}[R_i, R_j] = \frac{N(N-1)}{2} \overline{\text{Cov}}$$

Why this step follows: there are $\frac{N(N-1)}{2}$ pairs with $j > i$, and a sum is its number of terms times their average.

Step 4. The second sum as an average.

$$\frac{2}{N^2} \cdot \frac{N(N-1)}{2} \overline{\text{Cov}} = \left(1 - \frac{1}{N}\right) \overline{\text{Cov}}$$

Why this step follows: the 2s cancel, and $\frac{N(N-1)}{N^2} = \frac{N-1}{N} = 1 - \frac{1}{N}$.

Step 5. Put the two parts together.

$$\sigma_p^2 = \frac{1}{N} \overline{\sigma^2} + \left(1 - \frac{1}{N}\right) \overline{\text{Cov}}$$

Why this step follows: add Steps 2 and 4. This is 21 p. 12's formula, and Steps 1 to 4 show it needs $w_i = 1/N$ (03d p. 10 states the condition; 21 p. 12 does not print it).

Step 6. Uncorrelated assets.

$$\sigma_p^2 = \frac{1}{N} \overline{\sigma^2}$$

Why this step follows: every covariance is zero, so their average is zero and the second part of Step 5 drops out.

As N grows, a fixed average variance divided by N goes to zero: all the risk is diversified away.

Step 7. The general case.

$$\sigma_p^2 \rightarrow \overline{\text{Cov}} \quad (N \rightarrow \infty)$$

Why this step follows: in Step 5, $\frac{1}{N} \rightarrow 0$ and $1 - \frac{1}{N} \rightarrow 1$, while the two averages stay finite.

What is left is how the assets move together: risk that diversification cannot remove.

Step 8. A numerical check (the Lecture 7 note's illustration: identical assets with $\sigma = 40\%$ and $\rho = 0.25$, so $\overline{\sigma^2} = 0.16$ and $\overline{\text{Cov}} = 0.25 \times 0.16 = 0.04$).

N	$\frac{1}{N} \times 0.16$	$(1 - \frac{1}{N}) \times 0.04$	σ_p^2	σ_p
1	0.16	0	0.16	40%
10	0.016	0.036	0.052	22.80%
100	0.0016	0.0396	0.0412	20.30%
$\rightarrow \infty$	0	0.04	0.04	20%

With zero correlation (the same note's other curve, $\sigma = 20\%$), Step 6 gives $\sigma_p = 20\%/\sqrt{N}$: 10% at four assets, 2% at a hundred, and zero in the limit.

ANSWER

$$\sigma_p^2 = \frac{1}{N} \overline{\sigma^2} + \left(1 - \frac{1}{N}\right) \overline{\text{Cov}} \rightarrow \overline{\text{Cov}}$$

(variance, in return squared), for equal weights $w_i = 1/N$ only. **Uncorrelated assets: the variance goes to zero. In general: it goes to the average covariance.**

SANITY CHECK

- **One asset.** $N = 1$ gives $\sigma_p^2 = \sigma_1^2$: a one-asset portfolio is the asset. ✓
- **The workbook agrees.** WB 3.1 Q18 (p. 40): adding more and more uncorrelated securities, the variance goes to (c) zero; WB 3.1 Q19 (pp. 40–41): correlated securities, (d) the average covariance among the securities. ✓
- **The condition matters** (the note's own weights on the illustration). Hold two of the identical assets 75/25 instead of 50/50. Item 8 gives $0.75^2(0.16) + 0.25^2(0.16) + 2(0.75)(0.25)(0.04) = 0.115$ (33.91%), but the split with $N = 2$ gives 0.10 (31.62%), which is the 50/50 portfolio's variance. So 21 p. 12's formula describes the equally weighted portfolio only.

WHAT VARIES

Input	Direction	Size of the effect (illustration: $\sigma = 40\%$, $\rho = 0.25$)
The average covariance (decisive for large N)	a higher average covariance raises the floor that diversification cannot pass	$\rho = 0.5$: floor $\sqrt{0.5 \times 0.16} = 28.28\%$, against 20%
The number of assets N	more assets move σ_p toward the floor, fast at first, slowly later	22.80% at 10 assets, 20.30% at 100
The average variance	matters only while N is small: its weight $\frac{1}{N}$ vanishes	—

LINKS

- The same result taught in full, with its figure: 11. *Diversification with equal weights: independent returns* and 12. *Correlated returns: the average covariance* in the Lecture 7 note (29 Sep), where WB 3.1 Q18 and Q19 are the Self-checks.

Group 4. The CAPM's equations and consequences (21 pp. 17–18)

The numerical CAPM question of the review: the SML written once per stock gives two equations in the risk-free rate and the market risk premium; 21 p. 18 subtracts them. The problem carries the Lecture 8 note's rise-over-run route as its second way to the same numbers, and the check against WB 4.2 Q1's official answer. The review's second question — the three true/false claims, argued with diagrams — is a concept item and is in the Self-check, with the printed algebra for claim 1, the two diagrams described so you can draw them, and the Lecture 8 note's one-line identity as the other route.

21 p. 17 question 1 — the risk-free rate and the market risk premium from two stocks

Source: 21 p. 17, question 1, and 21 p. 18 (the printed answer); set up aloud, ⌚ 46:28 – ⌚ 47:14 . The same two stocks, with their labels swapped, are WB 4.2 Q1 (p. 73), worked in the Lecture 8 note (1 Oct).

SETUP

The CAPM holds; stock A has an expected return of 8% and a beta of 1, stock B an expected return of 10% and a beta of 1.5. Find the risk-free rate and the market risk premium.

GIVEN

Input	Value	Units	Source
Stock A	$\mathbb{E}[R_A] = 8\%, \beta_A = 1$	expected return per period; beta has no units	21 p. 17
Stock B	$\mathbb{E}[R_B] = 10\%, \beta_B = 1.5$	as above	21 p. 17
The SML	$\mathbb{E}[R_i] = R_f + \beta_i (\mathbb{E}[R_M] - R_f)$, for every security when the CAPM holds	—	21 p. 4 item 10; 21 p. 13
Unknowns	R_f and the market risk premium $\mathbb{E}[R_M] - R_f$	per period	21 p. 17

STRATEGY

21 p. 18's route, which Dávila set up aloud: "The SML gives one equation per stock", so two stocks give two equations in the two unknowns. Dávila left the algebra open ("you can solve it the way you want" ⌚ 46:56 ; "be creative on how you're going to use the equations" ⌚ 47:11); the printed answer subtracts the first equation from the second, which removes R_f and leaves the premium alone. R_f then follows from either equation, and $\mathbb{E}[R_M]$ from the two together.

SOLUTION

Step 1. The SML for stock A.

$$0.08 = R_f + 1 \times (\mathbb{E}[R_M] - R_f)$$

Why this step follows: the CAPM holds, so A lies on the SML; put in A's expected return and beta.

Step 2. The SML for stock B.

$$0.10 = R_f + 1.5 \times (\mathbb{E}[R_M] - R_f)$$

Why this step follows: the same line, with B's expected return and beta.

Step 3. Subtract Step 1 from Step 2.

$$0.10 - 0.08 = (1.5 - 1) \times (\mathbb{E}[R_M] - R_f)$$

Why this step follows: subtract left side from left side and right side from right side; R_f appears once on each right side and cancels.

Step 4. Evaluate the differences.

$$0.02 = 0.5 \times (\mathbb{E}[R_M] - R_f)$$

Why this step follows: $0.10 - 0.08 = 0.02$ and $1.5 - 1 = 0.5$ — 21 p. 18's line.

Step 5. The market risk premium.

$$\mathbb{E}[R_M] - R_f = \frac{0.02}{0.5} = 0.04$$

Why this step follows: divide both sides by 0.5.

Step 6. The risk-free rate.

$$R_f = 0.08 - 0.04 = 0.04$$

Why this step follows: Step 1 now reads $0.08 = R_f + 0.04$; subtract 0.04 from both sides.

Step 7. The market's expected return.

$$\mathbb{E}[R_M] = 0.04 + 0.04 = 0.08$$

Why this step follows: add the premium to R_f . Equivalently, stock A has $\beta_A = 1$, so it earns the market's expected return (21 p. 18).

Step 8. Check with stock B.

$$0.04 + 1.5 \times 0.04 = 0.10$$

Why this step follows: B's point must lie on the same line; it does. ✓

ANSWER

$\mathbb{E}[R_M] - R_f = 4\%, \quad R_f = 4\%, \quad \mathbb{E}[R_M] = 8\%$

per period — 21 p. 18's answer. **The workbook agrees:** WB 4.2 Q1 (p. 73) prints the same two stocks with the labels swapped (its A: 10% and beta 1.5; its B: 8% and beta 1), asks only for R_f , and answers " $R_f = 4\%$ ", adding that "the expected return on the market portfolio is $\mathbb{E}[R_M] = .04 + 1(.04) = 8\%$ ". (Its question prints " $\beta_A = 1$ " for the second stock; the Lecture 8 note rules on that slip.)

SANITY CHECK

- **The beta-1 stock earns the market's return.** Any asset with $\beta = 1$ sits at the market's point on the SML, so $\mathbb{E}[R_M] = \mathbb{E}[R_A] = 8\%$, whatever B is. ✓

- **Order.** $R_f = 4\% < \mathbb{E}[R_A] = 8\% < \mathbb{E}[R_B] = 10\%$, in the order of the betas 0, 1 and 1.5: the SML slopes upward, as it must with a positive premium. ✓
- **Size.** A 4% premium sits at the low end of 04a p. 14's "4% to 6% in practice". ✓

WHAT VARIES

Solving Steps 3 to 6 in general (*the note's own*):

$$\mathbb{E}[R_M] - R_f = \frac{\mathbb{E}[R_B] - \mathbb{E}[R_A]}{\beta_B - \beta_A}$$

Why this step follows: Step 3 in symbols, divided by $\beta_B - \beta_A$.

$$R_f = \frac{\beta_B \mathbb{E}[R_A] - \beta_A \mathbb{E}[R_B]}{\beta_B - \beta_A}$$

Why this step follows: substitute the premium into $R_f = \mathbb{E}[R_A] - \beta_A (\mathbb{E}[R_M] - R_f)$ and put both terms over $\beta_B - \beta_A$.

Input	Direction	Size of the effect
$\mathbb{E}[R_B]$	higher steepens the line: the premium rises 2 points and R_f falls 2 points per point on B	11%: premium 6%, R_f 2%
$\mathbb{E}[R_A]$	higher flattens the line: the premium falls 2 points and R_f rises 3 points per point on A	9%: premium 2%, R_f 7%
β_B (decisive when the betas are close)	a wider gap between the betas spreads the same 2-point rise over more beta	$\beta_B = 2$: premium 2%, R_f 6%

If the two betas were equal, Step 3 would read $0.02 = 0$: two stocks with the same beta and different expected returns cannot both lie on one SML.

TWO ROUTES

Subtracting the equations (the review) or rise over run (the Lecture 8 note).

- **Route A, the review's** (21 p. 18; Steps 1 to 6): write the SML twice and subtract.
- **Route B, the Lecture 8 note's** (its working of WB 4.2 Q1, following the workbook): every security lies on one straight line in the $(\beta, \mathbb{E}[R])$ plane, with intercept R_f and slope $\mathbb{E}[R_M] - R_f$. Two points fix the line:

$$\mathbb{E}[R_M] - R_f = \frac{0.10 - 0.08}{1.5 - 1} = 0.04$$

Why this step follows: the slope of a line through two points is rise over run.

Then the intercept from B's point: $R_f = 0.10 - 1.5 \times 0.04 = 0.04$.

Why they agree. Step 3's subtraction *is* rise over run written as an equation: $\mathbb{E}[R_B] - \mathbb{E}[R_A] = (\beta_B - \beta_A)(\mathbb{E}[R_M] - R_f)$, and dividing by $\beta_B - \beta_A$ is the slope formula. **Which the review used:** 21 p. 18 prints the subtraction, and Dávila accepted either route ("you can solve it the way you want" ⌚ 46:56).

LINKS

- WB 4.2 Q1, the same line with the labels swapped: Lecture 8 note (1 Oct), Worked problems.
- The second question on 21 p. 17, three claims about the same model: the next item in this note.

Group 5. The Block 5 key concept this meeting teaches

The workbook's Block 5 key-concepts list ends with "Do Mutual Funds beat the market?"; the eight concepts before it belong to the Lecture 11 note. The ninth, answered from 05a pp. 23–26, is in Key takeaways.

Common mistakes

Dates and clocks

- **Dating a perpetuity or annuity at its first payment.** The formula's value sits one period *before* the first payment: a perpetuity starting at year 9 is worth C/R at year 8, and the discount to today is $(1 + R)^8$, not $(1 + R)^9$ (21 p. 8). "The main issue always is how to clear the timings, like when is the value computed." ⌚ 23:47.
- **Using the annual rate on a two-year stream.** Payments every two years need the two-year rate, $R_b = (1 + R)^2 - 1$, in the annuity formula — "I should be computing a rate over two years" ⌚ 27:24 — and the formula's value then sits one *two-year* period before the first payment (year 1 for a first payment at year 3; "Some people are saying two" ⌚ 28:46).
- **Taking the last step on the wrong clock.** From year 1 to today is one year: divide by $1 + R$, "This r is not this ... This r is the original one" ⌚ 29:18. Dividing by $1 + R_b$, or dating the annuity at year 2 and dividing by $(1 + R)^2$, both land on the year -1 value — too low by a factor $1 + R$. 21 p. 8's "discount for one period" means one year.

- **Adding dollars from different dates.** "value of money at 0 and money at 1 are different things ... apples and bananas, different objects, I cannot put them together" ⌚ 30:30 . Move every amount to one date, then add.
- **Confusing bi-annual with semi-annual.** In this course "bi-annual" is once every two years (21 p. 8; 02b p. 10).

Formulas on and off the sheet

- **Expecting the equal-weight variance split on the paper.** 21 p. 4 does not carry 21 p. 12's formula; derive it from item 8 by setting $w_i = 1/N$ — "If you do one over n, you get exactly" the same equation ⌚ 37:49 .
- **Reading 21 p. 12's split as general.** It holds for equal weights only; the page dropped the condition (03d p. 10).
- **Treating "learn it by heart" as a list.** Results not on the sheet that you cannot derive from it are to be learned; no count of them is on record (§1.1).

Lines, slopes and prices of risk

- **Calling the SML's slope the "expected return on the market"** — the second name the lecturer gave it at ⌚ 43:16 . It is the market risk premium, $\mathbb{E}[R_M] - R_f$ (21 p. 15); the CML's slope is the market's Sharpe ratio, $(\mathbb{E}[R_M] - R_f)/\sigma_M$.
- **Putting a single stock on the CML.** Only the market and its mixes with the risk-free asset lie on the CML; every other security or portfolio lies below it, however high its expected return — Q and K on 21 p. 15 have the same expected return at different standard deviations and sit on the SML at the same beta.
- **Mistaking the SCL for a model equation.** The security characteristic line is a regression on data, one stock at a time, whose slope *estimates* beta; the CAL, CML and SML are what the model says ⌚ 45:12 .
- **Saying a CAL is never the efficient frontier, or always is.** With a risk-free asset the efficient frontier is the CAL through the tangency portfolio; a CAL through any other risky portfolio is dominated (03c p. 17; 03d p. 4; §17.1).
- **Ranking stocks by Sharpe ratio against the market.** No single stock's Sharpe ratio exceeds the market's; the stock's own volatility includes idiosyncratic risk that earns nothing. Rotate the line from R_f up to the tangency portfolio and every stock is on or below it (21 p. 18, claim 2).
- **Forgetting the sign condition in claim 3.** A negative Sharpe ratio forces a negative beta only when the market risk premium is positive (21 p. 18; §14.2's Warning).

Words that are not what they sound like

- **"Negative risk."** A negative-beta stock reduces a portfolio's risk; it is "a wonderful hedge", "Risk reducing" ⌚ 60:09 — not riskless and not negatively risky.
- **"Efficient", "optimal", "feasible", "desirable".** Feasible is what you can reach; efficient is the top of it; desirable is what you want; optimal is where the two meet — one point (03a p. 13; 03b p. 20; §15.4).
- **Alpha as an arbitrage.** It is not one: alpha is compensation for risk beyond the model's, and "you can lose money doing alpha investing" ⌚ 59:20 ; an arbitrage is riskless and costs nothing (01a pp. 3–7).
- **Low price, high return — without the fixed payoff.** The relation holds *given payoffs*; if the payoff moves too, "We don't know" ⌚ 63:56 .
- **Taking "5% of people are in the top 5%" ⌚ 06:41 as evidence about skill.** The slide's point is different: five of a hundred *unskilled* managers will look significant at the 5% level (05a p. 25).

Reading the paper

- **Answering true or false without the reason.** "just true or false comes zero, it's all about the explanation" ⌚ 13:03 ; one to three lines; no buzzword fishing.
- **Leaving a numerical item blank.** "if you don't write anything, you get a zero" ⌚ 12:32 ; partial credit is given (both recordings).
- **Assuming "the CAPM holds" means less than everything.** It means the assumptions, the CML and the SML are all true at once ⌚ 64:58 .
- **Expecting the printed "27/45 years" to match a 44-year chart.** It does not; the gap is a slide-internal discrepancy, not a fact to reconcile (05a p. 24).

Key takeaways

- **The paper:** Tuesday 20 October, in class; eight numerical items and twelve short questions at five points each, 100 points, 75 minutes; a calculator (bring two), no phones, tablets or computers; closed book (21 p. 2; syl p. 4). Short questions are the workbook's concept questions in short-answer form, and the explanation is the mark; numerical questions are like the class exercises and the practice problems (21 p. 3).
- **The sheet:** eleven equations are printed on the paper — 21 p. 4, word for word the formula sheet's Midterm list. Anything else you need and cannot derive from them, learn by heart; the equal-weight variance split is derived from item 8 with $w_i = 1/N$.
- **The scope the deck records:** Blocks 1–5, with the core "blocks three, four, and five" ⌚ 20:51 ; Block 5 through the mutual-fund pages. Said not to be asked: a question of the form *is this an arbitrage or not?*, the time-series test, a derivation of the SML, and 21 p. 17's first claim (followed by "maybe like

one or two questions of this" ⌚ 48:57). Later decks are absent from the deck; the equity-valuation video's status is open.

- **Block 5's last word:** after costs, index funds beat the average active fund by about a point a year and in most years; before costs, managers' picks beat the index by 1.5 points, but costs take 2.3; a handful of significant winners is what luck produces among a hundred unskilled managers. "if you don't have any insight into what to do with the money ... just to put in a well-diversified index fund, you're going to do well" ⌚ 08:57 .
- **Discounting is dating:** a formula's value sits one period before the first payment, on the clock the rate is quoted in; one move at the annual rate takes it to today, back if the date is after today, forward if before. $PV_0 = \frac{C/R}{(1+R)^8}$ for the deferred perpetuity; $PV_0 = \frac{1}{1+R} \frac{C}{R_b} (1 - (1 + R_b)^{-4})$ with $1 + R_b = (1 + R)^2$ for the two-yearly annuity.
- **Portfolio choice in three pictures:** the two-asset set bends further left the lower the correlation (a riskless mix at -1 , a straight line at $+1$); a risk-free asset turns the picture into capital allocation lines, the best of them tangent at the tangency portfolio, which everyone holds in a mix set only by risk aversion; with many assets the individual stocks fall inside the frontier, and an equally weighted portfolio's variance tends to the average covariance.
- **The CAPM in two lines:** tangency = market, so the CML runs through the market with slope SR_M ; and $\mathbb{E}[R_i] = R_f + \beta_i(\mathbb{E}[R_M] - R_f)$ — a high-beta stock is unpleasant to hold, so its price is low and its expected return high. Both slopes are prices of risk; beta is a quantity of risk; excess return is price times quantity.
- **Two CAPM questions:** two SML equations, subtract, $\mathbb{E}[R_M] - R_f = 4\%$, $R_f = 4\%$, $\mathbb{E}[R_M] = 8\%$; and three true claims — every stock trades return for marginal risk at the CML's rate, no stock's Sharpe ratio beats the market's (rotate the line to the tangency), a negative Sharpe ratio is a negative beta (below the R_f line on the SML, given a positive premium). "draw a diagram, draw a diagram" ⌚ 51:59 .
- **The students' questions, in one line each:** the MVP in general is $w_1^G = (\sigma_2^2 - \sigma_{12})/(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12})$ and "in principle yes" ⌚ 52:20 ; know the correlation cases; factor models multiply because every model's alphas invite a new factor, and data snooping is the limit; feasible–efficient–desirable–optimal are one picture; the four lines are three model equations and one regression; a negative beta is a hedge; only the cross-sectional test was reviewed; a CAL is the efficient frontier only when it is the best one; low price means high return given the payoff; "the CAPM holds" means everything is true; the SML comes from indifference at the margin, proved in the handout, not to be derived on the paper; the workbook is harder than class on purpose.

WB 5.3 — the Block 5 key concept this meeting covers

Source: Workbook Block 5.3, p. 93. The workbook closes Block 5 with nine terms and no definitions. The first eight are the Lecture 11 note (13 Oct)'s; **the ninth is this meeting's**. It is printed below with the statement that makes it usable, from the pages where the deck states it (05a pp. 23–26).

#	Key concept	The statement that makes it usable	Source
9	Do Mutual Funds beat the market?	Not after costs, on average. Active funds pick stocks and trade often (turnover about 100% a year, expense ratios about 1% plus 0.5 to 1% in trading costs); passive funds hold an index (turnover about zero, about 0.2%). After costs the index return averages 1% above the average fund's and beats the funds in 27 of 45 years. Before costs, the stocks funds hold do beat the index (1975–1994: 16.9% gross against the S&P 500's 15.4%), but investors receive 14.6% net. A few outperformers in a large group can be luck: 5 out of 100 unskilled managers will look significant at the 5% level.	05a pp. 23–26

WHERE IT IS PRACTISED

- The 05a p. 23 and 05a p. 25 self-checks, in this note.
- The guest lecture's question on 100 strategies tested at 5%, in the Lecture 10 note (8 Oct).

THE BULLETS THIS MEETING DOES NOT COVER

- **Bullets 1–8** (the efficient market hypothesis and its three forms, the random walk, long-run predictability, event studies, anomalies, the joint test problem, data-mining, the Grossman–Stiglitz paradox) are the Lecture 11 note (13 Oct)'s.

LINKS

- **The full Block 5 checklist** is assembled once, on the course review page, from the per-meeting subsets.

Connections

Backward. This meeting touches every earlier Fin note, and each section above links the one that teaches its page. The spine: the time value of money and the dating rule (Lecture 3, Section 2, Lecture 4); the statistics rules and the two-asset set (Lecture 5); preferences and the capital allocation line (Lecture 6); the tangency

portfolio, two-fund separation and diversification with many assets (Lecture 7); the CAPM's equilibrium, its two lines, beta and the variance split (Lecture 8); alpha, capital budgeting, the tests and the factor models (Lecture 9); the flat SML from the practitioner's side, betting against beta and data mining (Lecture 10); market efficiency to p. 22 (Lecture 11); market making, spreads, orders and short sales (Lecture 2); the course's rules and the four principles, no arbitrage among them (Lecture 1). Three ideas the review named as running through the course: **compensation for risk**, from the bid-ask spread to the SML; **"risk is always about covariance"** ⌚ 21:49, from the two-asset cross term to the average covariance to beta; and **the dating of a formula's value**, from the first perpetuity to the two-year clock.

What this meeting adds to the earlier notes (each a one-line correction or signal for its note, not applied there): the review's *back from year 1* route beside the *forward from year - 1* route of the Lecture 4 note (§5.3 and the cruise) and the Section 2 note's §14 — the back route itself first met in the Section 2 note's §9 (§5.2); 21 p. 17's question 1 as WB 4.2 Q1 with the labels swapped, solved by subtraction rather than slope (§14.1); WB 4.1 Q14 answered by diagrams rather than the identity $SR_i = \rho_{iM} SR_M$ (§14.2); the exam signals at each page, above all "I always ask some discounting questions" ⌚ 23:34, "there are always questions on this" for the two-asset case ⌚ 32:43, "Super important result" for two-fund separation ⌚ 35:59, "I could ask you this question" for the factor models ⌚ 53:16, and the four things said not to be asked (§1.5): an arbitrage question "like, oh, is this an arbitrage or not?" ⌚ 58:38; the time-series test ⌚ 61:28; a derivation of the SML ⌚ 65:35; 21 p. 17's first claim, "I'm not going to ask this one over" ⌚ 47:51 — followed by "maybe like one or two questions of this" ⌚ 48:57.

Forward. The midterm on Tuesday; then equity valuation by video (06a), arbitrage "after the break" (07a–07b), and fixed income (08a–08d) — Continues in.

Self-check

⌚ 03:49 · the meeting's concept items: the two printed questions of 05a pp. 23 and 25 that were put to the room, and the three true/false claims of 21 p. 17; each answer is hidden until revealed, and each says why the tempting wrong answer is wrong

The inline self-checks at the end of each section above test that section; these three are the meeting's own concept exercises, in lecture order. The first two carry the fund evidence of §2 — the comparison after costs, the comparison before costs, and the luck-against-skill count. The third is the review's second CAPM question (§14.2): the midterm's short-question form, "just true or false comes zero, it's all about the explanation" ⌚ 13:03, argued the way the room argued it — with a diagram first — and then checked with the Lecture 8 note's algebra.

Self-check — 05a p. 23: which type of fund normally does better?

Source: *05a p. 23*, its closing question, put to the room ⌚ 03:49 ; answered on *05a p. 24* and aloud,

⌚ 04:01 – ⌚ 05:23 .

QUESTION

05a p. 23 sets two kinds of fund side by side:

- **Active funds**, run by portfolio managers who bet on individual stocks, macro themes and style strategies; they trade often (turnover of about 100% a year) and pay for research and trading: expense ratios of about 1% a year, plus trading costs of about 0.5 to 1% a year. Example: Fidelity Magellan Fund.
- **Passive funds**, which mechanically hold a basket of stocks; turnover about zero, expense ratios of maybe 0.2% a year. Example: Vanguard S&P 500 Index Fund.

"Which type of fund normally does better?" Answer for the investor, after costs, and say what the evidence is.

ANSWER

Reveal the answer

Passive (index) funds, on average, after costs — by a small margin

The evidence (05a p. 24). A chart of the yearly returns to investors in active funds against the Wilshire 5000, a broad index of about 5,000 stocks, from 1971 to 2014, and its bullet: "Average index return is 1% above average fund return, and index beats funds in 27/45 years". Dávila stressed that the comparison is after costs ("these are comparisons once you've subtracted expenses" ⌚ 04:01) and drew the conclusion: "the index funds, they are marginally better" ⌚ 05:14 .

Why (explanation, from 05a pp. 23 and 26). The gap is the cost gap. Read together, p. 23's figures put an active fund's yearly costs at about 1.5 to 2% (1% in fees plus 0.5 to 1% in trading) against about 0.2% for an index fund. Before costs, the stocks active funds hold do beat the index (05a p. 26, the next self-check); after costs, the advantage is gone and then some.

A check on the chart. The chart shows 44 years of bars (1971 to 2014), not 45, so the bullet's "27/45" cannot be confirmed bar by bar. Read off the chart, the index is clearly ahead in 23 years and the funds in 16, with 5 years too close to call; the average gap, about 1 point a year in the index's favour, agrees with the bullet.

Why the other answers are wrong

Answer	Why it is wrong
Active funds, because their managers pick better stocks	Picking skill shows up only before costs. After fees and trading costs, which are what the investor actually pays, the average active fund trails the index by about 1 point a year (05a p. 24).
No difference: the yearly bars look alike	The difference is small and noisy — Dávila: "you don't see very strong difference" ⌚ 04:49 — but it is not zero and it runs one way: on average the index wins, and it wins in more years than it loses.

The course's theory points the same way: under two-fund separation and the CAPM every investor holds the market portfolio, which a passive index fund delivers at low cost (Dávila at ⌚ 03:13 , recalling those results).

LINKS

- Before costs, the other side of the comparison: the 05a p. 25 self-check, next in this note.

Self-check — 05a p. 25: can fund managers beat the index before costs, and do the few winners show skill?

Source: *05a p. 25*, its two numbered questions, put to the room ⌚ 05:35 and ⌚ 05:45 ; answered on *05a p. 26* and aloud, ⌚ 07:41 – ⌚ 10:12 , and on p. 25 itself.

QUESTION

1. "Can mutual fund managers pick stocks that beat the index before costs?"
 2. "Even if most active managers underperform, are there some that outperform?" And if a few funds do show statistically significant outperformance, should that impress you?
-

ANSWER

Reveal the answers

1. Yes, on average, before costs — but the investor does not see it after costs

Wermers (Journal of Finance, 2000) looks at the actual stocks active funds hold (05a p. 25). The bottom row of its Table II, for 1975–1994, in % a year (05a p. 26):

Return	% a year
S&P 500	15.4
Funds, gross (before fees and costs)	16.9
Funds, net (what investors receive)	14.6

(The fund rows are averages weighted by fund size, "TNA-Avg" in the table.)

$$16.9 - 15.4 = 1.5$$

Why this step follows: gross fund return minus the index: before costs, the stocks the funds pick beat the index by 1.5 points a year.

$$15.4 - 14.6 = 0.8$$

Why this step follows: the index minus the net fund return: after costs, investors trail the index by 0.8 points a year.

$$16.9 - 14.6 = 2.3$$

Why this step follows: gross minus net is what fees and costs take, the sum of the two gaps above ($1.5 + 0.8 = 2.3$).

05a p. 26 puts it in one line: "Stocks held by mutual funds beat the index, but the net fund returns are beaten by the index". Dávila on the 2.3: "That 2.3% that you're losing there, that's the money that the manager is getting"  10:12.

Why no is wrong. Before costs the funds' stocks beat the index by 1.5 points a year on average: there is some stock-picking skill. **Why yes, so buy active funds is wrong.** The skill is worth less than it costs: net of fees the funds earn 14.6% against the index's 15.4%.

2. Some may — but a few winners among many funds prove little

05a p. 25: individual funds may outperform the market ("e.g. Yale endowment"), but there is a "Luck vs. skill problem": in a large population of unskilled managers, a few will always perform particularly well by

luck, and "5 out of 100 will perform well enough to be significant at the 5% level". So the best few funds' significant outperformance should not impress.

$$100 \times 0.05 = 5$$

Why this step follows: each of 100 unskilled managers has a 5% chance of passing a test at the 5% level, and an expected count is the sum of the chances; no independence is needed for this.

If the 100 results are also independent, the chance that **at least one** passes is $1 - 0.95^{100} = 0.9941$: a winner that tests significant is almost certain to appear by luck alone (the guest lecture's 100 strategies, worked in the Lecture 10 note (8 Oct)).

Dávila made the point in different words: "always, 5% of people are in the top 5%, and they could just be lucky"  06:41. That is true of any ranking by construction, whatever the skill; the slide's claim is the sharper one, about false positives in a significance test. Both say that a top position alone is not evidence of skill. What would be: consistency — "if you're consistent over time, people say you're skilled"  07:33.

Why yes, the winners prove skill is wrong. Among 100 funds with no skill, about 5 will look significant at the 5% level, and in a single sample they look exactly like skilled winners. **Why no fund ever outperforms is wrong.** Some do: the slide's own example is the Yale endowment. The question is whether the outperformance persists, which a single good period cannot show.

LINKS

- After costs, the comparison of the previous self-check (05a p. 24): the index wins on average.
- The same count of lucky winners as a data-mining problem: the guest lecture's question on 100 strategies tested at 5%, Lecture 10 note (8 Oct).

Self-check — 21 p. 17 question 2: three claims about the CAPM, true or false

Source: 21 p. 17, question 2, and 21 p. 18 (the answers); argued aloud,  47:17 –  52:04. The three claims are WB 4.1 Q14's (p. 68), re-posed one at a time as true or false: the form of the midterm's short questions, where the explanation counts (21 p. 2: "You need to explain your answer (very important)").

QUESTION

Suppose the assumptions behind the Capital Asset Pricing Model (CAPM) hold. Which of the following statements are true or false? Explain each answer; a diagram is welcome.

1. If you hold the market portfolio, adding or subtracting a little of any stock should give the same increase in expected return per unit of additional standard deviation as the Capital Market Line.
2. All stocks must have Sharpe ratios that are less than or equal to that of the market portfolio.
3. A stock with a negative Sharpe ratio must have a negative market beta.

ANSWER

Reveal the answers

All three are true

21 p. 18 marks all three true; the workbook's answer to the same three claims is "(d) All of the above" (WB 4.1 Q14, p. 68). Notation: stock i 's Sharpe ratio is $SR_i = (\mathbb{E}[R_i] - R_f) / \sigma_i$; the market's is $SR_M = (\mathbb{E}[R_M] - R_f) / \sigma_M$, the slope of the CML (21 p. 16, "Sharpe Ratio of the Market").

Claim 1 — true: at the margin every stock trades return for risk at the CML's rate

How Dávila argued it (🕒 47:51 — 🕒 48:15). It is the condition the SML was derived from: "we essentially argued that these two things had to be the same". The derivation is 04a p. 11's: there, the ratio of a security's extra expected return to the extra variance it adds to an efficient portfolio "must be the same across all securities". And the better-deal test: if one stock broke the rule, "you have something that gives you more than the market, you would buy this thing". Everyone would tilt toward it, so the market could not be the best portfolio to hold; under the CAPM it is (21 p. 13, "Tangency = market").

The algebra 21 p. 18 prints, step by step. Hold the market and add a small amount Δ of stock i , financed by borrowing Δ at R_f .

$$\mathbb{E}[R_p] = \mathbb{E}[R_M] + \Delta (\mathbb{E}[R_i] - R_f)$$

Why this step follows: the stock earns $\mathbb{E}[R_i]$ on Δ and the loan costs R_f on Δ .

$$\sigma_p^2 = \sigma_M^2 + 2\Delta \text{Cov}[R_i, R_M] + \Delta^2 \sigma_i^2$$

Why this step follows: the covariance rule (21 p. 4 item 6) applied to $R_M + \Delta R_i - \Delta R_f$; the loan is riskless and adds no variance.

$$\left. \frac{d\sigma_p}{d\Delta} \right|_{\Delta=0} = \frac{\text{Cov}[R_i, R_M]}{\sigma_M}$$

Why this step follows: σ_p is the square root of the line above; its derivative is $\frac{2 \text{Cov}[R_i, R_M] + 2\Delta \sigma_i^2}{2\sigma_p}$, which at $\Delta = 0$ (where $\sigma_p = \sigma_M$) is the expression shown.

$$\frac{\text{Cov}[R_i, R_M]}{\sigma_M} = \beta_i \sigma_M$$

Why this step follows: $\beta_i = \text{Cov}[R_i, R_M] / \sigma_M^2$ (21 p. 4 item 10), so the covariance is $\beta_i \sigma_M^2$; divide by σ_M .

$$\frac{\mathbb{E}[R_i] - R_f}{\beta_i \sigma_M} = \frac{\mathbb{E}[R_M] - R_f}{\sigma_M}$$

Why this step follows: the extra return per unit of extra standard deviation is the ratio of the first line's increment to the last line's; the SML replaces $\mathbb{E}[R_i] - R_f$ by $\beta_i (\mathbb{E}[R_M] - R_f)$ and β_i cancels.

The right side is the CML's slope, the same for every stock: "This is the condition that makes the market the tangency portfolio" (21 p. 18). Taking a little away ($\Delta < 0$) gives the same ratio. It is a rate **at the margin**: exact for a small change, approximate for a large one (04a p. 12 writes β_i as the market's marginal risk, $\frac{1}{\sigma_M} \frac{\partial \sigma_M}{\partial \omega_i}$, where ω_i is stock i 's weight in the market portfolio). A stock with $\beta_i = 0$ adds neither return nor risk at the margin, so it has no rate to compare.

Why false is wrong. The tempting reason is that a stock with more expected return, or a higher Sharpe ratio of its own, should add more per unit of risk. But the risk a small position adds to the market is not the stock's own σ_i ; it is only the part that moves with the market, $\beta_i \sigma_M$, because the rest is diversified away inside the market. Measured that way, every stock's rate is the same.

Diagram (optional; the review argued this claim in words) (*reconstructed — no slide carries this figure*): in standard deviation (horizontal) against expected return (vertical), the CML from R_f touching the risky frontier at the market M ; through M , the short curve traced by the market plus a little of stock i touches the CML at M — at M it moves along the CML, whichever stock is added.

Claim 2 — true: no stock has a higher Sharpe ratio than the market

The diagram (*reconstructed from the class's answer at 49:32 and 21 p. 18 — no slide carries this figure*). A student's answer was to rotate the line from R_f , "increasing the slope until we get through the tangency portfolio". Draw:

- standard deviation on the horizontal axis, expected return on the vertical;
- R_f marked on the vertical axis;
- the frontier of risky portfolios (the curve of 21 pp. 11–12), with individual stocks as dots inside it;
- dotted lines from R_f through a few stocks: each line's slope is that stock's Sharpe ratio;
- the line rotated upward from R_f until it just touches the frontier: the touching point is the tangency portfolio, which under the CAPM is the market M (21 p. 13), and the line is the CML.

Every stock lies on or below the CML, so the line from R_f through any stock is no steeper than the CML: $SR_i \leq SR_M$. 21 p. 18: "The market is the tangency portfolio, so it has the highest Sharpe ratio". Equality holds only for a stock on the CML itself.

Why false is wrong. A high-beta stock can expect more than the market (21 p. 17's stock B, beta 1.5, expects 10% against the market's 8%), which tempts "false". But the Sharpe ratio divides by the stock's

total risk σ_i , which includes idiosyncratic risk that earns nothing under the CAPM. Per unit of total risk, no single stock beats the market portfolio, which has diversified that risk away.

Claim 3 — true, given a positive market risk premium

The diagram Dávila drew (🕒 50:54) (reconstructed — no photograph; 21 p. 15's right panel is its base):

- beta on the horizontal axis, expected return on the vertical;
- the SML through $(0, R_f)$ and $(1, \mathbb{E}[R_M])$, sloping upward;
- a dashed horizontal line at R_f ("right below the lower horizontal dash line");
- the region below that dashed line: an expected return below R_f , which is a negative Sharpe ratio ("It pays something less than the risk-free rate");
- on the SML, that region lies only to the left of $\beta = 0$: negative betas.

The algebra, and the condition it needs.

$$SR_i < 0 \iff \mathbb{E}[R_i] < R_f$$

Why this step follows: $\sigma_i > 0$, so the Sharpe ratio has the sign of its numerator (the room's answer, "It's less than the risk-free rate" 🕒 50:44).

$$\mathbb{E}[R_i] - R_f = \beta_i (\mathbb{E}[R_M] - R_f)$$

Why this step follows: the SML, rearranged by subtracting R_f .

With $\mathbb{E}[R_M] > R_f$, the right side is negative exactly when $\beta_i < 0$. 21 p. 18 does not print this condition, but the claim needs it: the CAPM's risk-averse investors hold the risky market only if it pays more than R_f **EXPLANATION**, and 04a p. 14 puts the premium at "4% to 6% in practice".

Why false is wrong. The tempting reason: a stock could earn less than R_f because it is very volatile, or badly run. Under the CAPM neither moves the sign: volatility sits only in the Sharpe ratio's denominator, and no stock is mispriced. A stock's expected excess return is set by its beta alone, so an expected return below R_f is accepted only for a stock that pays off when the market falls: a negative beta.

Two routes — diagrams (the review) or one identity (the Lecture 8 note)

The Lecture 8 note (1 Oct)'s Self-check on WB 4.1 Q14 proves claims 2 and 3 with one line of algebra:

$$SR_i = \frac{\beta_i (\mathbb{E}[R_M] - R_f)}{\sigma_i}$$

Why this step follows: the SML gives the Sharpe ratio's numerator.

$$SR_i = \frac{\rho_{iM} \sigma_i}{\sigma_M} \cdot \frac{\mathbb{E}[R_M] - R_f}{\sigma_i} = \rho_{iM} SR_M$$

Why this step follows: substitute $\beta_i = \rho_{iM} \sigma_i / \sigma_M$ (beta is covariance over variance, and the covariance is $\rho_{iM} \sigma_i \sigma_M$); σ_i cancels.

Claim 2 is $\rho_{iM} \leq 1$; claim 3 is $\rho_{iM} < 0$, which is $\beta_i < 0$, with $SR_M > 0$. **The two routes agree because the diagrams are this identity drawn:** the CML is the steepest line from R_f , and no stock's line can be steeper because no correlation exceeds one; the region below the dashed line is where the correlation with the market, and so the beta, is negative. **The review's route is the diagram:** "You can do pictures, by the way." (🕒 49:25); "draw a diagram, draw a diagram" (🕒 51:59).

LINKS

- The same three claims as a multiple-choice item, WB 4.1 Q14, with the algebra: Lecture 8 note (1 Oct), Self-check.
- The two panels claim 3's diagram comes from, the CML against the SML: 21 p. 15, earlier in this note.

What the materials don't settle

Questions the deck and the recording leave open, for office hours before Tuesday. Each is flagged where it arises; none is answered by anything on file.

1. **Partial credit on the numerical items:** how is it scored? One recording renders a score of "seven" for an answer that "makes sense", the other does not (§1.2).
2. **The formula sheet:** is formula sheet p. 1 handed out with the paper, as it was described on 10 September? The words "formula sheet" and "provided" were not said on 15 October, and 21 p. 4's title is the deck's only statement. And how many equations beyond it should be learned by heart — the recordings disagree (§1.1).
3. **The numerical part's shape:** how are the eight five-point items grouped — "three or four, like, questions, and then ABC" (§1.2)?
4. **21 p. 17's first claim:** "I'm not going to ask this one over" (🕒 47:51), then "maybe like one or two questions of this" (🕒 48:57) — which kind of question is meant (§1.6)?
5. **Equity valuation (06a):** "I'll just send you a video" — is any of it on the 20 October paper? Deck 21 lists Blocks 1–5 (§1.5).

6. **The 8 October guest lecture:** recalled three times as covered material; nothing was said about its place on the paper.
7. **21 p. 18, claim 3:** should an answer state that it needs a positive market risk premium (§14.2)?
8. **The CAL question (21 p. 21):** confirm the intended answer — only the CAL through the tangency portfolio is the efficient frontier (§17.1).
9. **Slide-only items** — 21 p. 13's risk decomposition and market-weight formula, 05a p. 23's cost figures, 21 p. 14's Grossman–Stiglitz paradox — are "fair game" by ⌚ 17:32 ; is any formula among them expected by heart?
10. **"The book I recommended"** at ⌚ 62:23 : which book (§16.3)?
11. **"The question yesterday"** at ⌚ 65:35 : which question (§17.4)?
12. **The three undecoded stretches** — the students' answers after ⌚ 49:38 and ⌚ 53:37 , and the arbitrage remark at ⌚ 58:51 : what was said?
13. **"The last two lectures"** (⌚ 22:17 , ⌚ 41:23): which meetings taught the testing of the CAPM — which decides what the reconstructed 6 October and 13 October notes should contain (§3).
14. **05a p. 24:** "27/45 years" against a chart of 44 years (§2.3).

Coverage audit

P9 (NOTE_SPEC Stage 4), fin-2026-10-15-L12 (the midterm review session, recorded, stitched from two recordings, the second in three parts, no hole), 2026-10-15, batch fin-review, full P9 in a fresh prep-model session (the write session had stopped; this session held no note text and no P8b conversation when it started). **Verdict: FAIL — 0 MISSING beats and a passing math check, but 20 fix items for P8b and 1 input item for the orchestrator** (the fix list closes this block). Nothing in the note's parts or drafts was edited by this audit.

Audited file: notes/fin/fin-2026-10-15-L12.md as assembled 19:15 from parts 00–10 and 8 includes (45,953 words), re-assembled with this part (--no-manifest). Inputs: the beat sheet (288 beats), the map (math inventory M01–M34, problem map §6, reading map §7.1 R21-01–R21-96), the gate (§§3–4, 9), exam-statements.md §1 (38 statements), the deck-21 visual read, the clean transcript and the stitched segment list, the 05a/04a/04b/03c visual reads, the workbook conversion (WB 2.3 Q3(b), 2.3 Q4, 4.2 Q1 answers re-found).

A1. Beat audit — 288 placed, 0 MISSING, 1 THIN (logistics X1–X3 excluded by the beat sheet)

Every beat's own τ is cited within 25 s in the note (script check), and each beat's content was read against its home.

Beats	Home
1–2	§1.5 S1 and its Gap 00:45; §2's opening Exam signal
3–9	§2.1 — 3 thin : the apology for the number of concepts and the hope that the class now has the finance economist's mindset are dropped
10–17	§2.2 (definition callout); 17 also the first Self-check
18–22	§2.3 (Gap 04:36; Warning 27/45 against 44 years)
23–32	§2.4 (Warning 06:41)
33–44	§2.5 (Exam signal 08:57; Intuition); 42–43 in §2.4's Yale paragraph
45–49	§2.6 (Gap: books not in the materials)
50	§1.7 S2
51–66	§1.2 M1–M5 (Warnings 12:03 and 12:47)
67–71	§1.4 R1–R2 (Warning 15:24)
72–77	§1.3 M6
78–83	§1.1 F1 and the p. 4 table (Warning 17:32)
84–86	§1.4 L1, R3
87	§1.7 L2 (Warning 19:43)
88–98	§3 (Exam signal 20:51; Warning 22:17)
99–104	§4 (Exam signal 22:30–23:28); §1.6 S4
105–114	§5 (Exam signal 23:34), §5.1, Worked problems group 2 (21 p. 7 example 1)
115–134	§5.2 (Warning on 21 p. 8 step 3; Warning 30:50; Exam signal; Two routes), Worked problems group 2 (example 2); §1.6 S6–S7; §1.3 S8
135–139	§5.3; §1.7 L3; §1.6 S9 (Warning 32:15)
140–143	§6 (Exam signal 32:21); §1.1 F2; §1.6 S10

Beats	Home
144– 151	§7 (two figures, slider, Exam signal)
152– 159	§8 (definition, Exam signal 35:59)
160– 169	§9 (Warning on the dropped condition; Exam signal 37:48–38:27; Gap 38:30); §1.1 F3 and Warning; Worked problems group 3
170– 182	§10
183– 186	§11
187– 196	§12 (Warning 43:16)
197– 205	§13
206– 211	§14.1; §1.7 L4; §1.2 M7; Worked problems group 4
212– 231	§14.2 (Gap 49:38; Warning on claim 3; Exam signal); §1.6 S11–S12 (Gap 48:57); §1.3 S13; Self-check (21 p. 17 question 2)
232– 253	§15.1–§15.4; §1.6 S14–S15
254– 269	§16 intro, §16.1 (Warning, Gap 58:51), §16.2, §16.3 (figure, Gap 62:23); §1.5 S16–S17
270– 288	§17.1 (Warning) – §17.5; §1.6 S18; §1.5 S19; §1.3 S20; §1.7 L5

A2. Exam statements (§1) against exam-statements.md and the clean transcript

38 of 38 present in §1, each timed in τ , the formula sheet first (§1.1: 21 p. 4 = formula sheet p. 1, the 17:20 and 32:29 statements, the 9/10 and 9/18 records). Clear runs quoted word for word; garbled runs and the other recording's wording given without quotation marks (gate Q-G1). Both readings where the recordings differ in meaning: X005, X028, X038, V1, V2, V3 (Warnings) and the 15:24, 17:32, 32:43, 23:28,

12:57, 14:42 and 23:42 items. **Defects:** S13 drops a clear sentence of the statement (the one at 51:44–51:59 saying the questions should be easy to understand); F3 paraphrases its clear lead-in to the rule; M2's last sentence is cited 16 s early (fix list 6). Deck 21 is followed in the gate's order (§1 with p. 4 first, then pp. 2–3; §2 05a; §3–§17 pp. 5–21 in order); every page 1–21 is carried.

A3. Problem audit

Record	Where	Template / form	Workbook check	Result
fin-ex-21-p7-1 (D1)	Worked problems group 2	7 headings in order	WB 2.3 Q3(b) 122,782 and 38,554 re-found	pass
fin-ex-21-p7-2 (D2)	Worked problems group 2	7 headings + Two routes	WB 2.3 Q4 15,427.34 re-found	pass (its LINKS line mislabels the Section 2 note's §9: fix 9)
fin-ex-l12-t3654 (D3)	Worked problems group 3	7 headings	WB 3.1 Q18 (c), Q19 (d)	pass
fin-ex-21-p17-1 (D4)	Worked problems group 4	7 headings + Two routes	WB 4.2 Q1 R_f 4% re-found (labels swapped)	pass
fin-ex-21-p17-2 (D5)	Self-check	concept self-check + Two routes	WB 4.1 Q14 (d)	fail: own-voice parabola (fix 10), undefined omega (fix 4)
fin-ex-05a-p23	Self-check	concept self-check	slide's own answer	pass
fin-ex-05a-p25	Self-check	concept self-check, two parts	slide's own answer	pass
fin-wb-5.3-keyconcepts-L12	Key takeaways	checklist	—	pass

Every draft included exactly once; `_index.md` ANSWER and Body-numbers lines agree with their drafts. Basis cited (syl p. 3). No exam worked; no target worked or drafted. **Links owned by other meetings (map §6.5): 27 of 32 present; 5 absent** — the Block 1 key concepts (Lecture 2), the Block 2 key concepts (Lectures 3, 4 and 5) and handout 04's Apple/Microsoft exercise (Lecture 9) (fix 8). P8a's link file `_LINKED-other-meetings.md`, which the gate required, was never written (input defect).

A4. Reading audit (map §7.1, deck 21, R21-01–R21-96; no textbook gap list: no textbook in the build)

R21	Pages	Home	Result
01	1	header	pass
02–13	2–3	§1.2, §1.3	pass
14–25	4	§1.1	pass
26	5	§3	pass
27–30	6	§4	pass
31–43	7–8	§5, §5.1–§5.3	pass
44–45	9	§6	pass
46–47	10	§7	pass
48–50	11	§8	pass
51–53	12	§9	pass
54–61	13	§10	pass
62–65	14	§11	pass
66	15	§12	pass
67–70	16	§13	pass
71–82	17–18	§14.1, §14.2, Self-check	pass
83–86	19	§15.1–§15.4	pass
87–91	20	§16	pass
92, 94–96	21	§17.1, §17.3– §17.5	pass
93	21	§17.2	thin: the comment's printed parenthesis (a fall in demand lowers the price and raises the return) is dropped (fix 7)

Pointed-to readings (map §7.2): handout 02 (§5.3), handout 04 (§17.4), the Excel optimizer (Gap), the three books (Gap), the newspaper articles (§2.6), the unnamed book (Gap, inference only) — all present.

A5. Math check

Runner work/mathcheck/fin-2026-10-15-L12.py : **3 part files, 682 checks, PASS** — P8a 472/472; P8b body 144/144; **P9 note check (new, __p9_note.py) 66/66** — the Wermers row cell by cell against the 05a p. 26 read, the 05a p. 24 bar counts and close years recomputed from the read, 04a p. 6's example, the three interactive blocks evaluated from the note's own expressions, the statement and comment counts, the self-checks and claims re-derived (a CAL is the frontier only through the tangency; the SCL slope; the MVP first-order condition). Its self-test plants three errors in the note text and catches all three (exit 1). Math inventory M01–M34: all present.

A6. Other checks

- **Quotations** (tools/p9_quotes.py 1.9.1): source self-test 563/563; planted 10/10; **809 quotations, 803 cite a time or page, 803 ok, 0 hits, 0 unpaired, 6 uncited (0 not found)**; no own-words file. What the tool cannot see: **4 quotations sit inside the low-confidence stretch, τ 02:44.9–03:13.7** (fix 1); no quotation crosses an unclear mark or a disputed mark (script, self-test 2/2).
- **No sealed content:** leak scan (leak_scan.py --no-default , the 11 parts, 8 drafts, _index.md , the preview and the assembled note: 22 files) **0 hits** (run 0, numbers 0, ref 0, label 0); positive control on the workbook 219. Every target appears only by display id with the adapted label; no paper, year, question or workbook number for a target; no original numbers or wording. **Fail (iv), minor:** two mentions of sealed papers in the note's own voice and three meta-remarks after the past-paper statements (fix 3).
- **KaTeX:** rendered with the local vendor copies, headless Edge DOM: 780 spans, **0 math-error ids**, 0 annotations starting with a greater-than sign.
- **Links:** 195 cross-note links to 14 notes and 116 in-note anchors, **0 unresolved**.
- **Pointer defects: 0. Gendered pronouns outside quotations: 0.** Clock times only in the header.
- **Reasons sweep (NOTE_SPEC §7): 4 items** (fix 11).
- **Figures:** 15 embeds (9 course pages, 6 redraws, labels at least 9.4 units) and 3 interactive blocks; each has alt text and a how-to-read line.

A7. NOTE_SPEC §8

Row	Result
Every beat maps to a section	pass (288/288; 1 thin)
Every symbol defined and in the table	fail (fix 4)
Every derivation step justified	pass
No jump of more than one manipulation	pass
Every lecture figure present, captioned, explained	pass
Every worked problem in the template	pass
Slide-only content included and flagged	pass
Gaps flagged, not filled	pass
Math inventory present and derived	pass
Spoken-vs-written conflicts and errata flagged	pass
Math check passes	pass (682)
Every problem-map item present	fail (fix 8)
Every assigned question worked, basis cited	pass
No exam question worked ahead of its exam	pass
Every quotation matches, right time and speaker	fail (fix 1)
No repetition, no padding	pass (the §1 statements repeated at their pages are the gate's design)
Core readings in full	fail (fix 7)
No sentence sends the reader to a source	pass
Reading it requires no other file	pass
No sealed content (batch row)	fail , minor (fix 3)

A8. Fix list for P8b (part file and line; source to check)

1. 02-mutual-funds.md l. 27 — four quotations sit inside the low-confidence segments (τ 02:44.9–03:13.7), one from the first segment and three from the second. Paraphrase without quotation marks (the sentence already says their sense is the page's). Clean transcript [02:26] paragraph; 05a p. 23.
2. 02-mutual-funds.md l. 88 — gate Q-G3: the lecturer's remark about an author's prose, which the gate's ruling excludes together with the aside before it, is quoted; drop it and keep the recommendation. Clean [10:25].
3. 00-header.md l. 17 and 03-blocks-discounting.md l. 120 — the note's own voice mentions sealed papers; use the generic adapted-from-a-past-exam wording or drop. 01-exam.md l. 52, l. 157 and 07-comments.md l. 19 — delete the three meta-remarks after the past-paper statements (gate §3f: nothing added, no comment).
4. 07-comments.md l. 142 and draft fin-ex-21-p17-2.md l. 87 — ω_i (and handout 04's lower-case m in σ_m , R_m) are never defined; define ω_i as security i 's weight in the market portfolio (handout 04 / 04a p. 12 notation), say the handout writes m for M , and add ω_i to the symbol table (00-header.md).
5. 01-exam.md l. 78 — S13 omits a clear sentence of the statement (the one between 51:44 and 51:59 on the questions being easy to understand); add it. exam-statements.md S13.
6. 01-exam.md l. 57 — the last sentence of M2 starts at τ 12:54 (segment 774.7 s), not 12:38; re-cite. Optional, l. 39: quote F3's clear lead-in to the rule before the Warning.
7. 07-comments.md l. 108 — §17.2's heading drops 21 p. 21's printed parenthesis on demand, prices and returns; print it in full (21 p. 21, deck-21 read).
8. 03-blocks-discounting.md l. 36 and l. 120; 07-comments.md l. 150 (or 06-capm-questions.md l. 47) — add the five missing links: Block 1 key concepts (Lecture 2), Block 2 key concepts (Lectures 3, 4, 5), handout 04's Apple/Microsoft exercise (Lecture 9). Map §6.5.
9. 03-blocks-discounting.md l. 110 and l. 132, 09-closing.md l. 59, draft fin-ex-21-p7-2.md l. 194 — the Section 2 note's §9 (and the Lecture 4 note's adapted four-yearly perpetuity) value the stream at year +1 and discount **back** one year, the review's own direction; it is not a forward-from-year-minus-1 example, and it is a four-year clock, not a three-year one. Cite only the Lecture 4 note's §5.3, the cruise and the Section 2 note's §14 as the forward route, and say the back route was already met there. Source: the Section 2 note §9.3–§9.4 and its §9 self-checks.
10. Draft fin-ex-21-p17-2.md l. 99 — the mean-standard-deviation frontier is called a sideways parabola in the note's own voice (gate W-G2); say curve or frontier.
11. Reasons sweep — label as explanation or give the source's own reason: 06-capm-questions.md l. 45 (the diagram route is the form the paper rewards: no source says so; cite 49:25 and 51:59 instead); 07-comments.md l. 30 (Carhart added momentum because of the 10% alpha: 04b p. 19

states the alpha, not the motive); 07-comments.md l. 59 (the spoken phrase is a gloss on free: the note's reading); 02-mutual-funds.md l. 16 (the spoken clause describes the trading, not the hypothesis: the note's reading).

12. 06-capm-questions.md l. 30 — gate I5: the number heard before *stocks* in the 47:51 sentence is quoted; quote with an ellipsis or not at all.
13. 01-exam.md l. 102 — says three statements narrow the deck, but the table below lists four (the first claim of 21 p. 17 as well); make them agree.
14. 00-header.md l. 76 — Toby is named four times on the recording, on three occasions; fix the count.
15. 03-blocks-discounting.md l. 14 (21:49) and 05-capm.md l. 18 (38:52, 39:29) and l. 25 (41:08) — gate §3e signals stated in prose only; give each an Exam-signal callout at its page.
16. 00-header.md l. 27 — explanations compulsory: 21 p. 2 prints the explain line under the short questions only (§1.2 says so); match §1.2.
17. 09-closing.md l. 37 — partial credit for something that makes sense leans on the disputed clause at 12:47; say only that partial credit is given (both recordings).
18. 02-mutual-funds.md l. 60 — the transcript says Swensen used to come here; visiting the class is an inference.
19. 02-mutual-funds.md l. 16 — beat 3 is thin: add the apology for the number of concepts and the hope that the class now thinks like finance economists (clean [01:03]).
20. 07-comments.md l. 28 — the 10.8-s undecoded stretch starts at 53:37 (cited 53:25) and, unlike the other two, has no Gap callout (gate G8).
21. Input defects for the orchestrator, not P8b's: P8a's _LINKED-other-meetings.md is missing (gate §3a item 3); the gate §3a and map §6.2 list the Section 2 note's §9 among forward-route examples (fix 9's source).

A9. Re-check — batch fin-review, 2026-10-15 20:12 (confirming, prep model, fresh session)

Verdict: FAIL, minor — 19 of the 20 fix items closed; item 13 closed at its place but not across the note; one new exam-statement omission. Audited: the parts and drafts as on disk after the fix round (last edit 19:57) and the note assembled from them. Nothing in the parts or drafts was edited by this re-check.

Item	Result	Where (part, line)
1	closed	02 I. 27: the low-confidence stretch is named and not quoted; scan: 0 quotations inside it
2	closed	02 I. 88: only the recommendation is quoted; nothing of the gate Q-G3 passage in any part or draft
3	closed	00 I. 17 (generic label only); 01 II. 52, 157 and 07 I. 19 end on the lecturer's own words; no own-voice mention left in the parts
4	closed	07 I. 145, draft p17-2 I. 87; symbol-table row 00 I. 61 (letters checked on the handout 04 p. 2 and 04a p. 12 reads)
5	closed	01 I. 78, cited at its segment start (51:56)
6	closed	01 I. 57 (12:54, segment start); I. 39 (38:19, ends before the disputed mark)
7	closed	07 I. 111 = 21 p. 21 as read
8	closed	03 II. 41, 125; 06 I. 47: the five links resolve to real headings
9	closed	03 II. 115, 137; 09 I. 59; draft p7-2 II. 194–195: the Section 2 note's §9 is the back route; the forward route is only the Lecture 4 note's §5.3 with the cruise and the Section 2 note's §14; both Two-routes blocks agree with those notes
10	closed	draft p17-2 I. 99
11	closed	06 I. 45; 07 II. 33, 62; 02 I. 16 (04b pp. 16, 19 and 05a p. 20 wording confirmed)
12	closed	06 I. 30 (cut marked); nowhere else
13	not closed across the note	01 I. 102 now gives four, as the table does; 09 I. 59 still gives three
14	closed	00 I. 77 (transcript: four mentions, three occasions)
15	closed	03 I. 16; 05 II. 18, 30
16	closed	00 I. 27
17	closed	09 I. 37
18	closed	02 I. 60

Item	Result	Where (part, line)
19	closed	02 I. 16 (01:09, segment start)
20	closed	07 II. 28–31 (53:26; the undecoded 10.8 s from 53:37; Gap callout)
21	orchestrator's	not re-checked

Checks. Math: runner 683 checks, 0 failing (P8a 472, P8b body 144, P9 note 67); the P9 note check's self-test still catches its 3 plants. Quotations: p9_quotes 1.9.1 source self-test 563/563; 818 quotations, 812 cite a time or page, 812 ok, 0 hits, 0 unpaired, 6 uncited (0 not found); planted misquotes 10/10. The re-check's own scan (self-test passing) of 820 quotation fragments in the parts, drafts and index: 0 inside the low-confidence segments, 0 across an unclear or disputed mark. Exam statements: 38 of 38 in §1, every quotation a contiguous run of the body reading in `exam-statements.md`. KaTeX (vendored render, headless Edge): 798 spans, 0 math-error ids, 0 annotations starting with a greater-than sign. Links: 203 cross-note links to 14 notes and 116 in-note anchors, 0 unresolved at heading level. Leak scan: 23 files, 0 hits; positive control on the workbook 219. Reasons sweep over every edited line: 8 reason words, each the source's own reason with its cite, labelled, or the note's own arithmetic. Pointer defects 0; gendered pronouns outside quotations 0. Assembler check: 12 parts, 8 includes, 49,208 words. NOTE_SPEC §8: every row passes, the five the full audit failed included.

Open — fix list for the next round.

1. `09-closing.md` l. 59 — the Connections paragraph counts *three things said not to be asked* and links §1.5, which (fix 13) now lists four; give §1.5's count or drop the number.
2. `01-exam.md` l. 62 (M3) — the clear clause that opens the 13:03 segment (the review workbook; both recordings word for word) is given in the note's words before the quotation that continues it; quote it, as S13 now is. Source: `exam-statements.md` M3.
3. For the orchestrator, not P8b: this audit's own fix list (A8 items 1, 2 and 12) restates in the assembled note the words the note must not carry; reword or accept as audit text.

A10. Re-check 2 — batch fin-review, 2026-10-15 20:22

Verdict: PASS. The three open items of A9 are closed: `09-closing.md` l. 59 (and Key takeaways l. 45) give the four things said not to be asked in §1.5's words, each timed; `01-exam.md` l. 62 quotes M3's 13:03 clause word for word (every quotation in §1 a run of the body reading; M3 now complete); A8 items 1, 2 and 12 describe by location and kind, and none of the gated words is in the assembled note. A8 items 10 and 16 (II. 142, 148) are not defects under the same test: they name the old wording of two fixed defects only to identify them, and neither wording is material the note must never carry. Checks: runner 683/0 (P9 note self-test catches its 3 plants); p9_quotes 1.9.1 source self-test 563/563, 822

quotations, 816 cited, 816 ok, 0 hits, 0 unpaired, planted 10/10; 0 quotations inside the low-confidence segments or across an unclear or disputed mark; KaTeX 798 spans, 0 math-error ids, 0 annotations starting with a greater-than sign; links 0 unresolved; leak scan 23 files, 0 hits. NOTE_SPEC §8: every row passes. Nothing in the parts or drafts was edited by this re-check.